Generation of flexural waves in a solid by an intense nanosecond electron beam

A. L. Bardenshteĭn, V. I. Bykov, and D. I. Vaĭsburd[1)]
Institute of High-Current Electronics, Siberian Branch of the Russian Academy of Sciences, 634055 Tomsk, Russia

(Submitted 21 November 1994)
Pis'ma Zh. Eksp. Teor. Fiz. **61**, No. 2, 96–100 (25 January 1995)

Intense flexural waves excited in a solid plate by an intense electron beam have been observed and studied. Experimental conditions under which flexural and longitudinal rarefaction waves can be observed simultaneously by laser interferometry have been determined. Corresponding experiments have been carried out. The results are used to determine elastic moduli and Grüneisen parameters of various solid materials, including composites. © *1995 American Institute of Physics.*

One would not be suprised to find that flexural waves result from the nonuniform pulsed bombardment of a solid plate by an electron beam. We were interested in determining why such waves had not been observed in some earlier, quite competent studies.[1–6] Analyzing the situation, we found some optimum experimental conditions under which intense flexural and longitudinal rarefaction waves can be observed simultaneously. This experimental situation proved favorable for accurate measurements of elastic constant and Grüneisen parameters of various solid materials, including composites.

A theory for flexural waves excited in a plate by an electron beam of nanosecond length has been derived on the basis of the linear theory of thermoelasticity under the following assumptions: The circular isotropic plate is bombarded in a coaxial geometry by a uniform cylindrical beam of smaller diameter. The energy input is instantaneous. Thermal conductivity is ignored; the corresponding time scale is several orders of magnitude longer than the period of the flexural vibrations. The latter period is in turn several orders of magnitude longer than the radiation pulse. In a cylindrical coordinate system, the equation of flexural elastic vibrations is, according to Refs. 7–9,

$$\frac{\partial^2\zeta}{\partial t^2}+\frac{h^2c_E^2}{12}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right)\right]^2\zeta=-\Gamma\frac{1-2\sigma}{1-\sigma}\left[\delta'(r_0-r)-\frac{1}{r}\delta(r_0-r)\right]M, \tag{1}$$

where $\zeta=\zeta(r,t)$ are the z coordinates of the points of the neutral surface (the buckling of the plate), $c_E=\sqrt{E/\rho(1-\sigma^2)}$ is the velocity of longitudinal waves propagating in a direction perpendicular to z, E is the Young's modulus, σ is the Poisson ratio, ρ is the density, h is the thickness of the plate, Γ is the Grüneisen parameter, r_0 is the beam radius, δ is the Dirac δ-function, δ' is its derivative, $D(z)$ is the longitudinal distribution (profile) of the absorbed dose in the plate, and

$$M = h^{-1} \int_{-h/2}^{h/2} zD(z)dz \tag{2}$$

is the first central moment of this distribution.

For a plate which is undeformed and at rest before the bombardment, and whose edges are pinned, the initial and boundary conditions are

$$\zeta(r,0+)=0, \quad (\partial\zeta/\partial t)|_{t=0+}=0, \quad \zeta(R_0,t)=0, \quad (\partial\zeta/\partial r)|_{r=R_0}=0,$$

$$|\zeta(0,t)|<\infty, \quad |[r^{-1}\partial(r\partial\zeta/\partial r)/\partial r]|_{r=0}<\infty, \tag{3}$$

where R_0 is the plate radius.

A solution of Eq. (1) which satisfies boundary conditions (3) is

$$\zeta(r,t)=A\left\{\zeta_0(r)+\sum_{n=1}^{\infty}\gamma_n[I_0(k_n)J_0(k_nr/R_0)-J_0(k_n)I_0(k_nr/R_0)]\cos\omega_n t\right\}. \tag{4}$$

Here

$$A=-12\Gamma\rho(1-2\sigma)(1+\sigma)MR_0^2/h^2E; \tag{5}$$

$$\zeta_0(r)=(2R_0)^{-2}\begin{cases}(1-r_0^2/R_0^2)r^2-2r_0^2\ln(R_0/r_0), & 0\leq r\leq r_0,\\ (1-r^2/R_0^2)r_0^2-2r_0^2\ln(R_0/r), & r_0<r\leq R_0;\end{cases} \tag{6}$$

$$\omega_n=hc_Ek_n^2/2\sqrt{3}R_0^2; \tag{7}$$

ω_n is the frequency of the nth harmonic of the flexural vibrations; k_n is the nth positive real root of the equation

$$J_0(k)I_1(k)+J_1(k)I_0(k)=0; \tag{8}$$

$J_l(k)$ and $I_l(k)$ are Bessel functions; and the coefficients γ_n are found from the following expansion of the constant component ζ_0:

$$\zeta_0(r)=-\sum_{n=1}^{\infty}\gamma_n[I_0(k_n)J_0(k_nr/R_0)-J_0(k_n)I_0(k_nr/R_0)]. \tag{9}$$

Figure 1 shows some typical time evolutions of the buckling of the center of the plate.

The calculations yield the following results on the flexural waves. 1) Their amplitude is proportional to the absorbed beam energy per unit volume according to (4)–(6). 2) It increases with the plate diameter, and it falls off with the plate thickness more rapidly than h^{-2} but more slowly than h^{-3}, according to (5) and (2). 3) The constant component ζ_0 (the static buckling), the first harmonic, and the second harmonic dominate the flexing of the plate. 4) The sign of the static buckling is determined by the first moment of the dose distribution, (2). If the range of the beam is much shorter than the plate thickness, the plate flexes in the direction opposite the beam. 5) The fine structure of the flexing is shaped by harmonics higher than the first. 6) Their amplitudes fall off in an approximately exponential manner with increasing harmonic index. 7) The flexural vibrations are

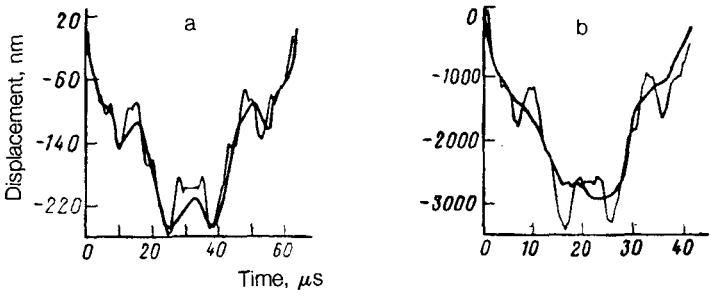

FIG. 1. Theoretical (light curves) and experimental (heavy curves) time dependence of the displacements of the centers of plates of quartz glass (a) and the alloy D16T (b). The diameter of the samples is 2 cm, and their thicknesses are 1 mm (a) and 0.94 mm (b). The beam diameter is 5 mm. The electron fluxes are 4.89×10^{13} cm^{-2} (a) and 1.88×10^{13} cm^{-2} (b).

not strictly periodic, since the frequencies of the harmonics are not multiples of each other, but the deviations from periodicity amount to less than 1%.

We are now in a position to state why flexural waves were not seen in earlier studies. The primary reasons are the large thickness of the samples and the short observation time interval. The apparatus was adjusted in such a way that it was possible to clearly observe the compression–expansion pulse of the longitudinal rarefaction wave, which propagated along the z axis at a velocity c_L. The average electron energy in the beams was 0.1–3 MeV. The average range of the beams was $Z_e=0.1-4$ mm. The duration of the longitudinal pulse was $Z_e/c_L=10^{-8}-10^{-6}$ s. The period of the flexural vibrations is on the order of $10^{-5}-10^{-4}$ s, according to (7). If the observation time interval were selected in such a way that the longitudinal pulse could be seen clearly, then one would simultaneously measure only a small fragment of the flexural wave, which plays the role of a nearly constant background. Furthermore, if the primary longitudinal pulse is not to be mixed up with reflected pulses, the thickness of the sample must be greater than the range of the beam. In the first studies,[1–4] the electron beams had comparatively high energies, 1–3 MeV. The range of the beam and the corresponding thickness of the sample were a few millimeters. The amplitudes of the flexural waves were accordingly small. In some later studies,[5,6] the beam electrons had a low energy, 0.1–0.4 MeV, and the samples were thin. The detector was a piezoelectric transducer. In this case the sample and the transducer form a very tight layered structure which has large total thickness and which greatly suppresses the generation of flexural waves, while effectively transmitting longitudinal waves.

Here are the most favorable conditions for exciting intense flexural waves by an intense electron beam in a solid. The sample must be thin. Correspondingly, the electron energy must be low, 0.1–0.3 MeV. The diameter of the unpinned surface of the sample should be as large as possible; at a minimum, it must be far greater than the thickness. The observation time must be longer than the period of the first harmonic of the flexural vibrations. The stresses in the longitudinal and flexural waves should be comparable in magnitude, while the displacement amplitudes in the flexural waves may be far larger.

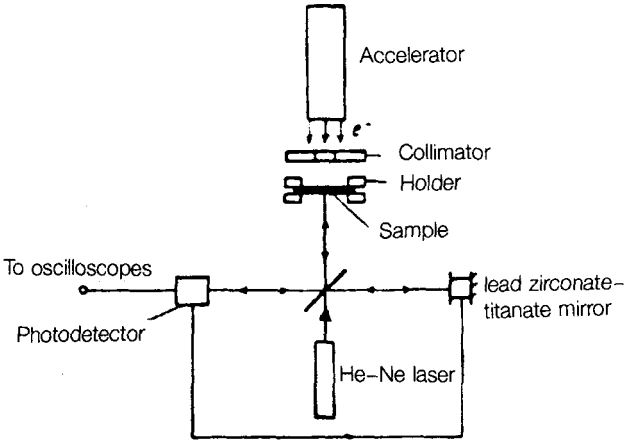

FIG. 2. Experimental layout.

Accordingly, an ideal detector of flexural waves would be a laser interferometer, which gives the displacement of the sample surface directly, in absolute units (wavelengths of the laser light).

Figure 2 shows the experimental layout. The samples are circular plates of copper, quartz glass, alkali halide crystals, silicon, the aluminum alloy D16T, and carbon-fiber composites. Their diameters are 2–3 cm, and their thicknesses 0.5–1 mm. The central part of a plate is bombarded through a collimator with a diameter of 3, 5, or 7 mm. A small, pulsed, high-current accelerator[10,11] is used. The parameters of the electron beam are varied over the intervals 0.20–0.35 MeV, 5–30 ns, and 0.1–1000 A/cm^2. The edge of the sample is pinned firmly along the entire circle. The free surface (subjected to polishing and ion-beam sputtering beforehand) serves as a mirror in the measurement arm of the Michelson laser interferometer. The laser used is an LG-79 ($\lambda = 0.63$ μm, average power of 15 mW). An important part of the apparatus is an optoelectronic unit for suppressing the effect of low-frequency vibrations of the apparatus. This unit includes a reference mirror, mounted on a ceramic piezoelectric cylinder, a photodetector, and a differential amplifier. The time evolution of the light intensity during the displacement of the sample surface is detected with a resolution of 2 ns by an FD-256 photodetector. The signal is then sent through a wide-band amplifier to the inputs of two oscilloscopes. The first, with a sweep time of 0.1–10 μs, is used to measure the longitudinal wave and the initial stage of the flexural wave. The second, with a sweep time of 20–200 μs, is used for full-scale observation of the flexural wave. The error of the displacement measurements is no worse than $\lambda/32 \approx 0.02$ μm.

Figure 1 compares theoretical and experimental results. We classified the materials studied as either elastic (solid, brittle) or plastic (soft, viscous). For the elastic materials (quartz glass, silicon, and alkali halide crystals), the discrepancy between theory and experiment is less than 10%. There is agreement in both the basic structure of the flexing, shaped by the static buckling in the first harmonic, and the fine structure, shaped by the higher harmonics (Fig. 1a). For the plastic materials (copper, the aluminum alloy, and the composites), theory and experiment agree only with regard to the basic structure of the

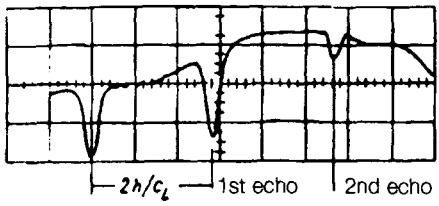

FIG. 3. Interferogram of displacements of the lateral surface (not bombarded) of a plate of the alloy D16T with a diameter of 2 cm and a thickness of 0.94 mm. The beam diameter is 5 mm, and the electron flux 1.88×10^{13} cm^{-2}. The horizontal scale is 0.1 μs per large division, and the vertical scale is 0.2 V per large division.

flexing. The fine structure is smoothed out experimentally (Fig. 1b). The reasons are the plastic deformation and the associated viscosity.[9] In general, the measurements confirm the basic properties of the flexural waves listed above: the dependence of the amplitude on the density of absorbed energy, the diameter of the sample, the thickness of the sample, etc. To the extent that the theoretical results agree with the experimental results, flexural vibrations of the plates excited by an electron beam can be regarded as thermoelastic and can be used to determine elastic constants of the materials. The procedure which we have developed exploits the advantages offered by laser interferometry of two types of rarefaction waves simultaneously: flexural waves and longitudinal waves, excited by a common radiation pulse. Here is the sequence of the basic operations: 1) The primary longitudinal pulse and several of its reflections are measured. The quantity $c_L = 2h/\tau_L$ is determined. Here τ_L is the time interval between the maxima of two neighboring pulses (Fig. 3). 2) The measured time evolution of the buckling, $\zeta_{\text{exp}}(r,t)$, is used to determine the period of the first harmonic, T_1, and also $c_E = 4\sqrt{3}\pi R_0^2/(k_1^2 h T_1)$. 3) Any two elastic moduli, e.g., the Poisson ratio and the Young's modulus, are expressed in terms of c_L and c_E: $\sigma = 1 - a^{-2}(1 - \sqrt{1-a^2})$, where $a = c_E/c_L$, and $E = \rho c_E^2(1-\sigma^2)$ (Ref. 7). 4) An integration of $\zeta(r,t)$ over the period of the flexural vibrations yields, according to (4) and (5),

$$\int_0^T \zeta(r,t)dt \approx -\frac{12R_0^2\rho\Gamma}{h^2E}(1-2\sigma)(1+\sigma)T_1 M\zeta_0(r), \tag{10}$$

within terms whose contribution is small (0.02–0.8%, depending on the value of r), because the flexural vibrations are quasiperiodic. The expression of the Grüneisen parameters is thus

$$\Gamma(r) = -\frac{h^2E}{12R_0^2\rho(1-2\sigma)(1+\sigma)T_1 M\zeta_0(r)}\int_0^{T_1}\zeta_{\text{exp}}(r,t)dt. \tag{11}$$

Measurements of the buckling of the sample at various distances from the center, r, yield values of Γ which are approximately the same: the discrepancy does not exceed 0.3%. The error in the determination of Γ is due primarily to the absolute calibration of the dose-distribution moment M. For a fixed experimental geometry, M is proportional to the surface density of absorbed beam energy over the pulse (in J/m^2). The latter is usually

TABLE I.

Material		E, GPa	σ	Γ
Quartz	(meas.)	49	0.33	0.042
glass	(tab.)	47	0.34	0.033
D16T	(meas.)	729	0.33	1.90
	(tab.)	720	0.32	2.13
Composite	(meas.)	339	0.46	0.52

measured with a calorimeter, within an error of about 12%. This error could be reduced substantially by using samples of materials with known values of Γ, e.g., alkali halide crystals, as references.

The procedure described above has been tested on samples of quartz glass, the alloy D16T, and a carbon-fiber composite with a density of 1.35 g/cm^3 (Table I). The measured properties of the first two of these materials agree well with handbook data.[12]

We are indebted to the Russian Fund for Fundamental Research for its support (Grant RFFI-94-02-05514-a).

[1)]E-mail: vaysburd@lnpspb.tomsk.su

[1] R. M. White, J. Appl. Phys. **34**, 2123 (1963).
[2] R. A. Graham and R. E. Hutchison, Appl. Phys. Lett. **11**, 69 (1967).
[3] F. C. Perry, J. Appl. Phys. **41**, 5017 (1970).
[4] R. B. Oswald *et al.*, J. Appl. Phys. **42**, 3463 (1971).
[5] A. A. Bespal'ko and G. I. Gering, Pis'ma Zh. Tekh. Fiz. **3**, 152 (1977) [Sov. Tech. Phys. Lett. **3**, 60 (1977)].
[6] A. A. Bespal'ko and G. I. Gering, Zh. Tekh. Fiz. **50**, 213 (1980) [Sov. Phys. Tech. Phys. **25**, 130 (1980)].
[7] L. D. Landau and E. M. Lifshitz, *Theory of Elasticity* [in Russian] (Nauka, Moscow, 1987).
[8] B. A. Boley and J. Weiner, *Theory of Thermal Stresses* (Wiley, New York, 1960).
[9] H. Parkus, *Nonstationary Thermal Stresses* [in German] (Springer–Verlag, Vienna, 1959).
[10] B. M. Koval'chuk *et al.*, Prib. Tekh. Eksp., No. 4, 15 (1981).
[11] D. I. Vaĭsburd *et al.*, *High-Energy Solid-State Electronics* [in Russian] (Nauka, Novosibirsk, 1982).
[12] *Handbook of Physical Quantities* [in Russian] ed. by I. S. Grigor'ev and E. Z. Meĭlikhov (Energoatomizdat, Moscow, 1991).

Translated by D. Parsons