Population inversion and amplification of far-IR radiation at cyclotron resonance of heavy hot holes in germanium

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A new mechanism for the creation of a population inversion of holes in germanium, involving a cyclotron resonance of heavy hot holes, is discussed. The coefficient of amplification far-IR radiation is derived. The distribution function found for light hot holes by the Monte Carlo method turns out to be inverted at low hole energies. This inversion may also lead to a population inversion of the light holes in terms of Landau levels. The possibility of an experimental observation of stimulated emission is discussed.

1. Introduction. Andronov *et al.*[1] have proposed a theoretical mechanism for the production of a hole population inversion and for amplification of far-IR radiation in germanium in a strong static electric field crossed with a strong static magnetic field. The hole population inversion would arise from a difference between the dynamics of the motion of hot heavy holes and light holes in momentum space in the fields $\mathbf{E}\perp\mathbf{H}$. The amplification of the far-IR radiation would arise in the course of direct transitions of light holes into the heavy-hole band upon the appearance of a hole population inversion (see the inset in Fig. 1). Soon after the hole population inversion was predicted in Ref. 1, it was observed experimentally[2] in *p*–Ge. Generation of far-IR radiation was achieved later.[3,4] Research in this direction has developed successfully in Russia and abroad[5] in recent years. Lasers of a new type have been developed: injection-free far-IR lasers using hot holes in germanium.

2. Population inversion at a cyclotron resonance of hot heavy holes. In this letter we wish to discuss a different mechanism for creating a population inversion of hot holes. The idea is quite simple. Let us consider the behavior of holes in germanium in an intense, circularly polarized microwave pump field of frequency ω_{pump}. Under conditions of cyclotron resonance of the heavy holes, with $\omega_{\text{pump}}=\omega_{c1}$ ($\omega_{c1}=eH/m_1c$ is the cyclotron frequency of the heavy holes), and under the condition $\omega_{c1}\tau_1\gg 1$ (τ_1 is the relaxation time of the heavy holes in the passive region, $\mathscr{E}<\hbar\omega_0$, where ω_0 is the frequency of an optical phonon), the heavy holes are heated by the microwave field. Since the mass of the light holes is considerably lower, the condition $\omega_{c2}\gg\omega_{\text{pump}}$ holds, so the light holes are heated to a lesser extent by the microwave field. The average energy of the heavy holes is thus greater than that of the light holes. Because of their larger average energy, the heavy holes are scattered more frequently than the light holes. As a result, the number of light holes, p_2, becomes greater than its equilibrium value p_{20} (Ref. 6). Both of these factors give rise to a hole population inversion. The mechanism is illustrated in the inset in Fig. 1.

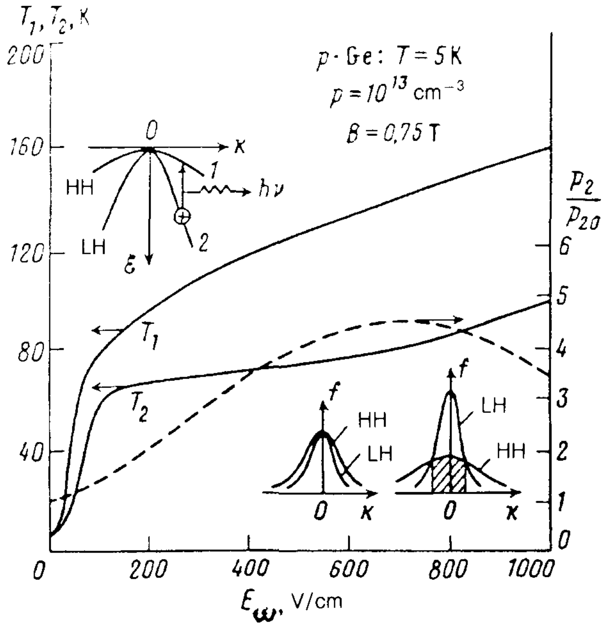

FIG. 1. Temperature of the heavy hot holes and the light hot holes, T_1 and T_2, and ratio of the concentration of light holes, p_2, to the equilibrium value p_{20} versus the microwave electric field. The inset shows the scheme of direct transitions of a hole from the light-hole band to the heavy-hole band accompanied by the emission of a photon. The inset so illustrates the mechanism for the population inversion of hot holes (the inversion region is hatched).

There is an important distinction between the dynamics of the motion of holes in the case of static crossed field $\mathbf{E} \perp \mathbf{H}$ and that in the case of cyclotron resonance. In the former case, the motion of the heavy holes in momentum space corresponds to streaming conditions,[1] while in the latter case the motion is along an cyclotron orbit with a radius which varies continuously in time. In the former case the momentum distribution of the heavy holes is approximately needle-shaped, while in the latter case the holes fill the entire passive region $\mathscr{E} < \hbar\omega_0$. With identical E and H in the former case, the heavy holes can penetrate deeper into the active region $\mathscr{E} > \hbar\omega_0$ than in the latter case ($\omega_{c1}\tau_0^- \ll 1$, where τ_0^- is the time scale of the emission of an optical phonon). After the scattering of a heavy hole into the light-hole band, accompanied by the emission of an optical phonon, the initial velocity of the light holes and thus the radius of the cyclotron-revolution orbit of the light holes are smaller in the latter case than in the former. In other words, the light holes in the latter case are in a lower-energy region in the passive region than in the latter case. As a result, the hole population inversion should be greater in the latter case than in the former.

Let us determine the gain coefficient for far-IR radiation in the case of intersubband direct transitions of holes under conditions of the cyclotron resonance of heavy holes.

We first find the temperatures and densities of the hot heavy holes and light holes, T_1, T_2, p_1, and p_2, working from the balance equations, and assuming that the distribution function of the hot heavy holes and light holes with respect to the momentum $\hbar k$ is a Boltzmann distribution:

$$f_i=\frac{2^{1/2}\pi^{3/2}\hbar^3 p_i}{(m_i k_B T_i)^{3/2}}\exp\frac{\hbar^2 k^2}{2m_i k_B T_i}, \quad i=1,2. \tag{1}$$

Although this is a crude assumption, calculations with the distribution function in (1) lead to average energies and gain coefficients for far-IR radiation which are fairly close to the experimental value, even under streaming conditions in static fields $\mathbf{E}\perp \mathbf{H}$ (Ref. 7).

In a circularly polarized microwave field and in a static magnetic field, the balance equations for the power and for the number of particles are

$$p_i\frac{e}{m_i}\frac{\langle\tau_i\rangle f_i E_\omega^2}{1+(\omega_{\text{pump}}-\omega_{ci})^2(\langle\tau_i\rangle f_i)^2}=p_i\left\langle\left(\frac{d\mathscr{E}}{dt}\right)^{i\to i}_{O+A}\right\rangle f_i$$

$$+p_i\left\langle\left(\frac{d\mathscr{E}}{dt}\right)^{i\to j}_{O+A+I}\right\rangle f_i-p_j\left\langle\left(\frac{d\mathscr{E}}{dt}\right)^{j\to i}_{O+A+I}\right\rangle f_j, \tag{2}$$

$$p_i\left\langle\left(\frac{1}{\tau}\right)^{i\to j}_{O+A+I}\right\rangle f_i=p_j\left\langle\left(\frac{1}{\tau}\right)^{j\to 1}_{O+A+I}\right\rangle f_j, \tag{3}$$

where i, $j=1$ for heavy holes and 2 for light holes. The angle brackets mean an average over the distribution function of the heavy or light holes. For the momentum relaxation time in Eq. (2) we have

$$\langle\tau_i\rangle f_i=1/(4/3\sqrt{\pi})\int_0^\infty \exp(-x_i)\tau_i^{-1}(x_i)x_i^{3/2}dx_i \tag{4}$$

for the scattering frequency $(1/\tau)$ in Eq. (3), and for the rate of scattering and energy transfer in Eq. (2) we have

$$\langle\varphi\rangle=2/\sqrt{\pi}\int_0^\infty \exp(-x_i)\varphi(x_i)x_i^{1/2}dx_i, \tag{5}$$

where $x_i=\mathscr{E}/k_B T_i$ and $\varphi=1/\tau$ or $d\mathscr{E}/dt$. The left side of Eq. (2) is the rate of energy acquisition, while the right side is the rate at which energy is scattered and transferred in the course of intrasubband and intersubband transitions of holes. Intrasubband and intersubband scattering by optical vibrations (O), acoustic vibrations (A), and impurities (I) are taken into account. In the numerical calculations we used the following parameter values for p–Ge: $m_1=0.33m_0$, $m_2=0.042m_0$, a sound velocity $u_l=5.4\times10^5$ cm/s, a density $\rho=5.32$ g/cm^3, an optical-vibration frequency $\omega_0=5.63\times10^{13}$ cm^{-1}, a permittivity $\kappa=16$, and strain-energy constants $\Xi_A=5.24$ eV and $\Xi_0=10.21$ eV ($D_tK=1.06\times10^9$ eV/cm).

The results of the calculations for heavy-hole cyclotron resonance are shown in Fig. 1. As expected, we find $T_1>T_2$ and $p_2/p_{20}>1$. At $\omega_{c1}=\omega_{\text{pump}}$ and $\omega_{c2}\gg\omega_{\text{pump}}$, the rate at which the heavy holes acquire energy is greater than that at which the light holes acquire energy by a factor of about $(m_2/m_1)(\omega_{c2}\tau_2)^2$. Since the condition $\omega_{c1}\tau_1 \gg 1$ holds for the heavy-hole cyclotron resonance, the condition $\omega_{c2}\tau_2 \gg 1$ holds even more strongly. Accordingly, despite the rapid exchange of energy in the course of intersubband scattering, in the course of which the heavy holes "tie" their temperature to the light holes, the condition $T_2 < T_1$ still holds. Since we have $T_1 > T_2$ and $p_2/p_{20} > 1$, a hole population inversion arises at small values of k (or at small values of $\mathscr{E}_2$, $\mathscr{E}_1$, and $h\nu$; see the inset in Fig. 1): $f_2(k) > f_1(k)$.

3. Amplification of far-IR radiation. According to Ref. 8, the optical absorption coefficient for direct intersubband transitions of holes is

$$\alpha_{12} = \frac{e^2\langle |\mathbf{e}_\nu \cdot \mathbf{p}_{12}|^2\rangle_\Omega k}{\pi c m_0^2 n\nu (d\mathscr{E}_2/dk^2 - d\mathscr{E}_1/dk^2)} [f_1(\mathscr{E}_1) - f_2(\mathscr{E}_2)], \tag{6}$$

where $\mathbf{e}_\nu$ is a unit vector along the field of the light wave, and $\mathbf{p}_{12}$ is a matrix element of the momentum operator. At small values of k we have, approximately,[8] $|\mathbf{e}_\nu \cdot \mathbf{p}_{12}|^2 \simeq \hbar^2 k^2$. We then find the following expression for the gain coefficient for direct intersubband transitions of holes:

$$\alpha_{21} = Bk^2 [f_2(\mathscr{E}_2) - f_1(\mathscr{E}_1)]/h\nu, \quad \mathscr{E}_2(\mathbf{k}) - \mathscr{E}_1(\mathbf{k}) = h\nu, \tag{7}$$

where the numerical coefficient B can be found by comparing theory and experiment for the optical absorption spectra of p-Ge (Ref. 9): $B = 1.24 \times 10^{-17}$ eV $\cdot$ cm^2.

In a calculation of the actual optical gain coefficient α_{gain} we need to consider the absorption of light by free hot holes in the course of indirect intrasubband and intersubband transitions of heavy and light holes involving optical and acoustic vibrations and impurities. The absorption coefficient α_{O+A+I} was calculated in second-order perturbation theory with a Boltzmann distribution function [see (1)]. The final result is $\alpha_{\text{gain}} = \alpha_{21} - \alpha_{O+A+I}$. The results of this calculation are shown in Fig. 2. We see that the gain in the long-wave part of the spectrum is limited by absorption of light by free holes. Shown for comparison here are results calculated for α_{gain} for the case of static crossed fields E and H. This calculation was carried out by a method similar to that described above (see Ref. 7 for more details). For $E = 500$ V/cm, α_{gain} turns out to be smaller than for the heavy-hole cyclotron resonance at the same fields E and H. The reason is that in the orientation $\mathbf{E} \perp \mathbf{H}$ the rate at which the heavy holes acquire energy is greater than that at which the light holes acquire energy by a factor of about $(m_2/m_1)[(\omega_{c2}\tau_2)^2/(\omega_{c1}\tau_1)^2]$. This difference is smaller by a factor of $(\omega_{c1}\tau_1)^2 \gg 1$ than at cyclotron resonance.

4. Population inversion and amplification of radiation in the Monte Carlo method. Inversion in terms of the light-hole Landau levels. The distribution function of the hot heavy holes and light holes and also α_{gain} can be found more accurately through a numerical simulation of the motion of holes in k space in a static magnetic field and in a circularly polarized microwave pump field (by the Monte Carlo method). The results are shown in Fig. 3. Shown for comparison is a Boltzmann distribution function for the light holes found from the solution for the balance equation (the dashed line). While the distribution functions found for the heavy holes by the Monte Carlo method and from the balance equations in the passive energy region,

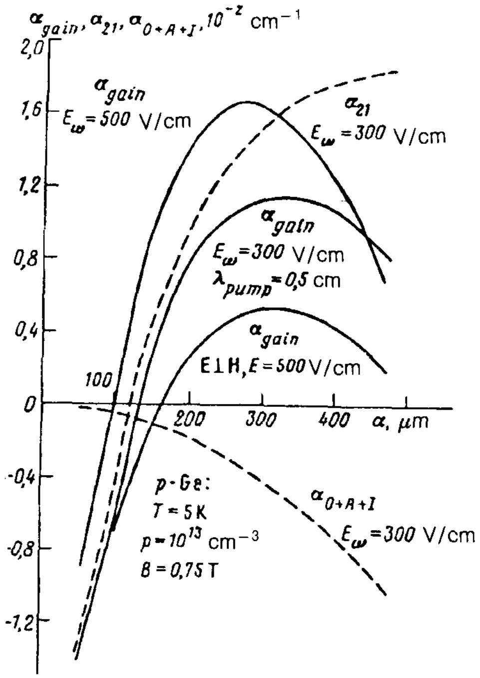

FIG. 2. Spectra of the optical gain coefficient for direct interband transitions of holes, α_{21}, of the optical absorption coefficient for indirect transitions of holes involving optical (O) and acoustic (A) lattice vibrations and impurities (I), α_{O+A+I}, and of the gain coefficient $\alpha_{\text{gain}}=\alpha_{21}-\alpha_{O+A+I}$ for two values of the microwave field and for constant crossed fields

$\mathscr{E} < \hbar\omega_0$, differ only slightly, the distribution functions for the light holes in these two cases are qualitatively different. At small values of k, a dip appears in the light-hole distribution. A similar dip has been found[10] in a simulation of the motion of heavy and light holes in static crossed fields E and H. It was explained in that earlier study as due

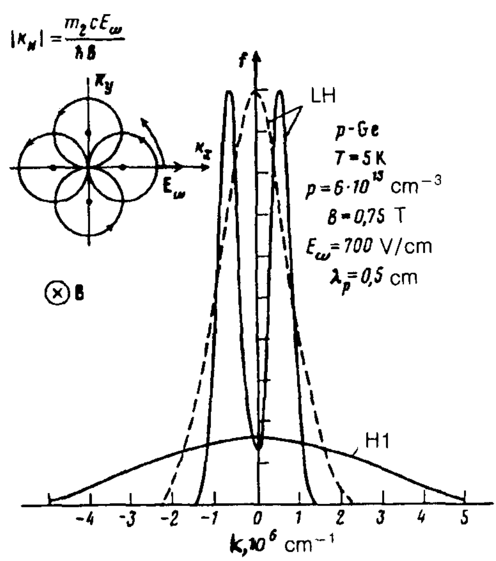

FIG. 3. Distribution functions of the heavy and light hot holes with respect to momentum $\hbar k$ as a function of the wave vector as found by the Monte Carlo method (solid lines) and from the balance equations for the power and number of particles (dashed line). The inset shows paths of the cyclotron revolution of light holes at various times during the microwave period with a circular polarization, with $\omega_{c1}=\omega_{\text{pump}}$ and $\omega_{c2}\gg\omega_{\text{pump}}$. The center of the cyclotron orbits, $k_{H2}=(cm_2/\hbar)E_\omega/H$, changes position slowly because of the rotation of E_ω.

to a scattering of light holes into the heavy-hole band in the course of impurity scattering of low-energy light holes. The dip at the cyclotron resonance of the heavy holes (which is deeper than for static fields E and H) is due to both a scattering of light holes in the course of impurity scattering into the heavy-hole band and the specific dynamics of the motion of holes in the circularly polarized microwave field and the static magnetic field.

It can be seen from Fig. 3 that after the minimum of the distribution function there is an interval of k (or of the energy $\mathscr{E}$) in which $[f_2(k) - f_1(k)]$ is greater than in the case of a Boltzmann distribution of the heavy and light holes. According to the calculations for $p=6\times 10^{13}$ cm^{-3}, $E_\omega=700$ V/cm, and $\lambda=200$ μm, we have $\alpha_{\text{gain}}=6\times 10^{-3}$ cm^{-1} for a Boltzmann distribution function (at this hole concentration and at $\lambda_{\text{pump}}=0.5$ cm, the maximum value is $\alpha_{\text{gain}}=6.7\times 10^{-3}$ cm^{-1} at $\lambda=175$ μm and $E_\omega=600$ V/cm) and $\alpha_{\text{gain}}=1.2\times 10^{-2}$ cm^{-1} in a calculation of the distribution function by the Monte Carlo method.

The light-hole distribution function turns out to be inverted at low energies; this inversion should lead to a population inversion of the light holes in terms of Landau levels. The number of Landau levels at $H=7.5$ kOe which lie in the inversion region is roughly 3, and the energy interval between them corresponds to $\lambda=600$ μm. There can thus be an amplification of radiation with $\lambda=600$ μm. In addition to the impurity scattering, a tunneling of holes can lead to an inversion in terms of Landau levels.[11]

5. Possibility of experimentally producing stimulated emission. In static crossed fields E and H in p–Ge at $T=4.2$ K, a generation in the course of intersubband transitions of holes arises even at $\alpha_{\text{gain}}\simeq 0.007$ cm^{-1} (Ref. 12). Accordingly, even at a hole concentration $p\lesssim 10^{13}$ cm^{-1} there can be a generation during microwave pumping. An important circumstance here is that there are no contacts, which would cause losses of far-IR radiation and which would make it difficult to observe generation. The pulsed power of the microwave pump must satisfy $P_{\text{pump}}>100$ W. Because of the pronounced absorption of the microwave radiation, the cross section of the sample must not exceed 2×2 mm. With increasing p, the gain falls off because of impurity scattering. According to calculations from balance equations (2) and (3), for example, with $p=6\times 10^{13}$ cm^{-3} and $\lambda_{\text{pump}}=0.5$ cm the maximum value is $\alpha_{\text{gain}}=6.7\times 10^{-3}$ cm^{-1} (at $\lambda=175$ μm and $E_\omega=600$ V/cm).

As λ_{pump} is increased, α_{gain} falls off. At $\lambda_{\text{pump}}=1$ cm ($H=3.75$ kOe) and at $p=10^{13}$ cm^{-3}, the value of α_{gain} reaches 9×10^{-3} cm^{-1} (at $\lambda=300$ μm and $E_\omega=300$ V/cm). It must be kept in mind, however, that the Monte Carlo calculations yield larger values of α_{gain}, as was shown above.

A generation in the course of transitions of light holes between Landau levels in static fields $\mathbf{E}\perp\mathbf{H}$ has been achieved[5] at $p=6\times 10^{12}$ cm^{-3}($\lambda=220$–350 μm). Accordingly, it is also possible to achieve stimulated emission with microwave pumping.

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