Properties of a hot hadron vacuum

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For nucleons, as for electrons in semiconductors at high temperatures, the energy gap separating nucleon states from antinucleon states is blurred. The blurring stems from a restructuring of the spectrum of nucleons in the dense medium of thermal pions. The equation of state of the hadron vacuum has an end point; i.e., there exists a limiting temperature T_k above which a hadron vacuum does not exist even in a metastable state. A deconfinement temperature is estimated.

1. According to the present theoretical understanding, a deconfinement, i.e., a phase transition from a gas of hadrons to a gas of quarks and gluons, occurs at high temperatures T (Ref. 1). The deconfinement temperature T_c has been estimated in the bag model[2] under the assumption that the density of nucleon–antinucleon pairs is exponentially small at $m < T < M$ because of the large difference between the mass of nucleons, M, and that of pions, m ($m \ll M$). On this basis one can ignore the effect of nucleons on the phase transition, and one can interpret deconfinement as a phase transition from a dense gas of pions to a gas of quarks. We show below that this approach requires refinement, since the gap $2M$ separating the nucleon and antinucleon states decreases at $T > m$. This decrease leads to a sharp temperature-induced increase in the densities of nucleons, n, and antinucleons, $\bar{n}$.

We consider a hadron vacuum, for which we have $n = \bar{n}$. The densities n and $\bar{n}$ are governed by the value of the effective nucleon mass, M^*, which in turn depends on T. Below we derive the T dependence of n and M^* for $T > m$,

$$n \simeq T_k^3 \exp\left[-\frac{M^*}{T}\right], \quad M^* = M\left(1 - \frac{T^2}{T_k^2}\right), \tag{1}$$

and we estimate an upper limit on the temperature T_k (we are using a pion system of units, with $\hbar = c = m = 1$),

$$T_k^2 < \frac{M(10)^{1/2}}{2\pi g} \equiv T_{k0}^2, \tag{2}$$

where $g \simeq 1$ is the pion–nucleon interaction constant. At $T \gg T_k$ a hadron phase of the vacuum cannot exist even in a metastable form; i.e., T_k is a temperature corresponding to an absolute instability of a hadron vacuum. Deconfinement occurs at a definitely lower temperature T_c, so the estimate in (2) also imposes a limit on T_c: $T_c < T_k < 250$ MeV.

Since the time scale for the collision of heavy ions is short, deconfinement is a highly nonequilibrium phase transition. From the experimental standpoint, we would

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be interested in determining not the equilibrium deconfinement temperature T_c but the temperature of the absolute instability of hadron matter, T_k. A rapid heating of hadrons to $T > T_k > T_c$ results in deconfinement without a prolonged stage involving the formation of large nucleation centers of the quark phase. The transition is of the nature of a thermal explosion or, more precisely, the breakdown of a hadron vacuum.

The pronounced increase in the density of nucleon–antinucleon pairs at $T > m$ suggests an interesting possibility for diagnostics of hot hadron matter, not at the time of its expansion[3] but in the early stages of the collision of nuclei, through measurements of the spectra of penetrating particles,[4] i.e., photons and leptons, which form as a result of the annihilation of these pairs. These annihilation particles are precursors of deconfinement; they carry information on how the parameter M^* depends on the energy at which the nuclei collide and on the temperature of the nuclear matter [see Eq. (1)].

2. To see the essence of the matter, it is convenient to look at the analogous problem of electrons and holes in a semiconductor at a high temperature T. When the electron–phonon interaction is taken into account, tails arise on the density of electron states deep in the band gap. The gap in the spectrum, which has clearly defined boundaries at $T=0$, becomes blurred as T increases.[5] The analogy here is complete: electrons → nucleons, holes → antinucleons, phonons → pions. A nucleon, as a particle with a spectrum $E_p^2 = M^2 + P^2$, is an elementary excitation against the background of zero-point vibrations of the pion field, i.e., against the background of a cold vacuum. At $T=0$ there exists a gas of real thermal pions, which a heavy nucleon can absorb without undergoing any substantial change in momentum P, by virtue of the condition $M \gg m$. The energy of the nucleon, E, on the other hand, changes dramatically, by an amount $\sim T$, in the course of each such absorption event; as a result, this energy departs from the mass shell $E = E_p$. In other words, a nucleon against the background of a hot vacuum is no longer characterized by a well-defined spectrum of E_p. However, crude characteristics of the nucleon such as the density of states $\rho(\omega)$ and the average occupation numbers n_p can be found correctly even at $T \neq 0$, by relating them to the retarded nucleon Green's function[6] $G^R(\omega)$:

$$G^R(\omega) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\rho(\epsilon)d\epsilon}{\omega - \epsilon + i\gamma}, \quad n_p = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\rho(\omega)d\omega}{\exp[(\omega + E_p)/T] + 1}. \tag{3}$$

The integration over p is determined by the nucleon density n:

$$n = 4\int n_p \frac{d^3p}{(2\pi)^3}, \quad n_0 = 4\left(\frac{MT}{2\pi}\right)^{3/2} \exp\left(-\frac{M}{T}\right), \tag{4}$$

where n_0 is the value of n at $T < M$, found without consideration of the pion–nucleon interaction, with $\rho(\omega) = \pi\delta(\omega)$. That interaction can be taken into account on the basis of the well-developed diagram technique for the analogous problem of the effect of thermal phonons on electrons in metals and semiconductors. Serving as the pion Green's function is the quantity $D(k,\omega)$, which is an analog of the phonon function[6]

$$D(k,\omega) = g^2 \frac{(\vec{\sigma}_1\mathbf{k})(\vec{\sigma}_2\mathbf{k})(\vec{\tau}_1\vec{\tau}_2)}{\omega^2 - \omega_k^2}, \tag{5}$$

where σ and τ are the spin and isospin matrices of the nucleons, and ω_k ($\omega_k^2=1+k^2$) is the spectrum of pions. The diagram technique developed to account for the electron–phonon interaction[6] simplifies substantially in this case, since we can ignore the recoil of the nucleon as a result of the emission or absorption of a pion ($M \gg m$). As a result, the nucleon Green's function G^R depends on essentially the one variable $\omega \equiv E-E_p$, not on E and p separately:

$$G^R(\omega)=\frac{1}{\omega-\Sigma^R(\omega)+i\gamma}, \tag{6}$$

$$\Sigma^R(\omega)=\frac{3g^2}{4\pi^2}\int_1 \frac{(\epsilon^2-1)^{3/2}d\epsilon}{\exp(\epsilon/T)-1}\,[G^R(\omega+\epsilon)+G^R(\omega-\epsilon)].$$

Expression (6) is valid in the linear approximation, in which the density of nucleon–antinucleon pairs is low, and we do not need to concern ourselves with the renormalization of the pion spectrum in the nucleon field.[7] In addition, expression (6) incorporates the contribution to Σ^R from only those diagrams which lack a crossing of pion lines. This approximation is justified in Ref. 8. The argument here is that the vertex representing the emission of a pion by a nucleon contains a factor $(\vec{\sigma}\mathbf{k})\vec{\tau}$ [see Eq. (5)], where s and τ are the spin and isospin matrices. Since these matrices do not commute, each diagram which has a crossing of pion lines is smaller by a factor of $3\times 3=9$ than the corresponding diagram without such a crossing. Expression (6) also ignores the presence of the Δ resonance in the amplitude for pion–nucleon scattering. All these effects have been analyzed. It has been found that incorporating them simply reduces T_k [see Eq. (2)] and increases the values given below for $\rho(\omega)$ [see Eqs. (8) and (13)] in the interior of the band gap.

Expression (6) is a closed nonlinear equation for the function $\Sigma^R(\omega)$. The density of states $\rho(\omega)$ is determined in terms of the solution of this equation:

$$\rho(\omega)=\frac{\gamma(\omega)}{[\omega-\mathrm{Re}\Sigma(\omega)]^2+\gamma^2(\omega)}, \quad \gamma=\mathrm{Im}\Sigma. \tag{7}$$

3. Solutions of Eq. (6) can be derived in the limits of low temperatures ($T<1$) and high ones ($T>1$). For low T, the density of the pion gas is exponentially small, and we can ignore Re Σ, along with ω, in (7). In this case the deep tail on the density of states satisfies the Urbach law[5]

$$\rho(\omega)=\pi\frac{\lambda^2}{6\omega^2}\varphi(|\omega|)\exp\left[-\frac{|\omega|}{T}\right], \quad \lambda\equiv\frac{3g}{2\pi}, \quad |\omega|\gg 1. \tag{8}$$

From (6) we find an equation for the pre-exponential factor φ:

$$\varphi(\omega)=\omega^3+\frac{\lambda^2}{6}\int_0^\omega (\omega-\epsilon)^3\frac{\varphi(\epsilon)d\epsilon}{\epsilon^2}, \quad \omega>0. \tag{9}$$

A solution of this equation is the power series

$$\varphi=\omega^3\left(1+\lambda^2\frac{\omega^2}{5!}+\lambda^4\frac{\omega^4 3!}{5!\,7!}+\ldots\right). \tag{10}$$

Substitution of (8) and (10) into (3) and (4) yields the density of nucleons, n:

$$n(T)=n_0(T)\left(1+\frac{\lambda^2M^2}{2!\,3!}+\frac{\lambda^4M^4}{4!\,5!}+\frac{\lambda^6M^6}{6!\,7!}+\dots\right). \tag{11}$$

In the limit $T\to 0$, the nucleon density n does not become equal to $n_0(T)$, so the "gas" expression for $n_0(T)$ has no range of validity. Expansion (11) is carried out in terms of the large literal parameter $M^2\gg 1$. A deep tail arises on the density of nucleon states $\rho(\omega)$ even in first-order perturbation theory in the pion–nucleon coupling constant g^2. An analogous effect occurs in solids: Even in first order in the electron–phonon coupling, a deep tail arises on the density of electron states in the band gap of a semiconductor. However, since the fine-structure constant is small ($\alpha=1/137$), this effect goes unseen against the background of competing multiphonon effects.

4. At high temperatures $T>1$, the characteristic values $\omega\approx T^2$ in (6) are higher than $\epsilon\approx T$. It is therefore legitimate to use a static approximation. The equation for G becomes an algebraic equation:

$$G^{-1}(\omega)=\omega-V_0^2G(\omega),$$

$$V_0^2(T)=\frac{2}{3}\lambda^2\int_0^\infty\frac{\omega^3d\omega}{\exp(\omega/T)-1}=g^2\frac{\pi^2T^4}{10}. \tag{12}$$

At $T>1$ the role played by the gas of pions is thus equivalent to a random, long-wave, spin–isospin potential. The functions $\rho(\omega)$ and $n(T)$ are found in terms of the solution of Eq. (12):

$$\rho(\omega)=\frac{1}{V_0}\left(1-\frac{\omega^2}{4V_0^2}\right)^{1/2},$$

$$n(T)=n_0(T)\frac{1}{\sqrt{\pi}}\left(\frac{T}{V_0}\right)^{3/2}\exp\frac{2V_0}{T}. \tag{13}$$

The rough estimate of T_k in (2) corresponds to the vanishing of the width of the band gap for the density of nucleon states, $\rho(\omega)$: $M=2V_0(T_{k0})$. When the width of this gap is on the order of T, i.e., at $T-T_{k0}\approx T_{k0}^2/M$, the density of nucleon–antinucleon pairs is so high that we must take into account the softening of the pion spectrum in the field of these pairs.[7] The problem becomes quite nonlinear, since the renormalization of the pion spectrum causes an even greater increase in the density $n(T)$; i.e., the creation of hadrons is an avalanche process. Using the technique developed in Refs. 6 and 7, we can find the change in the spectrum of pions in the field of nucleons by calculating the pion polarization operator. For the values of k of interest here, this change reduces to a replacement of ω_k in (5) by $\tilde{\omega}_k$:

$$\tilde{\omega}_k^2=1+(1-\nu)k^2,\quad \nu=\frac{4g^2n(T)}{V_0(n,T)}. \tag{14}$$

The T dependence of the density of nucleons at $T>1$ is given by expression (13) again, except that the parameter $V_0(T)$ in (12) is replaced by $V_0(T,n)$:

$$V_0(n,T)=\frac{V_0(T)}{(1-\nu)^{5/4}}. \tag{15}$$

Analysis of Eqs. (13) and (15) in light of (12) and (14) shows that an equation of state of the hadron vacuum, i.e., the functional dependence $n=n(T)$ for the density of nucleon–antinucleon pairs, exists only at $T<T_k$. The value of T_k is close to T_{k0} in (2):

$$T_k\simeq T_{k0}\left(1-\frac{T_{k0}}{M}\ln\frac{T_{k0}}{m}\right). \tag{16}$$

At $T=T_k$, the derivative of $n(T)$ with respect to T becomes infinite. Consequently, the temperature $T=T_k$ is the limiting temperature for the equation of state of a hadron vacuum. Here are the limiting values of the parameters n_k and ν_k [see Eq. (14)] at the point T_k:

$$n_k=n(T_k)=\frac{T_k}{10g^2},\quad \nu_k=\nu(T_k)=\frac{8}{10}\frac{T_k}{M}. \tag{17}$$

Using the inequality $T_k<M$ [see (2)], we conclude from (17) that the renormalization of the pion spectrum in the nucleon field, (14), is only slight up to the singular point $T=T_k$.

In summary, for a hadron vacuum there exists a limiting temperature T_k above which an equation of state $n=n(T)$ does not exist.

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