Radical change in spin precession frequency of ultrarelativistic electrons in a circularly polarized wave due to interaction with the radiation field

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The interaction with the radiation field leads to an increase in the spin precession frequency of ultrarelativistic electrons in a circularly polarized wave. The increase is by several orders of magnitude. This interaction also causes a substantial change in the dependence of this frequency on the electron energy and the wave frequency. This increase could actually be observed in intense laser and crystal fields, starting at energies of a few GeV. © *1995 American Institute of Physics.*

Effects of the quantum electrodynamics (QED) of intense fields[1,2] continue to attract research interest. The past decade has seen a huge number of studies of such effects in intense crystal fields.[3] There are plans to observe these effects in the fields of intense laser pulses.[4] One of the most important predictions[1,2] concerns a possible change in the anomalous magnetic moment of the electron (e^-)[or of the positron (e^+)] in a uniform field and in the field of a laser wave. This change could be detected by virtue of its effect on spin precession. Precession also arises when an electron is subjected to a circularly polarized wave, but the Bargmann–Michel–Telegdi equation tells us that the frequency of this precession is smaller by a factor of γ (γ is the Lorentz factor) than the precession frequency in a transverse uniform field of the same strength. Nevertheless, we will show that, if the energy of the scattered photon is comparable to the electron energy ϵ, the interaction with the radiation field leads to a significant increase (by a factor $\gamma \sim 10^5$) in the spin precession frequency. We will show that a description of the precession on the basis of the Bargmann–Michel–Telegdi equation is completely inadequate. At these energies, this equation[5] can be simplified:[1)]

$$d\boldsymbol{\zeta}/dt \simeq 2\mu'\{[\boldsymbol{\zeta}\times\mathbf{H}]+(\mathbf{v}\cdot\mathbf{H})[\mathbf{v}\times\boldsymbol{\zeta}]+[\boldsymbol{\zeta}\times[\mathbf{E}\times\mathbf{v}]]\}, \tag{1}$$

where $\mu' \simeq (\alpha/2\pi)e/2m$, e, and m are the anomalous magnetic moment, charge, and mass of the $e^{\pm}$; $\boldsymbol{\zeta}$ and $\mathbf{v}$ are its spin vector and velocity vector; $\alpha \simeq 1/137$; and $\mathbf{H}$ and $\mathbf{E}$ are the electric and magnetic fields.

We consider the interaction of ultrarelativistic $e^{\pm}$'s with a circularly polarized wave of frequency Ω propagating in the opposite direction. We impose a limitation on the transverse component of the velocity induced by the wave,[6] i.e., $\mathbf{v}_\perp = -e\mathbf{H}/\epsilon\Omega$; specifically, we impose the condition $v_\perp \leqslant 1/\gamma$ or $\xi = |e|H/m\Omega \leqslant 1$ (under the condition $v_\perp \gg 1/\gamma$, the interaction of the $e^{\pm}$ with the wave is described in the uniform-field approximation[1,2]). Substituting $\mathbf{v}_\perp$ into (1), we easily verify that the spin of the $e^{\pm}$ precesses around the direction of the velocity in the field of the circularly polarized wave. The frequency of this precession is $4\mu' eH^2/\Omega\epsilon$. With $\xi \sim 1$, this frequency is smaller by a factor $\Omega\epsilon/2|e|H \sim \gamma \sim 10^5$ than the frequency $2\mu' H$ $(2\mu' E)$ of the spin precession in a uniform transverse field of the same strength. It falls off with increasing Ω and ϵ.

However, there is reason to believe that relativistic suppression of this type does not always occur, and that the precession frequency in a circularly polarized wave becomes comparable to its value in a transverse field as the values of Ω and ϵ increase. Emission in the field of a circularly polarized wave is not suppressed in comparison with that in the case of a uniform transverse field. As in the latter case, the dependence of the emission probability in a circularly polarized wave on the longitudinal component of the $e^\pm$ spin is strongest when the energy of the emitted photons is comparable to the energy of the $e^\pm$. Under the condition $\xi \leqslant 1$, the characteristic emission frequency in the field of a wave propagating opposite the $e^\pm$ is estimated from $\omega \sim \epsilon s/(1+s)$, where

$$s = 4\epsilon\Omega/m^2 \simeq 0.019\epsilon(\mathrm{GeV})/\lambda(\mu\mathrm{m}). \tag{2}$$

Knowing the spin dependence of the emission probability, we can estimate the spin precession frequency. In a systematic analysis of this process, we should start from the real part of the spin-dependent contribution to the scattering amplitude, corresponding to the self-energy of the $e^\pm$ in the external field. A dispersion relation[1,2,7] can be used to express the real part of the amplitude in terms of an integral over the $e^\pm$ energy of a function proportional to the radiation probability. As mentioned earlier, in the region of the most obvious spin effects, $s \geqslant 1$, the spin-dependent part of the emission probability in the field of a circularly polarized wave is comparable to the corresponding quantity in a uniform transverse field. This region dominates the dispersion relations when the energy of the $e^\pm$ also satisfies the condition $s \geqslant 1$. As a result, the real part of the spin-dependent contribution to the scattering amplitude is on the order of the corresponding quantity in a uniform transverse field of the same strength. This is also true of the spin precession frequency in the circularly polarized wave, which is proportional to this real part. Under the condition $s \geqslant 1$, the relativistic suppression of spin precession in a circularly polarized wave which follows from the Bargmann–Michel–Telegdi equation should vanish.

To find the spin-dependent part of the amplitude, corresponding to the self-energy of the $e^\pm$ in the external field, we use a semiclassical expression derived in lowest-order perturbation theory in the interaction with the radiation field:[7]

$$\mathbf{T}_\zeta(\zeta) = \frac{i\alpha}{4\pi}\int dt \int_0^\epsilon d\omega \int_0^\infty d\tau \frac{\omega}{\epsilon}\zeta \times \left\{ \frac{1}{\gamma}[\mathbf{v}(t+\tau)\times\mathbf{v}(t)] + \left(1+\frac{\epsilon}{\epsilon'}\right)\left[\mathbf{v}(t+\tau)\int_0^\tau \mathbf{v}(t+\tau') \times \frac{d\tau'}{\tau}\right]\right\} \frac{1}{\tau}\exp\left(-i\frac{\omega\epsilon}{2\epsilon'}\left\{\frac{\tau}{\gamma^2} + \int_0^\tau\left[\mathbf{v}(t+\tau') - \int_0^\tau \mathbf{v}(t+\tau'')d\tau''/\tau\right]^2 d\tau'\right\}\right). \tag{3}$$

Here $\epsilon' = \epsilon - \omega$, ω is the photon energy (this would be a real photon in the case of emission, while it would be a virtual photon in an analysis of spin precession), and $\mathbf{v}(t)$ is the velocity of the $e^\pm$ at time t. The integration over t is carried out along the entire $e^\pm$ trajectory, while that over τ is carried out along only the part of this trajectory with $\tau > 0$. The spin vector on the right side of (3) is taken at time t, while on the left we have its value before entrance into the region with a field.

In uniform fields $\mathbf{H}$ and $\mathbf{E}$, the transverse component of the $e^\pm$ velocity can be written[7] $\mathbf{v}_\perp(t+\tau) = e\mathbf{F}_\perp \tau/\epsilon + \{\xi_1 \cos[2\Omega(t+\tau)+\varphi_0] + \xi_2 \sin[2\Omega(t+\tau)+\varphi_0]\}/\gamma$, where $\mathbf{F}_\perp$

$=\mathbf{E}-\mathbf{n}(\mathbf{n}\cdot\mathbf{E})+\mathbf{H}\times\mathbf{n}$, and φ_0 is a random phase, which depends on the time of entrance into the wave. The orthogonal vectors $\boldsymbol{\xi}_{1,2}$ describe the intensity $\xi^2=(\xi_1^2+\xi_2^2)/2$ and the polarization of the wave, which is characterized by the Stokes parameters λ_3 and $\lambda_2=\boldsymbol{\xi}_1\times\boldsymbol{\xi}_2\cdot\mathbf{n}/\xi^2$, where the unit vector $\mathbf{n}$ is directed opposite the $e^{\pm}$ velocity at the time $\tau=0$. For the case of circular polarization we have $\lambda_3=0$, $\lambda_2=1$, and $\xi_1=\xi_2=\xi=|e|H/m\Omega$.

In order to follow the disappearance of the relativistic suppression of the spin precession in the circularly polarized wave with increasing Ω and ϵ, we need to incorporate in (3) terms of various orders in the small parameter $1/\gamma$. Although only the leading terms in the expansion in $1/\gamma$ were retained in the derivation of (3), this circumstance does not disrupt our approach, since the discarded terms lead to only a negligible refinement of expression (3). Accordingly, to describe the relativistic suppression of precession which follows from the Bargmann–Michel–Telegdi equation, it is sufficient to consider, along with the contribution which is linear in the perturbation of the $e^{\pm}$ velocity by the wave, the contribution of next order in $1/\gamma$, which arises when this perturbation is taken into account simultaneously in the two factors of the first vector product in (3).

We have introduced a uniform field in order to compare the precession in it with that in a circularly polarized wave. Assuming that it is weak enough that we can expand the exponential function in (3) under the condition $\xi^2\ll 1$, and taking an average over the phase φ_0, we find, in first order in the small parameter ξ^2, the following expression for the real part of the amplitude for scattering by an isolated wave:

$$\frac{d\mathrm{Re}\mathbf{T}_\zeta}{dt}=\boldsymbol{\zeta}\times\mathbf{M}=\mu'\boldsymbol{\zeta}\times\left\{\mathbf{F}\times\mathbf{v}+\lambda_2\mathbf{v}\xi^2\frac{H_0}{2\gamma}\left[\frac{s}{\gamma}\left(\frac{1}{1-s^2}+\frac{2s^2\ln s}{(1-s^2)^2}\right)\right.\right.$$
$$-\left(1+\frac{2}{s}\right)\int_0^1\frac{dt}{t}\ln|1-t(1+s)|+\left(1-\frac{2}{s}\right)\int_0^1\frac{dt}{t}\ln[1-t(1-s)]$$
$$\left.\left.-\frac{2\pi^2}{3s}-\frac{s}{1-s^2}-\frac{2s^3\ln s}{(1-s^2)^2}+\ldots\right\}.\tag{4}$$

Here $z=1+s$, $y=1-s$, $H_0=m^2/|e|=4.41\times10^{13}$ G is a characteristic strength of the intense QED field, and the ellipsis represents terms which are proportional to the product $F\xi^2$, and which are of no interest here. We assume[7] that the effect of the radiation field on the spin evolution is incorporated in the relation $d\boldsymbol{\zeta}/dt=2\boldsymbol{\zeta}\times\mathbf{M}$. The vector product $\mathbf{F}\times\mathbf{v}$ describes spin precession in a uniform field [cf. (1)], while the term proportional to $\lambda_2\mathbf{v}$ describes that in a circularly polarized wave. The term which begins with a factor s/γ arises from the incorporation of the perturbation of the $e^{\pm}$ velocity by the wave in both factors in the first vector product in (3). Although this term is relativistically small, it is necessary in order to find a systematic description of spin precession in a circularly polarized wave. Specifically, we consider the limit $s\ll 1$:

$$\frac{d\mathrm{Re}\mathbf{T}_\zeta}{dt}\simeq\mu'\boldsymbol{\zeta}\times\left\{\mathbf{F}\times\mathbf{v}+\lambda_2\mathbf{v}\xi^2\frac{H_0}{2\gamma}s\left[\frac{1}{\gamma}+\frac{s^2}{36}\left(60\ln\frac{1}{s}-37\right)\right]\right\}.\tag{5}$$

From the relations among the terms on the right side of (5) we can establish the range of applicability and the accuracy of the Bargmann–Michel–Telegdi equation in the case at

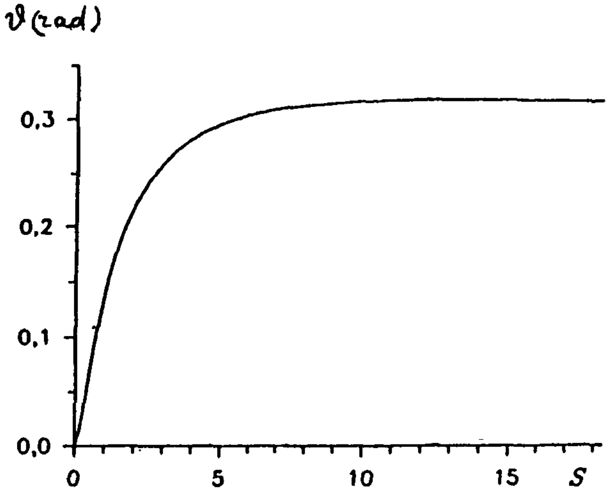

FIG. 1. Angle through which the spin rotates over a Compton scattering length versus the parameter in (2), which is proportional to the energy of the $e^{\pm}$ and to the frequency of the electromagnetic field.

hand. Under the condition $s \ll 1/\sqrt{\gamma}$, the relativistically small term is the leading term. It leads to the value of the precession frequency which follows from Eq. (1). With increasing ϵ, Ω, and s, however, the precession frequency increases, in flat contradiction of this equation; at $s \sim 1$ and $\xi \sim 1$, it becomes comparable to the precession frequency in a uniform field $F \sim H_0/\gamma$, characterized by the circumstance that it reaches H_0 in the proper frame of the $e^{\pm}$. Such fields have been achieved only recently, at the focus of subpicosecond laser pulses.[4] Studies of the interaction with effective fields of corresponding strength in crystals have been undertaken.[3] Accordingly, the spin precession frequency in the field of the circularly polarized wave at $s \sim 1$ can be (for example) a factor $\gamma \sim 10^5$ higher than that which follows from Eq. (1). It can reach levels corresponding to an extremely intense transverse uniform field. Speaking a bit loosely, we might interpret this result as a manifestation of an effective anomalous magnetic moment which is greater than the Schwinger moment by a factor of nearly γ.

In the most attractive version of an experiment to observe spin rotation, based on a selection of $e^{\pm}$'s which have not lost energy due to radiation, we should take the interaction length for the interaction of the $e^{\pm}$ with the wave to be the characteristic Compton scattering length[7] $l_C = 8\pi\alpha\epsilon/(m^4\xi^2 s\sigma) \simeq \lambda/(\alpha\pi\xi^2)$, where σ is the cross section for Compton scattering. Figure 1 shows the angle through which the transverse component of the spin rotates over this distance, $\vartheta = 2 l_C M$, versus the parameter s. Since the parameter s is given by expression (2), this plot also illustrates ϑ as a function of the energy of the $e^{\pm}$ or of the wave frequency. We wish to stress that incorporating the interaction with the radiation field has resulted in the replacement of the case in which the precession frequency decreases with increasing ϵ and Ω by a case in which it rapidly increases.

Since the spin rotation angle can be measured easily at the energies under consideration here by various methods, within an error ~ 0.01 rad, we easily see that in the optimum region, $s \geq 3-4$ ($\gamma \geq 10^5$, $\lambda = 0.2-0.3$ μm), the error in the observation of the effect would amount to a few percent. The precession frequency might exceed the value which follows from the Bargmann–Michel–Telegdi equation by a factor of $(0.2-0.4)\gamma$.

One might observe the effect at $s \geqslant 0.1$ or, with $\lambda \sim 0.2\ \mu$m, starting at an energy of a few GeV. By reducing λ, we could achieve an additional lowering of ϵ or an increase in the angle ϑ.

Although the parameter ξ does not appear in the expression for the angle ϑ, it determines the optimum interaction length l_C or the length of the laser pulse, $\tau(\text{fs}) = l_C/c \sim 150\lambda(\mu\text{m})/\xi^2$, and the corresponding energy flux, $S(\text{W/cm}^2) \simeq 2.75 \times 10^{18}\xi^2/\lambda^2(\mu\text{m})$. Using apparatus for amplifying subpicosecond pulses in wide-aperture excimer modules,[4] one could, for example, produce pulses with a length $\tau_p \sim 300$ fs and an energy $E \sim 0.3$ J at $\lambda = 0.2$–$0.3\ \mu$m. If the length of the focus and the geometric pulse length $c\tau_p$ were equal to the length l_C, these parameter values would lead to the optimum conditions for observing the effect with $\xi^2 = 0.1$ and $S \simeq 5 \times 10^{18}$ W/cm^2. In general, on the other hand, it would be possible to satisfy the necessary condition with existing laser apparatus in the interval $0.01 \leqslant \xi \leqslant 1$ [at $\xi \sim 1$, expression (4) is good only for estimates, but the method of Ref. 8 can be used to carry out numerical calculations from Eq. (3)]. In contrast with the change in the anomalous magnetic moment of an electromagnetic wave in the presence of a longitudinal magnetic field,[2] observation of a precession of $e^{\pm}$ spin in the field of a circularly polarized wave would not require the generation of an ultrastrong static field.

We also note that conditions equivalent to an interaction with a circularly polarized wave at optimum values of the parameters, $s \sim 1$ and $\xi \simeq 1$, are realized in the motion of an e^+ with $\gamma \sim 10^5$ in the regime[8] of planar channeling in a curved crystal at angles $\psi \sim 1$ mrad with respect to crystallographic axes.

It can thus be concluded that the effect discussed here can be reliably observed experimentally with existing apparatus.

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[1)]We are using a system of units with $\hbar = c = 1$.

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