Features of the 1/f noise in metals

R. O. Zaĭtsev
Kurchatov Institute Russian Science Center, 123182 Moscow, Russia

(Submitted 25 November 1993)
Pis'ma Zh. Eksp. Teor. Fiz. **58**, No. 12, 978–982 (25 December 1993)

A study of the interaction of diffusion modes and thermal-conductivity modes reveals the temperature dependence of the exponent α for the intensity of the $1/\omega^{\alpha}$ noise in metals.

The measured intensity of the $1/\omega^{\alpha}$ noise in metal films[1] is proportional to the square of the applied electric field E:

$$K_{\alpha\beta}(\omega)=\frac{1}{2}\langle\{\hat{j}_{\alpha}\hat{j}_{\beta}\}\rangle_{\omega}=S_{\alpha\beta\gamma\nu}(\omega)E_{\gamma}E_{\nu}\sim J_{\alpha\beta\gamma\nu}E_{\gamma}E_{\nu}\omega^{-\alpha}. \quad (1)$$

If the thermal insulation is good, the exponent α is a significant function of the temperature T. It thus becomes necessary to study the low-frequency singularities of the expectation value of the product of four current-density operators:

$$S_{\alpha\beta\gamma\nu}(t_1,t_2,t_3,t_4)=\frac{1}{2}\langle\hat{T}_{\Gamma}[\hat{j}_{\alpha}^{(+)}(t_1)\hat{j}_{\beta}^{(-)}(t_2)\hat{j}_{\gamma}(t_3)\hat{j}_{\nu}(t_4)+\hat{j}_{\beta}^{(+)}(t_2)\hat{j}_{\alpha}^{(-)}$$
$$\times(t_1)\hat{j}_{\gamma}(t_3)\hat{j}_{\nu}(t_4)]\rangle. \quad (2)$$

Here $\hat{j}_{\gamma}$ and $\hat{j}_{\nu}$ are the current operators associated with the voltage source, $\hat{j}^{(\pm)}$ are the current operators of the noise source, and $\hat{T}_{\Gamma}$ represents ordering on the time contour Γ, which is utilized in the diagram technique for nonequilibrium operators.[2]

The most convenient representation is that in which all the Green's functions have a single vanishing diagonal component:

$$\hat{D}_{\omega}(q)=\begin{pmatrix}0 & D_{\omega}^{A}(q)\\ D_{\omega}^{R}(q) & \mathscr{F}_{\omega}(q)\end{pmatrix},\quad \hat{j}_{\alpha}^{(\pm)}=\frac{1}{2}(1\pm\hat{\tau}^{x})\tilde{j}_{\alpha},\quad \hat{j}_{\gamma,\nu}=\tilde{j}_{\gamma,\nu}\hat{\tau}^{x}, \quad (3)$$

where $D_{\omega}^{R,A}(q)$ are retarded and advanced Green's functions, $\tilde{j}_{\alpha,\gamma}$ are current-density operators in the Heisenberg representation, and $\hat{\tau}^{x}$ is the Pauli matrix.

In an electron system in a crystal field, at a nonzero temperature, only the diffusion mode and the thermal-conductivity mode have a low-frequency singularity. For these modes we have

$$D_{\omega(q)}^{R,A}=(\pm i\omega-Dq^{2})^{-1},\quad \mathscr{F}_{\omega}(q)=\frac{i}{2}\coth\left(\frac{\omega}{2T}\right)[D_{\omega}^{R}(q)-D_{\omega}^{A}(q)]. \quad (4)$$

Here and below, D represents either the diffusion coefficient or the thermal conductivity χ.

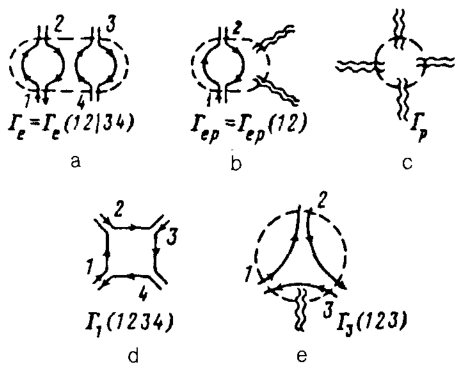

FIG. 1. Graphic representation of the scalar four-vertex parts in the zeroth approximation in the number of diffusion and thermal-conductivity propagators.

In the low-frequency limit $\omega \ll T$, the diagonal Green's function $\mathscr{F}_\omega(q)$ has a low-frequency singularity which is stronger than that in either the retarded or advanced one:

$$\mathscr{F}_\omega(q)=\frac{2T}{(\omega^2+D^2q^4)}. \tag{5}$$

In order to bring out the long-range and low-frequency singularities at a nonzero temperature, it is thus sufficient to single out the contribution which stems from the products of long-range Green's functions in (5) and which contains the smallest number of integrals over virtual momenta and frequencies. The largest contribution to the four-current correlation function in (2) arises from the integration of the product of the two correlation functions in (5) of diffusion origin:

$$S_{\alpha\beta\gamma\nu}(\omega)=\int_{|\omega|}^{\infty} d\omega' \int_0^{\infty} \frac{[(\Gamma_{\alpha\gamma}^{(+)}\Gamma_{\beta\nu}^{(-)}+\Gamma_{\beta\nu}^{(+)}\Gamma_{\alpha\gamma}^{(-)})+(\alpha\rightleftarrows\beta)]}{[(\omega')^2+D^2q^4]^2} q^{d-1}dq. \tag{6}$$

Here we have not written out the term proportional to the product of the two-current correlation functions $\Gamma_{\alpha\beta}$, which refer to the noise source, and the two-current correlation functions $\Gamma_{\gamma\nu}$, which are associated with the voltage source. In the long-wave limit $q^2\to 0$, their contribution turns out to be unimportant in comparison with that of the product of the mixed two-current correlation functions, $\Gamma_{\alpha\nu}^{(\pm)}$.

To find the law by which the quantities $\Gamma_{\alpha\nu}^{(\pm)}$ vanish, it is convenient to work in the limit $Dq^2 \gg \omega$ and to first integrate over frequency. In the space of critical dimensionality $d=6$, the integrals from the two Green's functions in (5) become logarithmic, and the renormalization-group equations for the four- and two-current correlation functions become

$$-\frac{\partial S_{\alpha\beta\gamma\nu}}{\partial t}=[(\Gamma_{\alpha\gamma}^{(+)}\Gamma_{\beta\nu}^{(-)}+\Gamma_{\beta\nu}^{(+)}\Gamma_{\alpha\gamma}^{(-)})+(\alpha\rightleftarrows\beta)]; \quad -\frac{\partial\Gamma_{\alpha\nu}^{(\pm)}}{\partial t}=[\Gamma_{\alpha\nu}^{(\pm)}\Gamma_e]. \tag{7}$$

Here $t=-\ln q$ is a logarithmic variable which becomes q^{-3} in 3D space, while Γ_e is a scalar electron vertex (Fig. 1a). The square brackets in (7) constitute a nonuniversal factor which can be made equal to one by means of a scale transformation.

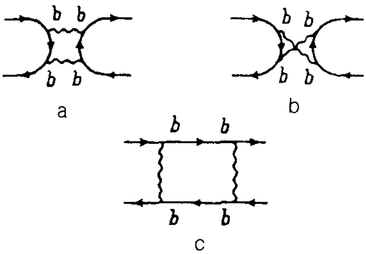

FIG. 2. Feynman diagrams whose ratio (a+b)/c determines the parameter φ. Here b–b represents a nonvanishing diagonal phonon or electron Green's function.

Calculations carried out in second order in the number of virtual diffusion modes and thermal-conductivity modes [see Eq. (5)] yield a closed system of equations which relate vertices of three types: Γ_e, Γ_{ep}, and Γ_p (as shown in Fig. 1). After three scale transformations, we find

$$-\frac{\partial\Gamma_e}{\partial t}=(3+g)\Gamma_e^2+\nu\Gamma_{ep}^2,$$

$$-\frac{\partial\Gamma_{ep}}{\partial t}=3\Gamma_e\Gamma_{ep}+2\Gamma_{ep}^2+\Gamma_p\Gamma_{ep},\quad -\frac{\partial\Gamma_p}{\partial t}=3\nu(g-1)\Gamma_{ep}^2+3\Gamma_p^2. \tag{8}$$

Here $\nu=(D+\chi)^2/4D_\chi$, $g=3-2\varphi+4\varphi^2$, and the quantity φ, which determines the temperature dependence of the thermal conductivity of the lattice, is equal to the ratio of the two-phonon intensity of thermal fluctuations to the two-electron intensity of these fluctuations (Fig. 2). In second order in the number of phonon lines we find $\varphi=N/R$, where

$$N=2\int\left(\frac{\omega_q}{v_0 q}\right)^2\Gamma_q^{-1}\frac{dq}{(2\pi)^3};\quad R=\int\left(\frac{\xi_p}{\Theta}\right)^4\tau_\xi\frac{mp_0}{2\pi^2}d\xi. \tag{9}$$

Here v_0 and p_0 are the velocity and momentum at the Fermi surface. In the spherical model which we are using here we have $\omega_q=cq$, and $\xi_p=p^2/2m-\mu$, and the integrals over the lengths of the vectors p and q are bounded by the conditions $q<\Theta/c$, $|\xi_p|<\Theta$, where c is the sound velocity, and Θ the Debye temperature.

In order of magnitude, the parameter φ is the ratio of the phonon mean free path $l_p\sim c/\Gamma_q$ to the electron mean free path τv_0. At low temperatures, $T\ll\Theta$, the attenuation of the phonons is, in order of magnitude, $\Gamma_\omega\sim\omega c/v_0$, and it is essentially independent of the temperature.[3] For this reason, the entire change in the parameter φ is determined by the temperature dependence of the reciprocal of the electron relaxation time:

$$\varphi\sim\frac{1}{\tau\Theta}\sim\frac{1}{\tau_0\Theta}+\varphi_1\left(\frac{T}{\Theta}\right)^3, \tag{10}$$

where τ_0 is the mean free time of an electron at $T=0$.

At high temperatures, $T\gg\Theta$, the mean free path of phonons having other than the maximum sound velocity is of the same order of magnitude and has the same temperature dependence as the electron length $l_e\sim l_1\sim a(Mc^2/T)$, where Mc^2 is a quantity on the order of the melting point, and a is the lattice constant. For longitudinal phonons with the maximum sound velocity we have[4] $l_p \sim a(Mc^2/T)^{5/4}$, so the parameter φ has the high-temperature expansion

$$\varphi = \varphi_2 + \varphi_3 (Mc^2/T)^{1/4}. \tag{11}$$

Although the constants φ_1 and φ_2 are on the order of unity, the small quantity φ_3 is proportional to the fraction of the phonons which have the maximum sound velocity. A comparison of (10) and (11) shows that the quantity φ reaches values on the order of unity quite rapidly as the temperature is raised, and it has a maximum at temperatures slightly above the Debye temperature.

We first consider the so-called B noise. In this case at least one surface of the metal film has a poor thermal insulation. Since under these conditions the heat flux does not vanish at the corresponding boundary, the contribution of the thermal-conductivity modes "cuts off" at frequencies on the order of $\chi\kappa_i^2/(\kappa_m L)^2$, where κ_m and κ_i are the thermal conductivities of the metal and the substrate, and L is the transverse dimension of the metal film. At lower frequencies, we should set $\varphi = 0$ and $\Gamma_p = \Gamma_{ep} = 0$ in Eqs. (8). As a result, in a space of dimensionality $d = 6 - \epsilon$ we find

$$S \sim q^{-2\epsilon/3} \sim \omega^{-\epsilon/3}. \tag{12}$$

In the $d=3$ case, this gives us a $1/f$ noise ($\epsilon = 3$, $\alpha = 1$).

In the case of the A noise the heat flux is zero along the entire surface of the metal film. In this case the thermal-conductivity mode is just as important as the diffusion mode, regardless of the frequency. Here we have $g > 11/4$, and the exponent α depends on the temperature through the function $\varphi(T)$.

In the asymptotic limit $t \gg 1$, we can seek a solution of Eqs. (7) and (8) in the form

$$\Gamma_s \simeq \gamma_s / t, \quad \Gamma_{\alpha v}^{(\pm)} \sim 1/t^{\gamma_e}. \tag{13}$$

In 3D space, all the scalar and two-current vertices thus vanish by a power law: $\Gamma_s \sim q^3$, $\Gamma_{\alpha v} \sim q^{3\gamma_\epsilon}$.

A stability analysis shows that for all $\varphi > 1.15$ or for $g > 6$ a solution with independent Γ_p and Γ_e, and with $\Gamma_{ep} = 0$ is stable:

$$\Gamma_e = 1/(3+g)t, \quad \Gamma_p = 1/3t, \quad \Gamma_{\alpha v}^{(\pm)} \sim t^{-1/(3+g)}; \quad S \sim t^{1-2/(3+g)}. \tag{14}$$

In 3D space we thus find

$$\alpha = \frac{3}{2}\left(1 - \frac{1}{3 - \varphi + 2\varphi^2}\right). \tag{15}$$

In the rather narrow interval $\varphi_c(\nu) < \varphi < 1.15$, solutions with a nonzero "entangling" vertex $\Gamma_{ep} = v\Gamma_p$, $\mathrm{Im}\, v = 0$, are stable. The parameter v is expressed in terms of ν and g by means of the equation

$$(g-1)^2\,[27\nu^2 v^4 - 6\nu v^3 - 2\nu v^2] - (g-1)\,[42\nu v^3 - (35\nu + 4)v^2 + 2v + 2]$$
$$+ (16+\nu)v^2 - 26v + 10 = 0. \tag{16}$$

Here

$$\Gamma_{av} \sim t^{-k}, \quad S \sim t^{1-2k}. \tag{17}$$

After making the substitution $t \to \omega^{-3/2}$, we thus find

$$\alpha = \frac{3}{2} - \frac{3(g-1)\nu v^2 - 2v + 2}{g[1+(g-1)\nu v^2]}. \tag{18}$$

Equations (16) and (17) determine the exponent α for all those values of the real parameter v for which real solutions of Eq. (16) in terms of the quantity $(g-1)$ exist. Corresponding to the maximum value $v=0.177$ are the minimum parameter values $\nu=1$ and $\alpha=1.117$. For $v=0$ or $g=6$ we have the intermediate value $\alpha=7/6$, which corresponds to $\varphi=1.15$. As the parameter φ is increased further, the value of the exponent α no longer depends on ν; it increases slowly according to (15).

The interval $0<\varphi<0.94$ (or $2.75<g<4.662$) of the parameter φ deserves a special analysis. For this interval, Eq. (16) has complex solutions. It can be shown that these solutions lead to non-power-law singularities in the four-current correlation function. However, we will not take up that possibility here; according to (10), a value $\varphi<1$ could be observed only in very pure metals, at sufficiently low temperatures.

According to (11) and (15), at $T \gg \Theta$ in the case of an A noise we have a slow decrease in the exponent α with the temperature. At low temperatures, and with a fairly short residual electron mean free path, the exponent α increases with the temperature according to (10) and (16). It reaches a maximum at a temperature slightly above Θ. The maximum value of the exponent α never reaches 3/2, but it is always greater than one. These conclusions are in qualitative agreement with the experiments of Ref. 5, in which a maximum value $\alpha=1.2$ was observed at 290 K for silver films, with $\Theta=215$ K. The origin of the maximum of the exponent α as a function of the temperature in the case of the A noise can thus be associated with the existence of an anomalous absorption of longitudinal phonons with the maximum sound velocity. This effect is weakened in anisotropic crystals. In the case of an A noise, we can thus expect a slow decrease in α with the temperature, toward values on the order of one.

The conclusion that the exponent α is independent of the temperature in the case of a B noise is in qualitative agreement with experiment.

This study had financial support from a Soros fund, awarded by the American Physical Society.

[1] J. Clarke and T. Hsiang, Phys. Rev. B **13**, 4790 (1976).
[2] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964) [Sov. Phys. JETP **20**, 1018 (1964)].
[3] A. B. Migdal, Zh. Eksp. Teor. Fiz. **32**, 399 (1957) [Sov. Phys. JETP **5**, 333 (1957)].
[4] I. Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. **12**, 419 (1942).
[5] P. Dutta and P. M. Horn, Rev. Mod. Phys. **53**, 497 (1981).

Translated by D. Parsons