Dilaton gravitation in $d=2$ with trivial quantum corrections

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(Submitted 14 October 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 11, 865–867 (10 December 1993)

A new type of classical potential of an induced (dilaton) gravitation in $d=2$ is proposed. This potential makes the quantum corrections trivial in first-order perturbation theory. The effective potential of the theory (including the Vilkovisky correction) is studied as a function of the gauge.

Two-dimensional dilaton gravitation[1] has been the subject of active research for several years now. This theory has attracted interest because of the close relationship with string theory and also because this theory is exactly solvable at the quantum level (see the citations in Ref. 2). In particular, exact solvability has been established for the local version of the theory,

$$S=\int d^2x\,\sqrt{g}\left\{\frac{1}{2}\,g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi+C_1\Phi R+V(\Phi)\right\}, \tag{1}$$

with the potentials $V=0$ and $V=\Omega\Phi$, where $\Omega=\text{const}$. It is thus natural to take up the question of seeking other exactly solvable models, e.g., models having a different form of the classical potential. Finding such models within the framework of perturbation theory is the primary topic of the present letter.

The theory in (1) is usually studied by nonperturbative methods, which require noncovariant gauges: conformal and light-cone. Nevertheless, the results of such research can be interpreted as the trivial nature of the quantum (loop) corrections to the action of exactly solvable models. Attempts at a perturbative treatment of theory (1) were recently undertaken in Refs. 2–6. A method was proposed in Ref. 2 for calculating the one-loop divergences in theory (1) on the basis of a covariant background gauge condition:

$$S_{gf}=-\frac{C_1}{2}\int d^2x\,\sqrt{g}\chi_\mu\left(\frac{\Phi}{\alpha}\right)\chi^\mu,$$

$$\chi_\mu=\nabla_\nu h^\nu_\mu-\frac{1}{2}\beta\partial_\mu h-\frac{1}{2}\gamma\partial_\mu\varphi\,. \tag{2}$$

Here $h_{\mu\nu}$ and φ are quantum fields, $g_{\mu\nu}$ and Φ are background fields, and α, β, and γ are gauge parameters.

Analysis of the part of the action $S+S_{gf}$ quadratic in the quantum fields shows that the corresponding quadratic form becomes minimal at $\beta=0$, $\gamma=\alpha$, for arbitrary α (Ref. 6). A natural metric in the configuration space of the fields in the theory in (1) (Ref. 7) thus contains an arbitrary parameter α, which can be identified as a gauge parameter. A direct calculation of the divergences of the effective action[2] and the effective potential[6] in gauge (2) with arbitrary α leads to the following results, respectively:

$$\Gamma_{\rm div}^{(1-\rm loop)}=\frac{1}{\epsilon}\int d^2x\,\sqrt{g}\left\{-\frac{\alpha}{2\Phi^2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi+\frac{\alpha V}{C_1\Phi}+\frac{V^1}{C_1}\right\}, \tag{3}$$

$$V_{\rm eff}^{(1-\rm loop)}=V-\frac{1}{4\pi}\left\{\frac{V^1}{C_1}\left[1-\ln\left(\frac{V^1}{C_1\mu^2}\right)\right]+\frac{\alpha V}{C_1\Phi}\left[1-\ln\left(\frac{\alpha V}{C_1\mu^2\Phi}\right)\right]\right\}. \tag{4}$$

Here ϵ is a dimensional regularization parameter, μ is a dimensional renormalization parameter, and $V^1=dV/d\Phi$. Expression (4) shows that $V_{\rm eff}$ effectively depends on the gauge parameter α. This dependence complicates the use of (4) to seek a theory in (1) with trivial quantum corrections.

An alternative calculation of the effective potential was carried out in Ref. 3, where a conformal gauge was used. In that gauge, the classical action of theory (1) is the action of a 2D σ model with a constant metric in the space of σ-model coordinates. That structure of the action leads to a trivial nature of the quantum corrections in the kinetic sector, but the effective potential becomes far more complex than (4). In particular, it depends on V''. We might note that a V'' dependence also arises in gauge (2) with $\beta\neq 0$ (Ref. 6).

We thus need either to construct a gauge-invariant effective action or to choose among various gauges. The concept of a gauge-invariant effective action is usually linked with Vilkovisky corrections.[8] The construction described in Ref. 8 is known to have some arbitrariness associated with the choice of metric on configuration space (see, for example, Ref. 9 and the papers cited there). In an effort to fix that arbitrariness, Vilkovisky[8] proposed an additional condition which fixes this metric, equal to the natural metric on the group. For the theory of (1), on the other hand, the natural metric depends on the gauge parameter α, as mentioned above (the special nature of 2D gravitation was also pointed out in Ref. 8). It follows from the calculations in Ref. 5 that an α dependence is also characteristic of an effective Vilkovisky potential. Accordingly, it is not possible to find a gauge-invariant effective potential within the framework of the method of Ref. 8.

The only possibility is thus to choose some specific gauge condition. The most reasonable approach is to use a gauge in which there is no divergence in the kinetic sector of the theory. It follows from (3) that in harmonic gauge (2) we need to take $\alpha=0$ for this purpose. Setting $\alpha=0$ in (4), we rewrite the condition under which the quantum corrections are trivial,

$$V_{\rm eff}=(1+\tau)V+\eta,\quad \tau,\eta=\text{const}, \tag{5}$$

as the differential equation

$$\tau V+\eta=\frac{V'}{4\pi C_1}\ln\left(\frac{V'}{eC_1\mu^2}\right). \tag{6}$$

It is simple to find a solution of (6) in the form

$$V=-\frac{\eta}{\tau}+\frac{\mu^2}{4\pi\tau}e^{a\sqrt{\Phi-\Phi^*}}[-1+a\sqrt{\Phi-\Phi^*}], \quad (7)$$

$$a=\pm\sqrt{8\pi C_1\tau}, \quad \Phi^*=\mathrm{const}, \quad \tau\neq 0, \quad V=\Omega_1\Phi+\Omega_2 \quad (8)$$

for $\tau=0$. The solution in (8) agrees with a solution found previously. The solution in (7) is a new form of a classical potential for which the single-loop corrections have the trivial form in (5). Without any loss of generality, we could set $\Phi^*=0$ in (7), since (1) is invariant under shifts of Φ.

Expression (7) incorporates a dependence on the renormalization parameter μ^2. To fix this parameter we need to introduce a normalization condition. For the normalization condition

$$V_{\mathrm{eff}}^{(1-\mathrm{loop})}|_{\Phi=0}=V(0), \quad (9)$$

the Φ-dependent part of the renormalized effective potential is zero. If we instead choose the more general condition

$$V_{\mathrm{eff}}^{(1-\mathrm{loop})}|_{\Phi=0}=\tau^* V|_{\Phi=0}+\eta^*, \quad (10)$$

where τ^* and η^* are constants (not necessarily the same as τ and η), then (10) has a nontrivial solution, and μ^2 can be expressed in terms of τ, η, τ^*, and η^*.

The trivial nature of the single-loop corrections in theory (1) with potential (7) of course does not by itself imply exact solvability. On the other hand, it is interesting to study such a theory by means of nonperturbative methods.

We wish to thank I. L. Bukhbinder for stimulating discussions and S. D. Odintsov and I. V. Tyutin for explaining certain aspects of a unified effective action.

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Translated by D. Parsons