New model for electron screening in an ion diode in an external magnetic field

A. V. Gordeev and A. V. Grechikha
Kurchatov Institute Russian Science Center, 123182 Moscow, Russia

(Submitted 30 November 1994; resubmitted 5 January 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **61**, No. 3, 193–200 (10 February 1995)

A basic mechanism for electron mixing is proposed. This mechanism would arise because the laminar structure of a magnetically insulated electron layer is disrupted by instabilities. The result would be the formation of a plasma-like medium with an equilibrium state $n_e \sim B$. This result would explain the decay of the voltage across the diode which has been observed experimentally and in numerical calculations. It would also explain the rapid decrease in the phase velocity of electromagnetic waves in the diode. Comparison of the theory derived here with experimental data yields the empirical observation that the rate at which electrons accumulate in the diode gap remains constant. © *1995 American Institute of Physics.*

1. Ion diodes are used to generate fast ion beams to be focused on targets in inertial-fusion work.[1–3] The basic effort has been to increase the energy and improve the quality of the beams. Experiment and theory have therefore focused on the current–voltage characteristics and stability of the diode. Practical experience shows that an ion diode is unstable, and that its current–voltage characteristics "float" over time, because of a single factor: the behavior of the magnetized electrons. Describing the electron component is the basic problem in the theory of the ion diode. The reason is that, in contrast with a classical Child–Langmuir diode,[4] in which the electrons are freely accelerated by the electric field, an ion diode requires magnetic insulation of the electrons, which would otherwise carry the entire current. Magnetic insulation, however, makes the behavior of the electrons extremely complicated. The first correct steps toward a description of this insulation were taken after a decade of theoretical attempts (see Refs. 5 and 6 and the papers cited there).

The theory proposed below is based on the obvious fact that the laminar structure of the magnetically insulated electron layer in an ion diode in an applied magnetic field is unavoidably disrupted by electromagnetic instabilities (see Refs. 1–3 and 5–7 and the papers cited there). This effect must lead to an accumulation and mixing of electrons in the magnetic-insulation region with drastic changes in the diode characteristics.

The reason why the diode impedance observed in the experiments of Refs. 1–3 was smaller than that expected on the basis of the Child–Langmuir law was explained at a quantitative level in Refs. 5 and 6, on the basis of the formation of a new, virtual cathode of electron-filled magnetic field lines penetrating the cathode metal (Fig. 1a). The balance struck between the magnetic pressure of the field lines squeezed toward the anode and the electrostatic attraction of the virtual cathode toward the anode sets the value of the vacuum gap length d.

If the initial magnetic field in the diode gap is B_0, and if L is the distance between the anode and the cathode foil, the magnetic flux (F_{vac}) cut out by an infinitely thin virtual cathode is $F_{vac}=B_{vac}d=B_0d_0$, where d_0 is the initial position of the virtual cathode. From the condition that the forces acting on the electron layer be in balance upon a substantial decrease in d,

$$B_{vac}^2-E_{vac}^2=B_{neut}^2 \tag{1}$$

[$B_{neut}=B_0(L-d_0)/(L-d)$ is the magnetic field in the charge-neutral region], we find $B_{vac}=E_{vac}$. This result means that the limit $d\to 0$ corresponds to a voltage $U^*=3F_{vac}/4$ and to an increasing ion current density

$$J_i=\sqrt{\frac{m_e}{m_i}}\frac{I_A}{18\pi d^2}\left(\frac{2eU^*}{m_ec^2}\right)^{3/2},$$

in accordance with the Child–Langmuir law. This model explains the impedance of the diode at maximum power.

Although the model of Refs. 5 and 6 gives an excellent explanation of the diode impedance at maximum power, some fundamental difficulties arise further on. The first is associated with the sharp voltage decay which occurs just after the phase with the maximum power. This effect was explained in Ref. 6 on the basis of an expansion of the anode plasma, in the course of which some of the vacuum gap is filled with plasma. The effect is to reduce the relative amount of magnetic flux which arrives from the accelerating gap. At a plasma thickness l, the limiting value of the voltage, $U^*=3F_{vac}(1-l/d)/4$, decreases. A second difficulty is that experiments[6] and numerical calculations[7] show that the frequency of the diode instabilities decreases sharply (by a factor of 5 to 10), in contradiction of the assumption that the diode is in steady-state operation.[5,6]

In the present letter we discuss a model which explains both of these effects, in a consistent way.

2. It has been recognized for a long time that instabilities cause a partial disruption of magnetic insulation.[8] A point that is not obvious is the distribution of the electron density in the vacuum gap which forms under these conditions. The case under consideration here makes it possible to resolve this point, because there are two small parameters. First, the characteristic frequencies are small in comparison with the characteristic Larmor frequency (or, equivalently, the electron inertial length scale is small in comparison with the vacuum gap; this situation corresponds to high voltages across the vacuum gap, $eU\gg m_ec^2$). It can thus be concluded that the drift approximation is valid for describing the electrons. This conclusion is verified by numerical calculations (Ref. 6, for example). We have a second small parameter because the influx of electrons from the walls to magnetic field lines of the virtual cathode is slow in comparison with the rapid mixing of electrons by the diocotron instability.

What will be the result of this mixing? Since the drift motion preserves the ratio B/n_e for all Lagrange particles, specifically this quantity should become uniform as a result of the mixing. A quasisteady equilibrium corresponds to a constant value of B/n_e over the entire vacuum gap. Accordingly, in contrast with the state $n_e=\text{const}$ postulated in Refs. 5 and 6, we find an adiabatic law B/n_e=const. The constant B/n_e varies slowly because of the influx of electrons due to the emission of electrons from the cathode metal.

A more general kinetic justification for the law B/n_e=const stems from the existence of the adiabatic invariant $\mu = p_\perp^2/2B$. Because of the mixing, the 2D kinetic electron distribution function $f_e(p_x, p_z, x, z)$, averaged over the longitudinal motion, thus depends on only the adiabatic invariant μ: $f_e = f_e(\mu)$. The density of electrons,

$$n_e(x,z) = \int f_e d^2p = 2\pi B \int f_e(\mu) d\mu,$$

is proportional to the magnetic field. An important point is that the adiabatic law B/n_e=const holds even in the limit $\mu \to 0$, in which we have $f_e(\mu) \sim \delta(\mu)$. This situation corresponds to the hydrodynamic approach taken in the present study. We should emphasize that the presence of yet another adiabatic invariant, associated with gyrations of the particles along B, must be taken into account for an accurate analysis of the longitudinal motion in the more general 3D geometry.

The idea of a mixing in a diode which leads to B/n_e = const—a fundamental idea for the entire subsequent analysis—was introduced in Ref. 9.

A similar idea, involving a restructuring of an equilibrium (in the particular case of quasilinear diffusion in an open confinement system), was discussed in Ref. 10, where the integral of a quasienergy of electrons interacting with a wave was used in addition to μ. Interestingly, a similar approach in tokamak physics recently led to a density profile which explains experiment, despite the presence of strong collisions, which disrupt conservation of the adiabatic invariant.[11] We wish to stress that collisions play a negligible role in a diode.

The accumulation of electrons in a vacuum gap leads to the formation of a plasma-like state with a screening of the electric field over the magnetic Debye length $r_B = B/4\pi e n_e$. The electron drift motion

$$\mathbf{E} + \frac{1}{c}\,\mathbf{v}_e \times \mathbf{B} = 0 \tag{2}$$

and the Faraday equation

$$\text{curl}\ \mathbf{B} = \frac{4\pi e}{c}\,(n_i \mathbf{v}_i - n_e \mathbf{v}_e), \tag{3}$$

where the slow ion motion is ignored, lead to $\mathbf{E} = -r_B \nabla B$. Substituting this relation into the Poisson equation

$$\text{div}\ \mathbf{E} = 4\pi e(n_i - n_e), \tag{4}$$

and noting that we have $4\pi e n_e = B/r_B$, we find an equation to describe the electrostatic screening over a length scale r_B:

$$r_B \nabla \cdot (r_B \nabla B) = -4\pi n_i r_B + B. \tag{5}$$

With r_B=const, this result is the electrostatic screening equation in which the ordinary Debye length is replaced by the magnetic Debye length r_B. Physically, the length scale r_B arises as a result of the equilibrium between the electrostatic attraction and the magnetic repulsion.

3. Let us describe the plasma-like state which arises in the gap. Substituting the expression for the ion charge density,

$$en_i=j_i/v_i=\frac{j_i}{\sqrt{(2e/m_i)(U-\Phi)}}, \tag{6}$$

into Eq. (5), and putting the variables in dimensionless form, using $b=\Phi/U=B/B_a=n_e/n_{ea}$, $\eta=x/r_B$, and $J=j_i/j_0$, where

$$j_0=\frac{B_a^{3/2}}{4\pi(2e/m_ir_B)^{1/2}},$$

we find the equation

$$b''_{\eta^2}=b-\frac{J}{\sqrt{1-b}}. \tag{7}$$

The subscript a here means the value of the corresponding quantity in the anode plane.

This equation is to be supplemented with the emission equation $b'_\eta=0$ at the anode (η=0) and by the pressure balance equation $b^2-b'^2_\eta=b_0^2$ ($b_0=B_{\text{neut}}/B_a$) at the virtual cathode ($\eta=d/r_B$). Integrating Eq. (7) twice, and imposing the boundary condition at the virtual cathode, we find

$$\frac{d}{r_B}=\int_{1-1/16J^2}^{1}[\Psi(J,b)]^{-1/2}db, \tag{8}$$

where $\Psi(J,b)\equiv b^2-1+4J(1-b)^{1/2}$.

The dimensionless voltage across the diode gap, $u=U/B_ar_B$, and the dimensionless magnetic flux per unit length of the gap, $f=F_{\text{vac}}/B_ar_B$, where

$$F_{\text{vac}}=\int_0^b B\,dx,$$

become

$$u(J)=\frac{1}{16J^2},\quad f(J)=\int_{1-1/16J^2}^{1}[\Psi(J,b)]^{-1/2}b\,db.$$

Since these expressions contain the implicit parameter J, it is convenient to use the dimensionless quantity

$$N=\frac{(8\pi)^{1/2}e^{5/4}N_e}{m_i^{1/4}j_i^{1/2}F_{\text{vac}}},\quad N_e=\int_0^d n_e dx,$$

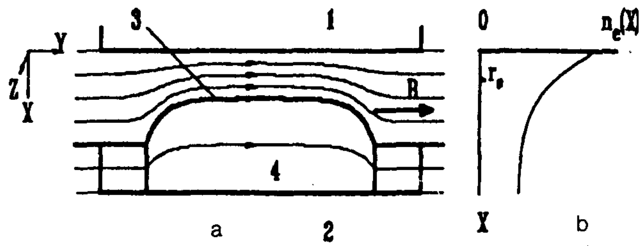

FIG. 1. a: Geometry of an ion diode. 1—Anode; 2—cathode foil; 3—virtual cathode; 4—region of quasineutral plasma. b: Profile of the electron density in the diode gap under the condition $N \gg 1$.

which contains only quantities which are given explicitly: the ion current density j_i, the magnetic flux F_{vac}, and the number of electrons over a unit length of the diode gap, N_e. It is simple to verify that the parameters N and J are related by

$$\frac{f^3(J)}{J^2} = \frac{N^4}{2}.$$

Figure 1b shows a characteristic profile of the magnetic field (and of the density) in the case $N \gg 1$. Within the framework of our model, with $B/n_e = \mathrm{const}$, the electron density and the magnetic field increase by a factor of 3 over a length scale r_B near the anode. This result is supported by numerical calculations which were recently carried out at Sandia Laboratory in the United States,[12] where the diode was described by a time-dependent 3D PIC code, and also by the calculations of Ref. 13, carried out at the Karlsruhe Research Center in Germany.

This model makes it a simple matter to calculate some quantities of importance for making a comparison with experiment and numerical calculations: the ratio of the limiting voltage U^* to the magnetic flux F_{vac}, $\alpha = U^*/F_{\mathrm{vac}} = u/f$ (curve a in Fig. 2), and the phase velocity of perturbations in the diode,

$$u_{\mathrm{ph}} = \mathrm{Re}\, \frac{\omega}{k} = c\,\mathrm{Re}\, \frac{\delta U}{\delta F_{\mathrm{vac}}},$$

which follows from the Faraday equation in integral form,

$$\partial F_{\mathrm{vac}}/\partial t = -c\,\partial U/\partial z.$$

Considering the dynamics slow in comparison with the typical ion transit time $\tau = (m_i d/2eB_0)^{1/2}$, we find the following relation under the condition $\omega\tau \ll 1$:

$$\frac{u_{\mathrm{ph}}}{c} = \mathrm{Re}\, \frac{-ikc\tau u'(J) + u - 3Ju'(J)/2}{-ikc\tau f'(J) + f - 3Jf'(J)}. \tag{9}$$

Curve b in Fig. 2 shows the behavior of u_{ph}/c as a function of N for the case $kc\tau \ll 1$; curve c shows it for the case $kc\tau \gg 1$.

Figure 2 reveals the voltage decay observed experimentally and also the sharp decrease in the perturbation phase velocity, which occurs in the 3D numerical calculations,

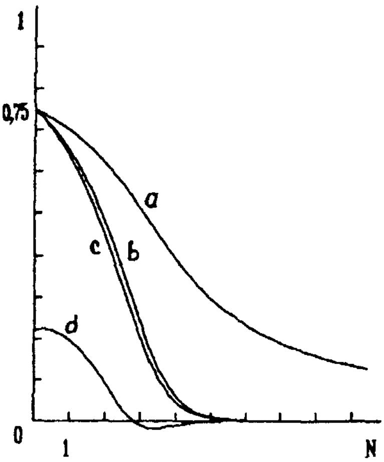

FIG. 2. Basic diode characteristics versus N. a—Dimensionless limiting voltage $\alpha = U^*/F_{\rm vac}$; b, c—dimensionless perturbation phase velocity $u_{\rm ph}/c$ for the cases $kc\tau \gg 1$ and $kc\tau \ll 1$, respectively; d—growth rate of long-wave perturbations, γ/k^2, in arbitrary units.

with increasing value of the parameter N. This behavior reflects an accumulation of electrons in the diode gap. Both effects are easy to understand: At a large value of N, the magnetic Debye length r_B becomes shorter than the gap length d, and the electric fields in the diode gap are screened. They persist only near the anode and the virtual cathode, over a distance on the order of r_B. This theory yields $d_{\rm eff} = 9r_B/8$; this result corresponds to the asymptotic behavior $u/f \sim N^{-4/3}$.

It is also a simple matter to explain the sharp decrease in the perturbation phase velocity at $N \gg 1$. In the vacuum case, $N \ll 1$, both $F_{\rm vac}$ and U^* are proportional to the gap size in the Faraday equation $F'_{{\rm vac}_t} + cU_z^{*\prime} = 0$: $F_{\rm vac} = B_a d$, $U^* = 3B_a d/4$. The perturbation phase velocity is $3c/4$. In the opposite case, $N \gg 1$, the situation is different because of the screening. The magnetic field varies only near the anode and the virtual cathode; elsewhere in the gap, the field deviates from the constant value $B = B_a/3$ by only an exponentially small value. The coefficient of 1/3 arises from the exact solution of screening equation (8) under the conditions $N \gg 1$ and $d/r_B \gg 1$. This circumstance can be seen from the fact that the integral in (9) diverges at the point $b = 1/3$. The electric field $\mathbf{E} = -r_B \nabla B$ is zero except in the region near the electrodes in which B changes. As a result, $F_{\rm vac}$ increases with increasing d in proportion to $B_a d/3$, and the voltage U^* deviates from a constant value by an amount which is exponentially small in terms of the parameter d/r_B. As a result, there is an exponentially small phase velocity $u_{\rm ph}/c = \delta U^*/\delta F_{\rm vac} \sim \exp(-\sqrt{3}d/2r_B)$.

4. In general, the perturbations which we have been discussing here are unstable near $N=0$ (Refs. 14, 15, and 10; this is the case of a vacuum gap). Curve d in Fig. 2 shows the N dependence of the growth rate calculated in the approximation $\omega\tau \ll 1$ (Refs. 10 and 15). That the perturbations are stable at $N \gg 1$ can be demonstrated analytically, by solving the following model problem. We take the virtual cathode to be a movable boundary between two charge-neutral regions. Using the continuity condition on the fluxes of momentum,

$$\left\{ m_i n_i (v_{i\perp}^2 - v_{vc}^2) + \frac{B^2}{8\pi} \right\} = 0, \tag{10}$$

that on the flux of matter,

$$\{ n_i (v_{i\perp} - v_{vc}) \} = 0, \tag{11}$$

and energy conservation as the ions move through the virtual cathode,

$$eU_{vac} = \left\{ \frac{m_i (v_i - v_{vc})^2}{2} \right\}, \tag{12}$$

we find an expression for the voltage across the diode:[16]

$$U_{vac} = \frac{B^2}{8\pi e n_{0i}} + \frac{(B^2/8\pi)^2}{2em_i^2 i_0^2}, \tag{13}$$

where $i_0^2 = n_{0i}^2 (v_{i0\perp} - v_{vc})^2$. The braces (curly brackets) here mean the abrupt change in the corresponding quantity at the surface of the virtual cathode; n_{i0} and $v_{i0\perp}$ are the density and normal velocity component of the ions arriving at the plane of the virtual cathode from the anode; and v_{vc} is the intrinsic velocity of the virtual cathode.

Introducing the function $d(z,t)$ under the condition $d \gg r_B$, and substituting $v_{vc} = d_z'/(1+d_z'^2)^{1/2}$ and $v_{i0\perp} = v_{i0}/(1+d_z'^2)^{1/2}$ into Eq. (13), we find a nonlinear equation for the evolution of the virtual cathode:

$$d_t' + \frac{c r_B}{16} \left[\frac{1 + d_z'^2}{(1 - d_t'/v_{i0})^2} \right]_z' = 0. \tag{14}$$

In the limit $N \gg 1$ this result corresponds to a damping rate

$$\mathrm{Im}\,\omega = -(k^2 r_B/8)(c/v_{i0}) u_{ph} < 0. \tag{15}$$

We have ignored effects associated with the motion of electrons along the magnetic field.[17]

5. Let us compare this model with experiment. Knowing the flux trapped in the vacuum, F_{vac}, and the voltage across the diode, $U(t)$, we can easily reconstruct the parameter $N(t)$ and thus the number of electrons N_e which have accumulated in the vacuum gap by a given time:

$$N_e = \mathrm{const}[j_i(t)]^{1/2} N(t).$$

Here j_i is the ion current density in the diode, and the constant can be found easily from the definition of the parameter N.

It is useful to compare the theoretical results found here with the results of experiments carried out on ion diodes with various configurations.[18,19] For this comparison we select the experimental results of Ref. 18, for which the geometry is approximately the same as that discussed above. Figure 3 shows oscilloscope traces of the current and of the voltage across the diode in the experiment of Ref. 18 on the PROTO-II device. Also shown here is the increase in $N_e(t)$, within a dimensional factor, as found from these traces. We see that beyond the power maximum the number of electrons in the gap

FIG. 3. Oscilloscope traces of the voltage U (1) and of the current I (2) in experiment 1047 at the PROTO- II device, along with a plot (3) of the total number of electrons in the gap, $N_e(t)$, in arbitrary units, constructed from these traces. The maximum voltage corresponds to 1.6 MV, and the maximum current to 2.4 MA.

increases linearly on the average, for the selected time evolution $\dot{U}(t)$ and $I(t)$. This result looks perfectly natural within the framework of the theory presented above.

We wish to thank H. Bluhm (Karlsruhe Research Center, Germany) for interest in this study and for useful discussions.

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Translated by D. Parsons