Is there an excited state of the ^{3}H nucleus?

A. L. Barabanov
Kurchatov Institute Russian Science Center, 123182 Moscow, Russia

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It is suggested that the excited state of the ^{3}H nucleus which was recently observed in the reaction H(^{6}He,α) [D. V. Aleksandrov *et al.*, JETP Lett. **59**, 320 (1994)] has a spin and parity of $1/2^+$ and the same configuration as the ^{6}He ground state. Since the amplitude for an electromagnetic transition to the ground state of the triton is strongly suppressed, this excited state cannot be observed in radiative capture of neutrons by deuterons. A resonance due to the excited state may not be seen in elastic *nd* scattering because of a destructive interference of the phase shifts of the potential and resonant scattering.

Aleksandrov *et al.*[1] have observed a maximum in the cross section for the reaction H(^{6}He,α). This maximum was interpreted as an excited state of the ^{3}H nucleus with an energy $E^*=7.0\pm0.3$ MeV and a width $\Gamma^*=0.6\pm0.3$ MeV. This is an extremely interesting result, because there had previously been no significant indications of the existence of excited states of the triton.[2] There are, on the other hand, grounds for skepticism regarding the interpretation offered in Ref. 1. The energy E^* lies about 0.7 MeV above the threshold ($E_b=6.26$ MeV) for the breakup of the ^{3}He nucleus into a neutron and a deuteron. The total cross section for the *nd* interaction, however, has no maximum near an energy of 0.7 MeV with a width on the scale of Γ^* (Refs. 3 and 4). Furthermore, no anomalies near the proposed excited level of the triton have been observed in the radiative capture of neutrons by deuterons.[5] Aleksandrov *et al.*[1] cited a review[6] of theoretical papers predicting the existence of an excited state of the triton which would lie 0.5 MeV above the $n+d$ breakup threshold. That citation is not completely correct, however, since the topic discussed in Ref. 6 was an energy ($\sim$0.5 MeV) of a virtual state of the *nd* system lying near the $n+d$ threshold. The purpose of the present letter is to show that the interpretation offered in Ref. 1 may nevertheless be correct.

We begin our discussion with the possible configuration of the excited state of the ^{3}He nucleus observed in Ref. 1. According to the naive shell model with an oscillator potential, the 0^+ ground state of the ^{6}He nucleus is a configuration of two protons and two neutrons, which are in a $1s$ state and which constitute an α particle, along with two neutrons in a $1p$ state with a zero total angular momentum. We denote by $\mathbf{r}_1$ and $\mathbf{r}_2$ the radius vectors of these neutrons with respect to the α particle. The vector $\mathbf{r}_{12}=\mathbf{r}_1-\mathbf{r}_2$ then gives the relative positions of the neutrons, while $\mathbf{r}$ gives the coordinate of their center of mass with respect to the α particle. For normalized oscillator wave functions we thus have the identity

$$\frac{1}{\sqrt{3}}\sum_m (-1)^m \psi_{1pm}(\mathbf{r}_1)\psi_{1p-m}(\mathbf{r}_2)=\frac{1}{\sqrt{2}}[\psi_{2s}(\mathbf{r})\psi_{1s}(\mathbf{r}_{12})-\psi_{1s}(\mathbf{r})\psi_{2s}(\mathbf{r}_{12})]. \quad (1)$$

The function $\psi_{1pm}(\mathbf{r}_i)$ on the left side is written for a potential $\mu\omega^2 r_i^2/2$, where $\mu=m_n m_\alpha/(m_n+m_\alpha)$ is the reduced mass of a neutron and an α particle. On the right side we have the functions $\psi_{ns}(\mathbf{r}_{12})$ in a potential $\mu_{12}\omega_{12}^2 r_{12}^2/2$, where $\mu_{12}=m_n/2$ and $\omega_{12}=\omega m_\alpha/(m_n+m_\alpha)$, along with functions $\psi_{ns}(\mathbf{r})$, which correspond to a potential $\mu_r\omega_r^2 r^2/2$, where $\mu_r=2m_n m_\alpha/(2m_n+m_\alpha)$ and $\omega_r=\omega(2m_n+m_\alpha)/(m_n+m_\alpha)$. According to (1), a state of two neutrons in a $1p$ shell with a zero total angular momentum is equivalent to a superposition of $1s$ and $2s$ states in terms of the relative variables $\mathbf{r}_{12}$ and $\mathbf{r}$. Danilin *et al.*[7] have shown that this simple picture aggrees well with the actual configuration of the ground state of the ^{6}He nucleus, which is a weakly bound system of three particles, $\alpha+n+n$.

Let us assume that the excited state of the $p+n+n$ system observed in Ref. 1 has the same configuration [see (1)] as the ground state of the ^{6}He nucleus (the α particle is replaced by a proton). This will obviously be an excited state of the ^{3}He nucleus with spin and parity of $1/2^+$. The $1/2^+$ ground state in this model corresponds to two neutrons in a $1s$ state with respect to the proton. For oscillator wave functions we have

$$\psi_{1s}(\mathbf{r}_1)\psi_{1s}(\mathbf{r}_2)=\psi_{1s}(\mathbf{r})\psi_{1s}(\mathbf{r}_{12}) \quad (2)$$

with the same relations between μ, μ_{12}, μ_r and ω, ω_{12}, and ω_r as before, aside from the obvious replacement of m_α by the proton mass m_p.

If we adopt this hypothesis, we see why an excited state is not manifested in the reaction $d(n,\gamma)$. Only an $M1$ transition is allowed in terms of quantum numbers, but the leading term in the transition amplitude is zero by virtue of the orthogonality of the $2s$ and $1s$ states. This situation here is analogous to a $2s\rightarrow 1s$ transition in the hydrogen atom, in which case, incidentally, a one-photon transition is less probable than a two-photon transition.

We turn now to an analysis of elastic nd scattering. The resultant spin of the neutron and the deuteron takes on values of 1/2 and 3/2, so one distinguishes doublet and quartet channels in each partial wave. The scattering lengths in these channels are $a_2=0.65\pm0.03$ fm and $a_4=6.34\pm0.02$ fm (Ref. 2).

In the absence of excited states of the ^{3}H nucleus we would naturally expect the scattering of neutrons by deuterons at energies below the deuteron breakup threshold $E_d=2.23$ MeV to be a purely potential scattering. The scattering length should be on the order of the radius of the potential. Let us assume that this is indeed the situation in the quartet channel. The radius of the nd potential can be very large, because of the smeared nature of the deuteron. The radial wave function of the deuteron falls off at long range as $\sim\exp(-\gamma r)$, where $1/\gamma\sim5$ fm (Ref. 9). Adding the radius of the nucleon–nucleon interaction (~2 fm), we take the radius of the neutron–deuteron potential to be $R=7$ fm. Assuming that the potential itself is a spherical square well, and adjusting its depth in the quartet channel to fit the value $a_4=6.34$ fm, we find $U_4=7.58$ MeV. In such a well there is a $1s$ bound level with an energy of -3.98 MeV, but the Pauli principle obviously forbids a filling of this level in the quartet channel.

We now assume that the triton has an excited state of the $p+n+n$ type [see (1)] in the doublet channel. This state should be thought of as a closed inelastic channel coupled with the $n+d$ elastic channel. To see how an inelastic channel of this sort might influence observables in the elastic channel, we adopt a simple model of two-channel scattering.[10,11] In this model, a particle interacts with a system which has two states: a ground state with a zero energy and an excited state with an energy ϵ. A resonance corresponds to an excitation of the inner system accompanied by a transition of the incident particle to a bound state.

We take all the potentials to be spherical square wells of radius $R=7$ fm. We assume that the depth of the well in the elastic channel is $U_2^{(0)}$, while that in the inelastic channel is $U_2^{(1)}$. We denote by W the depth of the potential coupling of these channels. The equations for the radial s-wave functions of the elastic channel, $F^{(0)}(r)$, and of the inelastic one, $F^{(1)}(r)$, at $r<R$ are

$$\begin{cases} d^2F^{(0)}/dr^2+(2mU_2^{(0)}/\hbar^2)F^{(0)}+(2mW/\hbar^2)F^{(1)}+k^2F^{(0)}=0, \\ d^2F^{(1)}/dr^2+(2mU_2^{(0)}/\hbar^2)F^{(1)}+(2mW/\hbar^2)F^{(0)}+k_1^2F^{(1)}=0. \end{cases} \tag{3}$$

Here k is the wave number in the elastic channel corresponding to an energy $E=\hbar^2k^2/2m$ (m is a reduced mass), and $k_1=[2m(E-\epsilon)/\hbar^2]^{1/2}$ is the wave number in the inelastic channel. If $E<\epsilon$, the inelastic channel is closed, so we have $k_1=iq_1$, where $q_1=[2m(\epsilon-E)/\hbar^2]^{1/2}$. Outside the interaction region, at $r>R$, we have

$$\begin{cases} F^{(0)}(r)=\exp[i\delta_2(k)]\sin[kr+\delta_2(k)]/k, \\ F^{(1)}(r)=-iS^{(1)}\exp(ik_1r)/2(kk_1)^{1/2}, \end{cases} \tag{4}$$

where $\delta_2(k)$ is the phase shift for elastic scattering in the doublet channel.

Solutions of Eqs. (3) in the region $r<R$ which are regular at the origin are

$$\begin{cases} F^{(0)}(r)=A\sin\kappa r+A'\sin\kappa' r, \\ F^{(1)}(r)=A(\Delta/W)\sin\kappa r-A'(W/\Delta)\sin\kappa' r, \end{cases} \tag{5}$$

where $\kappa=[2m(U_2^{(0)}+E+\Delta)/\hbar^2]^{1/2}$, $\kappa'=[2m(U_2^{(1)}+E-\epsilon-\Delta)/\hbar^2]^{1/2}$, and $\Delta=[(U_2^{(0)}-U_2^{(1)}+\epsilon)^2/4+W^2]^{1/2}-(U_2^{(0)}-U_2^{(1)}+\epsilon)/2$. Joining the functions in (4) and (5) and also their derivatives at the point $r=R$, we can determine the coefficients of the radial wave functions [$A(E)$, $A'(E)$, and $S^{(1)}(E)$] and also the elastic-scattering phase shift $\delta_2(E)$.

The $s-$wave phase shift for elastic scattering can, as we know, be expressed in the following way in terms of the logarithmic derivative $\Phi_2(E)=R(dF^{(0)}/dr)/F^{(0)}$ of the elastic-channel wave function at the joining point:

$$\exp[2i\delta_2(E)]=\exp(-2ikR)\frac{\Phi_2(E)+ikR}{\Phi_2(E)-ikR}. \tag{6}$$

According to the usual definition,[11] a resonance occurs in a doublet channel at an energy E_2 such that we have $\Phi_2(E_2)=0$. Writing $\Phi_2(E)$ in the form $\Phi_2(E)=(E_2-E)/\gamma_2$ near this energy, we find a Breit–Wigner description of the resonance with a reduced width γ_2 and an energy-dependent width $\Gamma_2(E)=2kR\gamma_2$. Assuming that this parametrization of the logarithmic derivative is valid at energies $E\to 0$, we find $a_2=R(1-\gamma_2/E_2)$ for the

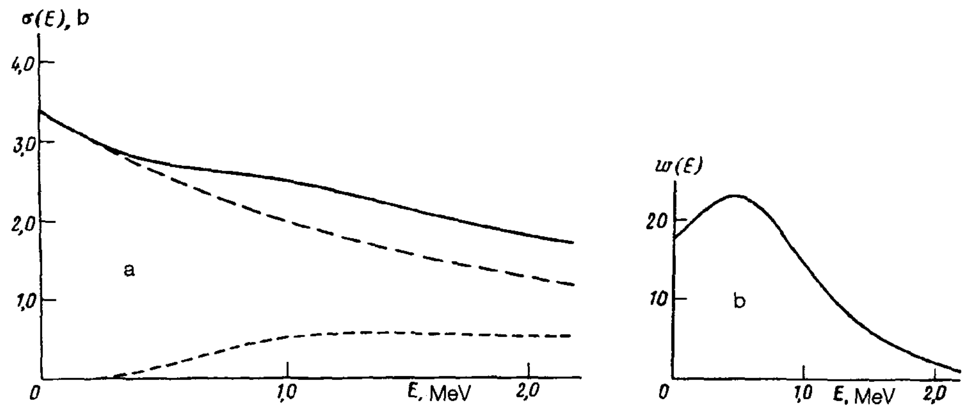

FIG. 1. The s-wave cross sections (a) and the square of the integral of the inelastic-channel wave function (b), as calculated in the model of an nd interaction, versus the energy E. In part a: Short-dash line—The cross section $\sigma_2(E)$ in the doublet channel; long-dash line—the cross section $\sigma_4(E)$, in the quartet channel; solid line—the total cross section.

scattering length. If we have $\gamma_2 \sim E_2$, we find a value of a_2 which is small in comparison with R . For the energy $E_2 \simeq 0.7$ MeV reported in Ref. 1, we thus find $\gamma_2 \simeq 0.6$ MeV and $\Gamma_2 \simeq 1.3$ MeV. The reduced width of a purely potential resonance in the model of a spherical square well, $\gamma^{\text{pot}} = \hbar^2/mR^2 \simeq 1.3$ MeV, is roughly twice the value found for γ_2 here. On the other hand, the width Γ_2 is slightly larger than the experimental estimate[1] Γ^*.

According to (6), the phase shift for elastic scattering in the doublet channel is the sum of the negative phase shift for potential scattering, $\delta^{\text{pot}}(E) = -kR$, and the positive shift for resonance scattering, $\delta^{\text{res}}(E) = \arctan[kR/\Phi_2(E)]$. If the radius of the nd interaction is indeed as large as we have assumed, then at an energy $E_2 \simeq 0.7$ MeV the potential phase shift would reach a value $\delta^{\text{pot}}(E_2) \simeq -1.05$, which is comparable in absolute value to the resonant phase shift $\delta^{\text{res}}(E_2) = \pi/2$. As a result, the total phase shift $\delta_2(E)$ definitely does not cross $\pi/2$ at the point of the resonance, telling us why there is no maximum in the nd-scattering cross section near the energy (E_2) of the proposed level.

To illustrate these qualitative arguments, we will go through a calculation on the basis of the model constructed above. We take ϵ to be the binding energy of the deuteron, 2.23 MeV, so that the energy interval $E < \epsilon$, in which there is purely elastic scattering in this model, coincides with the corresponding interval in the nd reaction. The remaining adjustable parameters in the doublet channel—the potential depths $U_2^{(0)}$, $U_2^{(1)}$ and W—are more than sufficient to reproduce the experimental value of the scattering length. Let us adopt $U_2^{(0)} = 5$ MeV and $U_2^{(1)} = U_4$ as an example. Fitting a_2 to the value of 0.65 fm, we find the value $W = 3.68$ MeV for the one remaining parameter. In this case the logarithmic derivative $\Phi_2(E)$ vanishes at $E_2 = 0.76$ MeV. The reduced and total widths of the resonance are $\gamma_2 = 0.57$ MeV and $\Gamma_2^0 = \Gamma_2(E_2) = 1.24$ MeV. Figure 1a shows the scattering cross sections $\sigma_2(E) = (4\pi/3k^2)\sin^2\delta_2(E)$ and $\sigma_4(E) = (8\pi/3k^2)\sin^2\delta_4(E)$ in the doublet and quartet channels, respectively, along with their sum, i.e., the total s-wave scattering cross section. The cross section $\sigma_2(E)$ reaches its largest values at energies above 1 MeV; it causes a slight smoothing of the cross section. This smoothing is quite clear on the experimental curve of Ref. 3 and is a consequence of an additional increase in the contributions of p and d waves with increasing energy.

In this model, the hypothetical resonance is thus not manifested as a clearly defined maximum in the elastic channel, because of the large phase shift for potential scattering. In the inelastic channel, in contrast, there is an increase in the function $F^{(1)}(r)$ near the energy E_2. The probability for a filling of the excited state of the triton in the reaction $H(^6He,\alpha)$ is proportional to the square modulus of the matrix element for the overlap of wave functions of the type in (1) for two neutrons in ^6He and ^3H. In the triton, a function of the type in (1) is the inelastic-channel wave function in the reaction $n+d$. The increase of this function at energies close to E_2 should therefore correspond to a maximum in the cross section for the reaction $H(^6He,\alpha)$. Part b of Fig. 1 illustrates the situation with the square integral of the function $F^{(1)}(r)$ according to a calculation in our model:

$$w(E)=\left(\int_0^R F^{(1)}(r)dr\right)^2. \tag{7}$$

The independent variable is the energy E. Interestingly, the position and width of the maximum agree qualitatively with the values of E^* and Γ^* found in Ref. 1.

Finally, we turn to the correspondence between the possible existence of a $1/2^+$ excited state of the ^3H nucleus and a virtual state of the nd system in the double channel.[6] One speaks in terms of a virtual state when there is a pole in the elastic-scattering amplitude in (6) in the lower half-plane of the complex variable k, on the imaginary axis. If the energy of the resonance, $E_2=\hbar^2k_2^2/2m$, is small, then the Breit–Wigner approximation $\Phi_2(k)=\hbar^2(k_2^2-k^2)/2m\gamma_2$ may be valid in some region of the complex k plane including the origin. The poles k_r of the amplitude in (6) which lie near the origin are in this case roots of the quadratic equation $(k_r)_{1,2}=-ik_2(\Gamma_2^0/4E_2)\pm k_2[1-(\Gamma_2^0/4E_2)^2]^{1/2}$. We see that, in this approximation, virtual states correspond to broad resonances, with $\Gamma_2^0>4E_2$ (both values of k_r lie on the imaginary axis in the lower half-plane). Incidentally, this is the situation which prevails in the singlet channel for nucleon–nucleon scattering. Resonances with widths Γ_2^0 comparable to E_2 correspond, as in the case we are discussing here, to poles on both sides of the imaginary axis.

In reality, even at a small value of E_2, the deviation of $\Phi_2(k)$ from a Breit–Wigner form may grow rapidly as we move away from the real k axis. Accordingly, a resonance with a width $\Gamma_2^0\sim E_2$ may correspond to a pole on the imaginary axis, i.e., to a virtual state. In the model which we are discussing here, however, this is not the case. Using the parameter values fixed above for the two-channel model for doublet nd scattering, we find, through a direct calculation, the poles of amplitude (6), i.e., the zeros of the expression $\Phi_2(k)-ikR$ in the complex k plane. Like the roots of the quadratic equation written above, they turn out to lie in the lower half-plane, in symmetric positions with respect to the imaginary axis. Corresponding to these poles are complex energies $(E_r)_{1,2}=\hbar^2(k_r)_{1,2}^2/2m$ on a nonphysical sheet; here we have $\mathrm{Re}(E_r)_{1,2}=0.54$ MeV and $\mathrm{Im}(E_r)_{1,2}=\pm$ 0.76 MeV. In this model, a resonance in nd scattering (Fig. 1b) thus corresponds to poles of the S matrix which are close to the imaginary axis. The energies on the nonphysical sheet are on the order of ~ 0.5 MeV at these poles. In view of the extreme simplicity of the model which we have been using here, it is fair to say that there is a qualitative agreement with the results of the papers discussed in Ref. 6.

In summary, it has been shown here that the interpretation of the maximum in the cross section for the $H(^6He,\alpha)$ reaction which was proposed in Ref. 1—an excited state of the ^3H nucleus—contradicts neither existing experimental data nor theoretical ideas regarding the poles of the S matrix in the double channel of the nd interaction. In the elastic nd scattering channel, this excited state is apparently not seen because of a destructive interference of the phase shifts for potential and resonant scattering due to the smeared nature of the deuteron and, correspondingly, the anomalously large range of the nd potential. According to ideas which have been offered regarding the structure of the excited state, the cross section for the $d(n,\gamma)$ reaction is suppressed because the amplitude of the $M1$ transition to the ground state of the triton is strongly forbidden. At the same time, the hypothesis that there is a similarity between the structure of the excited state of the ^3H nucleus and the structure of the ground state of the ^6He nucleus leads to a natural explanation for the observed[1] sensitivity of the cross section for the $H(^6He,\alpha)$ reaction to this state.

Our basic conclusion here is that systematic three-body calculations on nd scattering should incorporate the energy dependence of the wave functions of the inelastic channels, which are structurally similar to the configuration in (1). There is also obvious interest in evaluating the possibility of an excitation of the triton in inelastic scattering of electrons by virtue of $E0$ transitions and in extending this analysis to the $p+d$ system and the ^3He nucleus.

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