Two-dimensional vortices in a stack of thin anisotropic superconducting films

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The solution for the problem of a two-dimensional (2D) vortex in a stack of thin anisotropic superconducting films is presented. The effect of an anisotropic 2D vortex structure on the 2D and 3D vortex interaction and on the general peculiarities of current-voltage characteristics near the Kosterlitz–Thouless transition is discussed.

1. Magnetic anisotropy of the new high-T_c superconductors is parallel to the CuO_2 layers. In the c direction, i.e., in the perpendicular direction, it is suppressed in regions between superconducting layers. Apart from that whether screen estimated that strong anisotropic Bi and Tl high-T_c compounds and artificial high-T_c multilayers are the stacks of magnetic coupling superconducting planes in a wide range of temperatures below T_c. The transverse coherence length ξ_c in these materials is less than the interlayer distance s, so the Josephson coupling between superconducting layers is vanishing. The current-voltage characteristics of extremely anisotropic high-T_c superconductors[1,2] are evidence that the superconductivity in separate CuO_2 layers exists in a 2D sense.

The 2D magnetic vortices in such systems have been described earlier in numerous papers (see, for example, Refs. 3–6). In all these works use was made of a simplifying assumption that superconducting layers are isotropic in the basal plane. However, it is well known that there is an anisotropy in the CuO_2 plane of $YBa_2Cu_3O_x$ crystals.[7,8] The effective electron masses along the **a** and **b** crystal axes, μ_a and μ_b, respectively, differ from each other, so that $\mu_b \cong 1.4\mu_a$. The precision of μ_b/μ_a evaluation in Bi high-T_c crystals is not enough to confirm the equality of the masses μ_a and μ_b.[8,9]

2. Let us consider a stack of thin superconducting layers of thickness d, separated from each other by the distance s. We tie the Cartesian coordinate system to the crystal axes: the $\mathbf{x}_3$ axis is oriented along the c axis and along the normal $\mathbf{n}$ to the layers and $\mathbf{x}_1$ and $\mathbf{x}_2$ axes are parallel to the symmetry axes in the superconducting planes a and b, respectively.

First of all, we consider an isolated 2D vortex situated in the coordinate center of the superconducting plane with the number $k=0$. The distributions of the vector potential $\mathbf{A}(\mathbf{x})$ throughout all space and the screening currents $\mathbf{I}_k(\mathbf{x})$ in the superconducting kth layers will be described within the Pearl limit of very thin layers,[4] such that the current $\mathbf{I}_k(\mathbf{x})$ will flow in the **ab** plane only. The complete system of equations is

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$$\mathbf{A}(\mathbf{x})=\frac{4\pi}{c}\sum_k \mathbf{I}_k(\mathbf{x})=\frac{2}{\Lambda}\hat{\mu}^{-1}\sum_k\left(\frac{\phi_o}{2\pi}\vec{\nabla}\theta-\mathbf{A}\right)\delta(x_3-ks), \qquad (1.1)$$

$$\mathrm{div}\ \mathbf{I}_k=0, \qquad (1.2)$$

$$\mathbf{n}\mathbf{I}_k=0. \qquad (1.3)$$

Here ϕ_0 is the flux quantum, and $\Lambda=2\lambda^2/\mathbf{d}$. Note that the length parameter is equal to $\Lambda=2\lambda^2/s$ in the Lawrence–Doniach model for the Josephson-decoupled layers. We assume the normalizing second rank tensor $\hat{\mu}$ to be $1=\mu_1\mu_2$. A vector potential gauge we define by the condition div $\vec{\nabla}\theta=0$, when θ is a phase of the order parameter. Since this condition does not depend on the anisotropy parameters $\hat{\mu}$, the phase gradient of the 2D vortex has the same space distribution as in the case of isotropic layers.

To solve Eqs. (1), we have defined the Fourier transforms:

$$\mathbf{A}_k(\mathbf{q})=\int d^3\mathbf{q}e^{-i\mathbf{q}\mathbf{x}}\mathbf{A}(\mathbf{x})\delta(x_3-ks),$$

$$\mathbf{S}(\mathbf{q})=i\phi_o\frac{\mathbf{q}\times\mathbf{n}}{q^2}\delta_k0,$$

with

$$\mathbf{q}=(q_1q_2,0).$$

From Eqs. (1.2) and (1.3) we obtain the orientation dependence of the current distribution,

$$\mathbf{I}_k(\mathbf{q})=(\mathbf{q}\times\mathbf{n})i_k(\mathbf{q}), \qquad (2)$$

where $i_k(\mathbf{q})$ is the scalar function of the vector $\mathbf{q}$.

From this expression and from Eq. (1.1) we obtain the anisotropic form of the vector potential,

$$\mathbf{A}_k(\mathbf{q})=\mathbf{S}(\mathbf{q})+\hat{u}(\mathbf{q}\times\mathbf{n})\frac{i\phi_o\Lambda}{qK(\mathbf{q})}[W_{k0}-\delta_{k0}]. \qquad (3)$$

Here

$$W_{km}=\frac{\sinh(qs)[G-(G^2-1)^{1/2}]k-m}{K(\mathbf{q})(G^2-1)^{1/2}},$$

$$G=\cosh(qs)+K^{-1}(\mathbf{q})\sinh(qs),\quad K(\mathbf{q})=\frac{\Lambda}{q}(\mathbf{q}\times\mathbf{n})\hat{\mu}(\mathbf{q}\times\mathbf{n}).$$

The exact solution (3) generalizes Clem's[4] results for the anisotropic superconducting layers.

In the limit of "large" distance, $qs<1$, the well approximation of W_{km} is

$$W_{km}=\frac{\exp\left[-|k-m|sq\left(1+\frac{2}{sqK(\mathbf{q})}\right)^{1/2}\right]}{K(\mathbf{q})\left(1+\frac{2}{sqK(\mathbf{q})}\right)^{1/2}}.$$

This expression describes the screening of the anisotropic vortex fields and the currents by the stack of superconducting layers.[4]

Equation (3) tends to the Fisher's solution[5] in the Lawrence–Doniach model for Josephson-decoupled layers by vanishing anisotropy.

The distribution of the 2D vortex screening current is given by

$$\mathbf{I}_k(\mathbf{q})=\frac{ic\phi_o}{2\pi}\frac{\mathbf{q}\times\mathbf{n}}{qK(\mathbf{q})}[\delta_{k0}-W_{k0}]. \tag{4}$$

Note that by describing the current in the central layer which consists of the 2D vortex, W_{k0} can be ignored with respect to unity by natural condition $\Lambda\gg s$. As a result, we obtain the strongly anisotropic current distribution central plane,

$$\mathbf{I}_0(\mathbf{x})=\frac{c\phi_o}{4\pi^2\Lambda}\frac{\mathbf{n}\times\mathbf{x}}{x\hat{\mu}\mathbf{x}}. \tag{5}$$

3. The free energy of an arbitrary 2D vortex configuration is

$$F=\frac{\phi_o^2}{16\pi^3}\sum_k\sum_m\int\frac{d^2\mathbf{q}}{qK(\mathbf{q})}[\delta_{km}-W_{km}]S_k(\mathbf{q})S_m^*(\mathbf{q}). \tag{6}$$

Here $S_k(\mathbf{q})=\Sigma_l\exp(i\mathbf{q}\mathbf{x}_{kl}^o)$ is the structure factor of the 2D vortices of kth layer, and $\mathbf{x}_{kl}^o$ is the coordinate of the lth vortex center.

The self-energy of a pair of 2D vortices and antivortices

$$F_o(\mathbf{R})=\left(\frac{\phi_o}{4\pi}\right)^2\frac{2}{\pi\Lambda}\int\frac{d^2\mathbf{q}}{(\mathbf{q}\times\mathbf{n})\hat{\mu}(\mathbf{q}\times\mathbf{n})}(1-e^{i\mathbf{q}\mathbf{R}})$$

can be reduced to the isotropic shape by the scale transformations

$$(\mathbf{q}\times\mathbf{n})\hat{\mu}(\mathbf{q}\times\mathbf{n})=(\mathbf{q}')^2,\quad \mathbf{R}\hat{\mu}\mathbf{R}=(\mathbf{R}')^2, \tag{7}$$

where $\mathbf{R}$ is the intervortex distance. Note that the self energy of a vortex pair has just the isotropic form in the prime coordinate system. This is possible due to the anisotropy of the coherence length $\vec{\xi}$, which is determined by the relation $\vec{\xi}\cdot\hat{\mu}\cdot\vec{\xi}=(\vec{\xi}')^2$, where the length $\vec{\xi}'$does not depend on the vector $\mathbf{R}'$ orientation.

The scale transformation opposite to (7) leads to

$$F_o(\mathbf{R})=\left(\frac{\phi_o}{4\pi}\right)^2\frac{1}{\Lambda}\ln\left(\frac{\mathbf{R}\hat{\mu}\mathbf{R}}{\xi\hat{\mu}\xi}\right)=\left(\frac{\phi_o}{4\pi}\right)^2\frac{2}{\Lambda}\ln\left(\frac{R'}{\xi'}\right). \tag{8}$$

This means that the 2D vortex-anivortex interaction in a stack of anisotropic superconducting films coincides with the well-known expression in the isotropic case.[5]

Let us now examine the free energy of the stacks of 2D vortices, whose centers lie on the c axis and whose relative position is arbitrary. After summation over k in W_{km} of Eq. (6), we obtain the value of the free energy per layer,

$$F=\left(\frac{\phi_o}{4\pi}\right)^2 \frac{s}{2\pi}\int \frac{d^2\mathbf{q}\,|S(\mathbf{q})|^2}{\lambda_{\parallel}^2\,(\mathbf{q}\times\mathbf{n})\hat{\mu}(\mathbf{q}\times\mathbf{n})+\frac{qs}{2}\coth\left(\frac{qs}{2}\right)}. \tag{9}$$

Here $S(\mathbf{q})$ is the structure factor of the system of 3D vortices. $\lambda_{\parallel}^2 = s\Lambda/2$ is the penetration depth of the 3D vortex magnetic field along the superconducting layer.

In the limit $s\to\infty$ and when $\coth(qs/2)$ tends to unity, Eq. (9) describes the vortices in an isolated anisotropic film.[10]

For $s\to 0$ (in another words, for a "large" distance, $R\gg s$, between vortices) the second term in the denominator of Eq. (9) tends to unity. Equation (9) describes the 3D vortices in a bulk anisotropic superconductor.[11] In the prime coordinate system (7), Eq. (9) has an isotropic form, so the hexagonal vortex lattice[12] with isotropic deformation moduli[11] holds to its absolute minimum. The scale transformation opposite to (7) makes it possible to easily obtain all of the stable vortex lattice structures and anisotropy of the shear modulus.[14]

4. In conclusion we discuss the effect of a superconducting layer anisotropy on the general peculiarities of the current-voltage (I-V) characteristics of Josephson-decoupled layered superconductors.

In the Meissner phase the dissociation of the 2D vortex-anivortex pairs takes place at the Kosterlitz–Thouless temperature:[15]

$$T_{KT}=\frac{\phi_o^2}{16\pi^2\Lambda}. \tag{10}$$

In a stack of anisotropic layers the energy of vortex pair (8) has a strictly isotropic form. An entropy contribution to the free energy does not depend on the choice of any coordinate system, so the Kosterlitz–Thouless transition occurs at the temperature T_{KT} in the case of anisotropy.

At a small value of the external current I, the energy of the vortex pair (8) decreases:

$$F=F_o(\mathbf{R})-\frac{\phi_o}{c}(\mathbf{n}\times\mathbf{I})\mathbf{R}. \tag{11}$$

It does not have an anisotropy parameter. Thus, the I-V characteristics of the anisotropic multilayers in the Meissner phase must be the same as isotropic multilayers if the vortex mobility is isotropic too. Otherwise, a voltage scaling will make it possible to obtain the dependence of the 2D vortex mobility on the direction and anisotropy.

Let us now consider the case of nonzero magnetic field and thermal disruption of the 3D vortex. At the displacement of any 2D vortex on the distance $\mathbf{R}$ from one stack axis, the free energy excess is given by

$$F_o(\mathbf{R})=\frac{\phi_o^2}{\delta\pi^3\Lambda}\int\frac{d^2\mathbf{q}[1-\cos(\mathbf{qR})]}{(\mathbf{q}\times\mathbf{n})\hat{\mu}(\mathbf{q}\times\mathbf{n})[1+\lambda_\parallel^2(\mathbf{q}\times\mathbf{n})\hat{\mu}(\mathbf{q}\times\mathbf{n})]}. \tag{12}$$

In the primed coordinate system (7) this equation is isotropic and

$$F_o(\mathbf{R}')=\frac{\phi_o^2}{4\pi^2\Lambda}[\gamma+\ln(R'/2\lambda_\parallel)+K_o(R'/\lambda_\parallel)], \tag{13}$$

where $\gamma=0.5772...$ is Euler's constant.[4]

Following Clem[4] and using Eq. (13), we can assert that thermal decoupling of the 3D vortex lattice does not depend on the anisotropy parameters in the primed coordinate system.

The effect of the current **I** on the disruption of the 3D vortex is described by Eq. (11), where $F_0(R)$ is given by Eq. (12) or (13). To transform potential (11) into fully isotropic form, we give the coordinate scaling (7) together with the current transformation

$$\mathbf{I}\hat{\mu}\mathbf{I}=(\mathbf{I}')^2. \tag{14}$$

From this expression the dependence of the I-V characteristics on the direction of the current **I** follows immediately. For example, the critical current $\mathbf{I}_c$ of the lattice melting varies as $(\mathbf{I}\cdot\mu\cdot\mathbf{I})^{1/2}$ by the rotation of the current **I**. The maximum values of $\mathbf{I}_c$ differ from each other by a factor of μ_a. The value μ_a is roughly equal to 1.2 for artificial Y high-T_c multilayers. The possible difference in the critical currents in a Bi or Tl high-T_c compound may be evidence of anisotropy of the CuO_2 layers in this crystal.

Thus the I-V characteristics of anisotropic multilayers depend on the current direction and anisotropy parameters only in the presence of a nonzero magnetic field.

I believe that experimentally noticeable difference in the values of I_c at different current orientations can be obtained by the method of Ref. 2 and in artificial multilayers of polysulfur nitride $(CN)_x$, whose long molecules can be easily situated in fixed planes.[16] Because of this circumstance, a strong anisotropy in the basal plane can be reached and the obtained results can be easily examined.

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[1] A. K. Asadov, Yu. A. Genenko, G. G. Levchenko *et al.*, Physica C**206**, 119 (1993).
[2] Y. M. Wan, S. E. Hebboul, D. S. Harris, and J. C. Harland, preprint, submitted to Phys. Rev. Lett. (1993).
[3] K. B. Efetov, Zh. Eksp. Teor. Fiz. **76**, 1781 (1979) [Sov. Phys. JETP **49**, 905 (1979)].
[4] J. R. Clem, Phys. Rev. B **43**, 7837 (1991).
[5] K. H. Fisher, Physica C **178**, 161 (1991).
[6] D. Feinberg, Physica C **194**, 126 (1992).
[7] L. Ya. Vinnikov, L. A. Gurevich, G. A. Emel'chenko, and Yu. A. Osip'yan, Pis'ma Zh. Eksp. Teor. Fiz. **47**, 109 (1988) [JETP Lett. **47**, 131 (1988)].
[8] G. J. Dolan, F. Holtzberg, C. Field, and T. R. Dinger, Phys. Rev. Lett. **62**, 2184 (1989).
[9] D. J. Bishop, P. L. Gammel, D. A. Huse, and C. A. Murray, Science **255**, 165 (1992).
[10] A. Yu. Martynovich, submitted to Zh. Eksp. Teor. Fiz. (1993).
[11] L. J. Gampbell, M. M. Doria, and V. G. Kogan, Phys. Rev. B **38**, 2439 (1988).

[12] J. Matricon, Phys. Lett. **9**, 289 (1964).
[13] E. H. Brandt, J. Low Temp. Phys. **26**, 709 (1977).
[14] A. M. Grishin, A. Yu. Martynovich, and S. V. Yampolskii, Zh. Eksp. Teor. Fiz. **101**, 649 (1992) [Sov. Phys. JETP **74**, 345 (1992)].
[15] V. L. Berezinskii, Zh. Eksp. Teor. Fiz. **61**, 1144 (1971) [Sov. Phys. JETP **34**, 610 (1971)]; J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).
[16] Y. Oda, H. Takenaka, H. Nagano, and I. Nakada, Solid St. Commun. **32**, 659 (1979).

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