Vector particle in the model of an instanton vacuum

S. V. Esaĭbegyan and S. N. Tamaryan
Erevan Physics Institute, 375036 Erevan, Armenia

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In an instanton "liquid" there is a mechanism for momentum transfer between quarks via this medium. This interaction allows one to describe the low-energy characteristics of the ρ meson (m_ρ, f_ρ) without going beyond the approximation which incorporates only zero modes. © *1995 American Institute of Physics.*

Large-scale instanton fluctuations of vacuum may explain the mechanism for the spontaneous breaking of chiral invariance.[1–8] The explanation is based on the idea that a mixing and a delocalization of instanton zero modes I and anti-instanton zero modes $\bar{I}$ can lead to a spontaneous breaking of chiral invariance. A vacuum–(charged $I\bar{I}$ "liquid") model has been constructed. However, the approach opened up in those earlier studies ignores the fundamental fact that the quarks are in a continuous medium, so they necessarily exchange momentum.

Here is the physical picture: The medium is treated as a discrete set of instanton–anti-instanton pairs. A transition to the continuous limit is taken only in the final stage of the calculations. That picture is valid for the propagation of a quark in an instanton vacuum, since the "slowing" effects due to the medium, which gives rise to an effective quark mass $M(\rho)$, do not depend on the order in which the thermodynamic limit is taken ($N\rightarrow\infty$, $V\rightarrow\infty$, $N/V=\text{const}$, where N is the number of pseudoparticles, and V is the 4-volume). The same approach is valid for describing a Goldstone mode (a pion). In this case a fundamental role is played by the interaction of quarks with vacuum, and the exchange of momentum between quarks is unimportant.

In an analysis of other (non-Goldstone) modes, e.g., vector particles, the continuity of the medium plays a governing role, since it is through this continuous medium that the quarks exchange momentum and may form a bound state. The problem of dealing correctly with both effects—the interaction of the quark with large-scale instanton fluctuations of vacuum and the interaction of quarks with each other—can be solved by taking the expectation value of the complete quantum chromodynamic (QCD) Lagrangian in a statistical ensemble of pseudoparticles, rather than of individual correlation functions, as in Refs. 3–5 and 8. This problem was partly solved in an earlier paper by the present authors.[9]

In this letter we wish to demonstrate that the effective action which arises in the course of such an averaging procedure leads to a correct description of a low-lying bound state in the vector channel (a ρ meson).

The modified QCD partition function in Euclidean space is

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$$Z=\int D\psi D\psi^{+}D\chi D\chi^{+}\exp\left[-S_0+N_{+}\ln\frac{1}{V}\int d^4zK_{+}+N_{-}\ln\frac{1}{V}\int d^4zK_{-}\right], \quad (1)$$

$$S_0=-\int[\psi^{+}i\hat{\partial}\psi+\chi^{+}i\hat{\partial}\chi]d^4x ,$$

$$K_{+}=\frac{1+\frac{i}{mN_c}(\psi_L^{+}\psi_L)(z)}{1+\frac{1}{mN_c}(\chi_L^{+}\chi_L)(z)}+\frac{\frac{i^2}{m^2N_c^2}(\psi_L^{+}\chi_L)(z)x(\chi_L^{+}\psi_L)(z)}{\left[1+\frac{1}{mN_c}(\chi_L^{+}\chi_L)(z)\right]^2}, \quad (2)$$

where $K_{-}=K_{+}(L\rightarrow R)$, $N_{+(-)}$ is the number of instantons (of anti-instantons), and the spin-1/2 boson fields χ and χ^{+} in (1) are actually new variables, into which the collective variables transform when an average is taken in the statistical ensemble of pseudoparticles. The parentheses here have the meaning

$$(A_i^{a+}B_j^b)(z)\equiv\int A_i^{a+}(k)B_j^b(q)\exp[iz(k-q)]a(k)a(q)\frac{d^4kd^4q}{(2\pi)^8},$$

where $a(k)=|k|\Phi(k)$. The function $\Phi(k)$ is associated with the Fourier transform of zero modes and has the following asymptotic regime:[3]

$$\phi(k)=\begin{cases}-2\pi\rho/|k|, & k\rho\ll 1,\\ -12\pi/k^4\rho^2, & k\rho\gg 1.\end{cases} \quad (3)$$

As was shown in Ref. 9, the action contains an effective mass $M(p)\sim(N_c)^0$:

$$M(p)=\frac{N\epsilon}{2VN}a^2(p), \quad (4)$$

where ϵ is determined by the self-consistency condition[3,9]

$$1=\frac{4VN_c}{N}\int\frac{d^4p}{(2\pi)^4}\frac{M^2(p)}{p^2+M^2(p)}, \quad (5)$$

and has a value[3] $\epsilon\simeq 85\ \mathrm{MeV}^{-1}$.

The expectation value $(\chi_L^{+}\chi_L)(0)$ is defined by[9]

$$\left\langle\left(1-\frac{(\chi_L^{+}\chi_L)}{mN_c}\right)^{-1}\right\rangle=m\epsilon. \quad (6)$$

From this point on, the procedure of singling out the interaction potential is different from that of Ref. 9. It is more conventional (Ref. 10, for example) and can be summarized as follows. We transform to the new fields χ'^{+}, χ', which have a zero vacuum expectation value:

$$\chi_{Li}^{a+}\chi_{Lj}^{b}=\chi_{Li}^{\prime a+}\chi_{Lj}^{\prime b}+\frac{1}{4N_c}\langle\chi_L^{+}\chi_L\rangle\delta_{ab}\delta_{ij}.$$

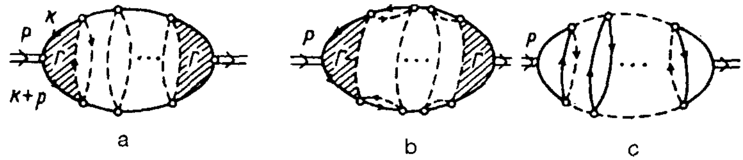

FIG. 1.

Ignoring the z dependence in the denominators in (2) (Ref. 9), expanding the ln in the leading approximation in $1/N_c$, and using (6) in the chiral limit $m \to 0$, we find the following result for the effective action in the instanton medium:

$$S=\int [\psi^+[\hat{k}-iM(k)]\psi+\chi^+k\chi]\frac{d^4k}{(2\pi)^4}$$

$$+2i\frac{V}{N}\int \{[\psi_L^+(k)\psi_L(q)][\chi_L^+(p)\chi_L(l)]-[\psi_L^+(k)\chi_L(q)][\chi_L^+(p)\psi_L(l)]\}$$

$$\times[M(k)M(q)M(p)M(l)]^{1/2}\delta^4(k+p-q-l)\frac{d^4kd^4pd^4qd^4l}{(2\pi)^{12}}+(L\to R). \quad (7)$$

We have omitted the primes from the fields χ and χ^+ in (7). Our reason for retaining the potential $\sim 1/N_c$ (we recall that we have $N \sim N_c$) is that it leads to results $\sim N_c$ for the correlation functions of colorless currents (the suppression of $1/N_c$ is offset by powers of N_c from fermion ghost loops).

The two different terms in potential (7) differ in physical nature. The first term, in which the quark fields and ghost fields have been factored, describes an interaction with large-scale vacuum fluctuations. This first term is similar to the "longitudinal forces" of the Stokes resistance which arise in a liquid (Ref. 11, for example). It can be shown that these forces are responsible for the onset of a Goldstone mode (a pion; the results of the calculations agree with those of Ref. 3). The second term, not factored, makes it possible to describe a transfer of momentum between quarks. This second term is therefore responsible for the formation of a bound state. This term is reminiscent in nature of the "transverse friction forces" which arise in a viscous liquid in the course of the motion of two solid objects.[11]

Figure 1 shows the type of diagram which arises when the transverse transfer, which contributes to the correlation function in the vector channel, is taken into account (the "longitudinal" forces are suppressed as $1/N_c$). Our purpose here, however, is to single out one-meson states. Accordingly, if we arrange conditions such that a cut of any diagram would contain only two quarks, and the ghost ends should not form a white subsystem, i.e., if we recall the diagram-planarity rule plus the minimal number of quark loops in the limit $N_c \to \infty$, which is of a general nature,[12] then the entire class of diagrams of the type in Fig. 1c can be discarded. This approach actually corresponds to ignoring the continuum contribution to the correlation function.

The coupling part of the correlation function is thus determined by the diagrams in Fig. 1, a and b, which can be summed in the standard way with the help of the Fredholm equation. The quantity $\Pi_{\mu\nu}(P)$ is of the form

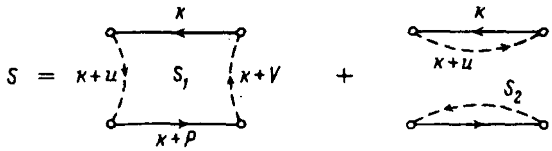

FIG. 2.

$$\prod_{\mu\nu}(P)=-\int\frac{d^4k_1d^4k_2}{(2\pi)^4}\,\delta(k_1-k_2+p)j_\mu(k_1)j_\nu(k_2)e^{-S}=2N_c\lambda\int\Gamma_\mu(u,P)$$

$$\times\Gamma_\nu^+(u,P)\frac{d^4u}{(2\pi)^4}+2N_c\lambda^2\int\Gamma_\mu(u,P)R(u,v)\Gamma_\nu^+(v,P)\frac{d^4ud^4v}{(2\pi)^8},\qquad(8)$$

where $\lambda=4V^2N_c/N^2$. The vertex Γ_μ is defined in Fig. 1 (see the hatching):

$$\Gamma_{\mu\pm}(u,P)=\int\mathrm{Sp}\left(\frac{\hat{k}+iM}{k^2+M^2}\,\gamma_\mu\frac{\hat{k}+\hat{P}+iM}{(k+P)^2+M^2}\,\frac{1\mp\gamma_5}{2}\,\frac{\hat{u}+\hat{k}}{(u+k)^2}\,\frac{1\pm\gamma_5}{2}\right)$$

$$\times\sqrt{M(k)M(k+P)}M(u+k)\frac{d^4k}{(2\pi)^4},\quad\Gamma_{\mu+}=\Gamma_{\mu-}=\Gamma_\mu,$$

where the function $R(u,v)$ satisfies the Fredholm equation

$$R(u,v)=S(u,v)+\lambda\int S(u,k)R(k,v)\frac{d^4k}{(2\pi)^4}.\qquad(9)$$

The kernel $S(u,v)$ is shown in Fig. 2:

$$S_1\simeq\int\mathrm{Sp}\left(\frac{\hat{k}}{k^2+M^2}\,\frac{1\pm\gamma_5}{2}\,\frac{\hat{u}+\hat{k}}{(u+k)^2}\,\frac{1\mp\gamma_5}{2}\,\frac{\hat{k}+\hat{P}}{(k+P)^2+M^2}\,\frac{1\pm\gamma_5}{2}\,\frac{\hat{v}+\hat{k}}{(v+k)^2}\,\frac{1\mp\gamma_5}{2}\right)$$

$$\times M(k)M(k+P)M(u+k)M(v+k)\frac{d^4k}{(2\pi)^4},$$

$$S_2\simeq N_c\left(\int M(k)M(u+k)\mathrm{Sp}\frac{\hat{k}}{k^2+M^2}\,\frac{1\pm\gamma_5}{2}\,\frac{\hat{u}+\hat{k}}{(u+k)^2}\,\frac{1\mp\gamma_5}{2}\,\frac{d^4k}{(2\pi)^4}\right)^2.\qquad(10)$$

We will conduct the calculation in the leading approximation in the packing parameter ρ/R of the medium, where R is the average distance between pseudoparticles ($\rho/R\approx1/3$; Ref. 2). In this approximation we have $\rho M\ll1$, $\rho u\ll1$, and $\rho v\ll1$. Since $M(k)$ is a rapidly decreasing function [see (3)] at large values of k, we make the substitution $M(k_i)\to M(0)$ in S_1, and we introduce a cutoff at the upper limit of the integration, $k\leqslant1/\rho$ (Ref. 3).

When the external momentum of the particle is $P\to0$, we find

$$S(u-v)_{P\to 0}=\frac{M^4(0)}{8\pi^2}\ln[\rho^2(u-v)^2]+\frac{1}{\lambda}(2\pi)^4\delta^4(u-v),$$

$$\Gamma_\nu(u,P)_{P\to 0}=\frac{M^2(0)}{16\pi^2}\left[(2u_\nu-P_\nu)\ln(\rho^2u^2+\rho^2M^2)+\tfrac{2}{3}P_\nu\frac{Pu}{u^2}\ln\frac{M^2}{u^2+M^2}\right]. \quad (11)$$

Since the kernel S is a function of the difference $(u-v)$, Eq. (9) can be solved by Fourier transforms:

$$R(x)=\mathrm{P}\,\frac{S(x)}{1-\lambda S(x)},$$

where P means the principal value, and

$$\prod_{\mu\nu}(P)=2N_c\lambda\int\Gamma_\mu(P,x)\Gamma_\nu^+(P,x)d^4x$$

$$+2N_c\lambda^2P\int\Gamma_\mu(P,x)R(x,P)\Gamma_\nu^+(x,P)d^4x.$$

Using (11), we find the following expression for the correlation function in the vector channel as $P\to 0$:

$$\prod{}_{\mu\nu}(P)=2N_c\lambda P\int\frac{\Gamma_\mu(P,x)\Gamma_\nu^+(P,x)}{1-\lambda S(x)}d^4x=-2N_cP\int\frac{\Gamma_\mu(P,x)\Gamma_\nu^+(P,x)}{S_1(x)}d^4x$$

$$=-\frac{N_c}{16\rho^2\pi^2}\{-I\delta_{\mu\nu}+P_\mu P_\nu[\tfrac{8}{3}\rho^2(\ln\rho^2M^2)^2]\},$$

$$I=\mathrm{P}\int_0^\infty\frac{zJ_2^2(z)dz}{2J_0(z)+zJ_1(z)-2}=-5.5. \quad (12)$$

Here $J_i(z)$ are the Bessel functions of the first kind. The integral was evaluated numerically.

In general, the correlation function in the vector channel is transverse. However, we have ignored the continuum contribution, retaining the contribution from only low-lying bound states.

It can also be shown (Ref. 10, Vol. 1) that in the limit $P\to 0$, (which corresponds to $x\to\infty$) the dominant contribution in the Källén–Lehmann representation is determined by one-particle states, and the correlation function in our case (in Euclidean space) is of the form[13]

$$\prod_{\mu\nu}(P)=-\Sigma\left(\delta_{\mu\nu}+\frac{P_\mu P_\nu}{m_i^2}\right)f_i^2. \quad (13)$$

We find the current matrix element in the following way: $\langle 0\bar{d}\gamma_\mu u\rho\rangle=\epsilon_\mu^\lambda f_\rho m_\rho$, where $f_{\rho\exp}=200$ MeV, and ϵ_μ is the polarization vector of the ρ meson. Taking into account that the f_i are small for excited states[14] $(\rho'...)$, we finally can write

$$\Pi_{\mu\nu}(P) = -\left(\delta_{\mu\nu} + \frac{P_\mu P_\nu}{m_\rho^2}\right) f_\rho^2. \tag{14}$$

Comparing (12) and (14), we find

$$f_\rho^2 = \frac{5.5N_c}{16\rho^2\pi^2}, \quad m_\rho^2 = \frac{3\times 5.5}{8\rho^2(\ln\rho^2M^2)^2}. \tag{15}$$

In other words, both the constant f_ρ and m_ρ are determined by quantities which are inversely proportional to the average sizes of a nonperturbative fluctuation ($1/\rho = 600$ MeV). They are in good numerical agreement with experiment: $f_\rho = 193$ MeV, $m_\rho = 797$ MeV. However, we need to bear in mind that the assumptions made here introduce an error on the order of 30% in the estimates. We would thus not place much emphasis on an exceptional numerical agreement with experimental data . We would repeat that the effective action derived here, (7), makes it possible to describe physically observable particles. The vector channel is described on the same basis as the pseudoscalar octet. Actually, our approach differs from approaches discussed previously[3,7] in that we have incorporated momentum transport due to the "viscosity" of the instanton "liquid." The vector channel in our approach has a nonzero contribution because of effects of permanent scattering in a continuous medium. Since our interaction mediators are a set of nonperturbative gluons, the interaction is an attraction. Because of this attraction, bound states can form.

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