Spinor structures in the superstring theory

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(Submitted 6 November 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 10, 790–793 (25 November 1993)

Superconformal Schottky groups which are suitable for the description of all the superstring spinor structures are built.

In the Neveu-Schwarz-Ramond[1] superstring theory the multiloop amplitudes are usually written[2–4] as sums over spin structures integrated over Riemann moduli. This form of the amplitudes mentioned above arises after integration over odd moduli which are performed in accordance with the prescription given in Refs. 2 and 3. However, in the above scheme[2–4] the multiloop amplitudes turn out to be depended on the choice of the basis of the gravitino zero modes.[2,4] This means that the two-dimensional supersymmetry is lost in this scheme. In the superstring theory the "vierbein" field and the gravitino field are the gauge fields. Because of the gauge invariance, the "true" superstring amplitudes are independent of the choice of the gauge of the above gauge fields. Therefore, they have no dependence on the choice of the basis of the gravitino zero modes.

The dependence on the choice of the basis of the gravitino zero modes appears to be a serious difficulty in the considered scheme. The above difficulty is absent in the formulation[5] which has the indicated two-dimensional supersymmetry. In this scheme the n-loop superstring amplitudes A_n turn out to be[6–8] the integral over the $(3n-3|2n-2)$ complex moduli q_N and over their complex conjugated $\bar{q}_N$:

$$A_n=\int \prod_N dq_N d\bar{q}_N \sum_{L,L'} \hat{Z}^{(n)}_{L,L'} \langle V\rangle_{L,L'}, \qquad (1)$$

where $\hat{Z}^{(n)}_{L,L'}$ are the partition functions, and $\langle V\rangle_{L,L'}$ denote the vacuum expectations of the vertex products. The index L (L') labels "superspin" structures of the right (left) fields. The above superspin structures are defined for superfields which live on the complex $(1|1)$ supermanifolds.[5] Being twisted around the (A,B) cycles, the superfields are changed by mappings which are the superconformal versions of the fractional linear transformations. In general, every considered mapping depends on the $(3|2)$ parameters.[5] For the odd parameters to be arbitrary, the above mappings also include the fermion-boson mixing. It sets the superspin structures apart from the ordinary spin structures. The ordinary spin structures imply that boson fields are single-valued on the Riemann surfaces, and that under 2π-twists about the (A,B) cycles the fermion fields can be multiplied by (-1). For all odd parameters to be equal to zero every genus-n superspin structure $L=(l_1, l_2)$ is reduced to the ordinary (l_1, l_2) spin structure. Here l_1 and l_2 are the theta function characteristics: $l_i=\{l_{is}\}$, where $l_{is}=0, 1/2$ with $i=1$ or 2 and $s=1, 2,\dots n$.

In the scheme which we are discussing the partition functions can be computed from equations[7,8] which are the Ward identities. These equations satisfy the condition that the superstring amplitudes be independent of the vierbein field and the gravitino field. The multiloop amplitudes which are calculated in terms of the superspin structures therefore turn out to be consistent with the requirement that the superstring be gauge invariant.

In Refs. 3 and 6–8 only the superspin structures with all l_{1s} equal to zero were studied. For the description of these superspin structures, the superconformal versions of the Schottky groups[9,10] have been employed. In this paper our goal is to build the superconformal Schottky groups which are suitable for all the superspin structures, including those where $l_{1s}\neq 0$. In this formulation the superfields which are associated with the above superspin structures turn out to be branched on the complex z-plane, where the Riemann surfaces are mapped. This makes it rather difficult to carry out the calculations for the superspin structures in question. This is nevertheless the only formulation which allows us to perform the explicit calculation in terms of the even and odd moduli of the partition functions and the vacuum superfield correlators. This formulation is therefore worth noting.

In general, every superspin structure given on a genus-n complex $(1|1)$ supermanifold is defined by the transformations $[\Gamma_{a,s}(l_{1s}),\, \Gamma_{b,s}(l_{2s})]$ which are associated with runs about the $(A_s,\, B_s)$ cycles, respectively. The above supermanifolds are mapped by the supercoordinate $t=(z|\theta)$, where z is a local complex coordinate and θ is its odd partner.

To build all the $\Gamma_{a,s}(l_{1s}),\, \Gamma_{b,s}(l_{2s})$ mappings, we note that for genus $n=1$ there are no odd moduli. Indeed, the genus-1 amplitudes are obtained in terms of ordinary spin structures.[11] For every particular s, all the odd parameters in both $\Gamma_{a,s}(l_{1s})$ and $\Gamma_{b,s}(l_{2s})$ can be reduced to zero by a suitable transformation $\tilde{\Gamma}_s$, which is the same for both the above transformations:

$$\Gamma_{a,s}(l_{1s})=\tilde{\Gamma}_s^{-1}\Gamma_{a,s}^{(o)}(l_{1s})\tilde{\Gamma}_s, \quad \Gamma_{b,s}(l_{2s})=\tilde{\Gamma}_s^{-1}\Gamma_{b,s}^{(o)}(l_{2s})\tilde{\Gamma}_s, \tag{2}$$

where $[\Gamma_{a,s}^{(o)}(l_{1s}),\, \Gamma_{b,s}^{(o)}(l_{2s})]$ are equal to $[\Gamma_{a,s}(l_{1s}),\Gamma_{b,s}(l_{2s})]$ which at all the odd moduli were found to be equal to zero.

For the $\Gamma_{b,s}^{(o)}(l_{2s})$ mappings we employ the Schottky transformations. At the same time, the θ spinor receives the $(c_s z+d_s)^{-1}$ factor. In addition, for $l_{2s}=0$ the spinors have the sign.[3] Therefore,

$$\Gamma_{b,s}^{(o)}(l_{2s})=\{z\to(a_s z+b_s)(c_s z+d_s)^{-1},\ \theta\to-\theta(-1)^{2l_2}(c_s z+d_s)^{-1}\}, \tag{3}$$

where a_s, b_s, c_s, and d_s are complex parameters, and $a_s d_s-b_s c_s=1$. The $\Gamma_{b,s}(l_{2s})$ discussed above are the superconformal versions of the above $\Gamma_{b,s}^{(o)}(l_{2s})$ transformations.

We assume $\Gamma_{b,s}(l_{2s}=1/2)$ to be the same as in Refs. 3 and 6–8:

$$\Gamma_{b,s}(l_{2s}=1/2)=\left\{z\to\frac{a_s(z+\theta\varepsilon_s)+b_s}{c_s(z+\theta\varepsilon_s)+d_s},\quad \theta\to\frac{\theta+\varepsilon_s}{c_s(z+\theta\varepsilon_s)+d_s}\right\}, \tag{4}$$

$$\text{with}\quad \varepsilon_s=\alpha_s(c_s z+d_s)+\beta_s,\quad a_s d_s-b_s c_s=1-\varepsilon_s\partial_z\varepsilon_s.$$

In (4) the even a_s, b_s, c_s, and d_s and the odd α_s and β_s parameters can be expressed[3,8] in terms of two fixed points, $(u_s|\mu_s)$ and $(v_s|\nu_s)$, on the complex $(1|1)$ supermanifold, together with the multiplier k_s as follows:

$$a=\frac{u-kv-\sqrt{k}\mu\nu}{\sqrt{k}(u-v-\mu\nu)},\quad d=\frac{ku-v-\sqrt{k}\mu\nu}{\sqrt{k}(u-v-\mu\nu)},\quad c=\frac{1-k}{\sqrt{k}(u-v-\mu\nu)},$$
$$\alpha=(\mu+\sqrt{k}\nu)(1+\sqrt{k})^{-1},\quad \beta=-(\nu+\sqrt{k}\mu)(1+\sqrt{k})^{-1}. \tag{5}$$

Here the index s is omitted. To obtain the above $\Gamma_{b,s}(l_{2s}=1/2)$ mappings in the form (4), we chose the $\widetilde{\Gamma}_s$ mapping in (2) as follows:

$$\widetilde{\Gamma}_s:\ z\to z_s+\theta_s\widetilde{\varepsilon}_s(z_s),\quad \theta\to\theta_s(1+\widetilde{\varepsilon}_s\widetilde{\varepsilon}_s'/2)+\widetilde{\varepsilon}_s(z_s);$$
$$\widetilde{\varepsilon}_s'=\partial_z\widetilde{\varepsilon}_s(z),\quad \widetilde{\varepsilon}_s(z)=[\mu_s(z-v_s)-\nu_s(z-u_s)](u_s-v_s)^{-1}. \tag{6}$$

Furthermore, we chose $(3|2)$ of the u_s, v_s, μ_s and ν_s parameters to be the same for all the genus-n supermanifolds; the rest of them, together with the k_s multipliers, are the $(3n-3|2n-2)$ complex moduli q_N in (1). We assume that $|k_s|<1$. For the isomorphism to exist between (4) and (5), we fix the branch of $\sqrt{k_s}$, for example, as $|\arg k_s|\leqslant\pi$. Then $\Gamma_{b,s}(l_{2s}=0)$ can be obtained from (4) and (5) by the $\arg k_s\to\arg k_s+2\pi$ replacement. The $\Gamma_{b,s}(l_{2s}=0)$ discussed above appear to be slightly different from those in Refs. 3 and 6.

In fact, the above $\arg k_s\to\arg k_s+2\pi$ replacement presents the (super)modular transformation which turns $(l_{1s}=0,\ l_{2s}=1/2)$ into $(l_{1s}=0,\ l_{2s}=0)$. To prove this statement, it is sufficient to check it for the genus $n=1$. For $n=1$ the period ω is given by[3,10] $\omega=(2\pi i)^{-1}\ln k$. We see, therefore, that ω becomes $\omega+1$ under the replacement which we discussed. Employing the explicit form of the θ functions, we can verify that this transformation of ω is accompanied by the replacement $(l_1=0,\ l_2=1/2)\to(l_1=0,\ l_2=0)$. Therefore, in our scheme the $|\arg k_s|\leqslant\pi$ condition provides in Eq. (1) the separation of the $(l_{1s}=0,\ l_{2s}=1/2)$ and $(l_{1s}=0,\ l_{2s}=0)$ superspin structures from each other.

Every mapping (3) turns the circle $C_s^{(-)}=\{z:|c_sz+d_s|=1\}$ into $C_s^{(+)}=\{z:|-c_sz+a_s|=1\}$. The run about the $C_s^{(-)}$ or $C_s^{(+)}$ circle corresponds to the 2π twist about the A_s cycle. For $l_{1s}=1/2$ the spinors have the sign of the above round.[3] The $\Gamma_{a,s}(l_{1s}=1/2)$ mappings then appear to be

$$\Gamma_{a,s}(l_{1s}=1/2)=\{z\to z-2\theta\widetilde{\varepsilon}_s(z),\quad \theta\to-\theta(1+2\widetilde{\varepsilon}_s\widetilde{\varepsilon}_s')+2\widetilde{\varepsilon}_s(z)\}, \tag{7}$$

where $\widetilde{\varepsilon}_s$ is defined by Eq. (6). In this case the cut $\widetilde{C}_s$ therefore appears on the z plane. With regard to $\Gamma_{a,s}(l_{1s}=1/2)^2=I$, its endcut points are the square-root branched points. One of them is placed inside the $C_s^{(-)}$ circle and the other is placed inside $C_s^{(+)}$.

Taking the superconformal p-tensors $F_p(t)$ into account, $\Gamma_{a,s}(l_{1s}=1/2)$ relate $F_p(t)$ to its value $F_p^{(s)}(t)$ obtained from $F_p(t)$ by the 2π twist about the $C_s^{(-)}$ or $C_s^{(+)}$ circle. Therefore, $F_p(t)$ changes under the $\Gamma_{a,s}(l_{1s})=\{t\to t_s^a\}$ and $\Gamma_{b,s}=\{t\to t_s^b\}$ mappings as

$$F_p(t_s^a)=F_p^{(s)}(t)Q^p_{\Gamma_{a,s}}(t), \quad F_p(t_s^b)=F_p(t)Q^p_{\Gamma_{b,s}}(t). \tag{8}$$

For $l_{1s}\neq 0$ the above $F_p(t)$ cannot be obtained by a simple supersymmetrization of the conformal p-tensors in the boson string theory. The construction of the above superconformal p-tensors will be considered in another paper.

This work was supported, in part, by a Soros Foundation Grant awarded by the American Physical Society.

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