Edge barrier and structure of the critical state in superconducting thin films

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The structure of the critical state in a superconducting thin film with a high edge barrier and bulk pinning of vortices is analyzed. Analytic expressions describing the distributions of the current density and the vortex density in the system are derived. A critical state with a fundamentally new structure is observed in the system. In this state, magnetic flux is concentrated at the middle of the film, and Meissner currents are concentrated at the periphery. © *1995 American Institute of Physics.*

Several papers have been devoted to an analysis of the structure of the critical state in thin films (e.g., Refs. 1–4). In the overwhelming majority of these papers, the critical state has been described on the basis of Bean's model, which assumes that the Abrikosov vortices which penetrate into the film are concentrated primarily at the periphery of the sample, while a Meissner state persists at the middle of the film. Essentially, a situation of this sort arises when a surface barrier, which determines the conditions for the penetration of vortices into the film, is substantially suppressed, and the vortices enter even when the magnetic field strength near the edge satisfies $H_k \geqslant H_{c1}$. A penetration regime of this sort prevails, for example, in plates whose characteristic thickness d is considerably larger than the London depth λ: $d \gg \lambda$. Thin films with $d \ll \lambda$ are distinguised by a high edge barrier.[1] This barrier, combined with the nonlocal nature of the vortex–vortex interaction, leads to the onset of a fundamentally new structure of the critical state, as we show below.

In this letter we examine the structure of the critical state in a superconducting thin film with a high edge barrier and bulk vortex pinning. We derive analytic expressions describing the distributions of the current density and the vortex density in the system. We observe a fundamentally new structure of the critical state in the system: The magnetic flux is concentrated at the middle of the film, while the Meissner currents are concentrated at its periphery. Some of the results derived in this study were reported in Ref. 5.

We consider a long film strip in an external magnetic field $H=(0,0,H)$, directed perpendicular to the surface of the strip. We assume that the width W and thickness d of the film satisfy $d \ll \lambda_{\text{eff}} \ll W$, where $\lambda_{\text{eff}} = \lambda^2/d$. We are interested in the distribution of the current induced by the magnetic field and in the distribution of the vortex density along the width of the film. An equation relating these two quantities can be found from the London equation, which takes the following form after an average is taken over the positions of the vortices:[1,2]

$$2\pi \frac{d}{w}\frac{di(y)}{dy}+2\int_{-1}^{1}\frac{i(\tau)d\tau}{\tau-y}=H-\Phi_0 n(y). \tag{1}$$

Here H is the external magnetic field, Φ_0 is the magnetic flux quantum, $n(y)$ is the density of vortices, and $i(y)$ is the current per unit length, defined by

$$i(y)=\int_0^d j_x(y,z)dz.$$

Below we will make use of the dimensionless variable $y=2Y/W$, where Y is the distance from the center of the strip.

For wide thin films ($W \gg \lambda_{\text{eff}}$) we can ignore the derivative on the left side of Eq. (1) for the inner part of the film. As a result, we find the basic equation of the problem:

$$2\int_{-1}^{1}\frac{i(\tau)d\tau}{\tau-y}=H-\Phi_0 n(y). \tag{2}$$

A Meissner state $[n(y)=0]$ exists up to a field $H_k \leqslant H_s$, where H_s is the barrier suppression field.[1] At $H_k>H_s$, the vortices entering the film are distributed in accordance with (2). We assume that the number of penetrating vortices is macroscopic, so that the continuum approach we are using here is justified. The two unknown functions $i(y)$ and $n(y)$ are related by only a single equation. We must therefore formulate some additional conditions in order to determine them unambiguously. Simple physical considerations allow us to write such conditions:

$$i(y)=i_p\,\text{sign}(y), \quad n(y)\neq 0, \quad |y|<\theta, \tag{3a}$$

$$i(y)\neq 0, \quad n(y)=0, \quad |y|>\theta. \tag{3b}$$

Condition (3a) reflects the circumstance that the equilibrium vortex density must lead to a local current per unit length $i(y)$ which is equal to the depinning current per unit length i_p: $|i(y)|=i_p$ (Ref. 6). Here we have noted that the current in the system is induced by the external field, so the current density is an odd function of the coordinate: $i(y)=-i(-y)$. Condition (3b) incorporates the repulsion of vortices out of the region $|y|>\theta$, occupied by the Meissner currents due to the Lorentz force $f_L=i(y)\Phi_0$, which exceeds the pinning force $f_p=i_p\Phi_0$: $f_L>f_p$.

It is convenient to rewrite Eq. (2) as

$$i(y)=\frac{1}{2\pi^2\sqrt{1-y^2}}\left\{\pi H y+\Phi_0\int_{-\theta}^{\theta} n(\tau)\frac{\sqrt{1-\tau^2}}{\tau-y}\,d\tau\right\}, \tag{4}$$

which allows us to directly apply the method of the inversion of Cauchy integrals.[7] After inverting (4), using (3a), we find $n(y)$ and $i(y)$ in the inner region, $|y|<\theta$,

$$n(y)=-\frac{4i_p}{\Phi_0}\,\text{Arctanh}\left(\theta\frac{\sqrt{1-y^2}}{\sqrt{\theta^2-y^2}}\right)+\frac{1}{\Phi_0}\frac{\sqrt{\theta^2-y^2}}{\sqrt{1-y^2}}(H+4i_p\,\text{Arctanh}\,\theta),$$

$$i(y)=i_p\,\text{sign}(y), \tag{5}$$

and also in the region $|y|>\theta$, occupied by Meissner currents,

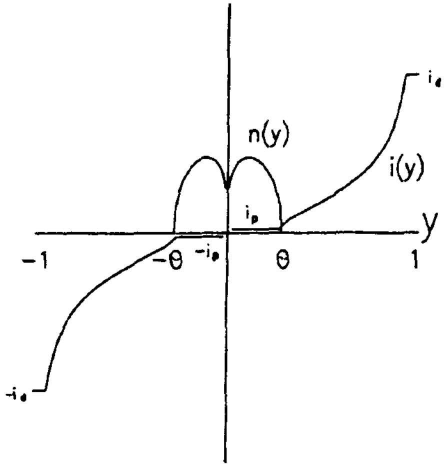

FIG. 1.

$$n(y)=0,$$

$$i(y)=\frac{H}{2\pi}\frac{\sqrt{y^2-\theta^2}}{\sqrt{1-y^2}}\left(1+\frac{4i_p}{H}\operatorname{Arctanh}\,\theta\right)\operatorname{sign}(y)$$

$$-i_p\left(1-\frac{2}{\pi}\operatorname{Arctan}\left(\frac{1}{\theta}\frac{\sqrt{y^2-\theta^2}}{\sqrt{1-y^2}}\right)\right)\operatorname{sign}(y). \tag{6}$$

The distributions $n(y)$ and $i(y)$ are shown in Fig. 1. The equilibrium width of the region occupied by vortices, $\theta=\theta(H)$, is found from the condition that the current per unit length near the edges of the film, $i(1-\xi)$ (here $\xi=\lambda_{\mathrm{eff}}/W$), be the same as the depairing current per unit length i_d: $i(1-\xi)=i_d$ (Ref. 1). The corresponding equation is

$$\frac{H}{H_1}\left(1+\frac{4i_p}{H}\operatorname{Arctanh}\,\theta\right)\sqrt{1-\theta^2}=1, \tag{7}$$

where $H_1=2\pi i_d\sqrt{2\xi}=H_s\sqrt{\xi}$.

Analysis of (7) shows that just above the penetration field, $H=H_1+\delta H$, the magnetic flux occupies a region of finite dimensions at the middle of the film: $\theta(H_1+0)\simeq 8i_p/H_1\ll 1$. As H is increased, the region in which the magnetic flux is concentrated expands to the edges of the film: $\theta\rightarrow 1$ at $H\gg H_1$.

The total flux (per unit length of the film) which penetrates into the sample at $H>H_1$ is given by

$$\Phi_p(H)=2\mu_0 W\{(H+4i_p\operatorname{Arctanh}\,\theta)[E(\theta)-(1-\theta^2)K(\theta)]-4i_p\theta K(\theta)\}, \tag{8}$$

where K and E are the elliptic integrals of the first and second kinds, respectively.

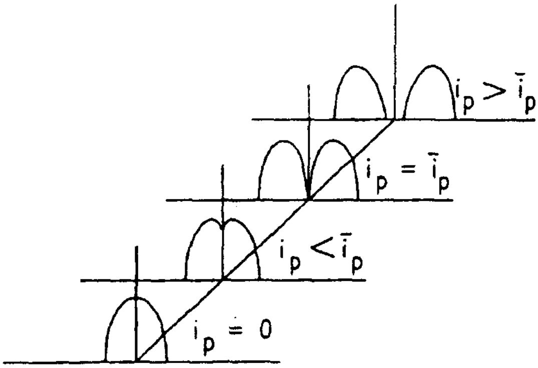

FIG. 2.

At $H \geqslant H_1$, the magnetic induction averaged over the sample, $\bar{B}$, is a linear function of H:

$$\bar{B}(H) = \tfrac{1}{2}\mu_0 \pi (H - H_1). \tag{9}$$

At $H \gg H_1$ we have

$$\bar{B}(H) = \mu_0 \{H - 4i_p \ln 2 - H_1^2/[H + 4i_p \ln(2H/H_1)]\}. \tag{10}$$

In the case $i_p = 0$, Eqs. (5) and (6) become the result of Ref. 8, which was derived by ignoring vortex pinning. Figure 2 shows the evolution of the distribution of vortices as a function of the quantity i_p (at a fixed value of H). We see the onset of a doubly connected region in the vortex distribution at $i_p = \bar{i}_p$. A detailed analysis of this case will be published separately. Strictly speaking, if there is any pinning at all, no matter how weak, there exists a region $|y| < y_0$ in which there are no vortices. The physical reason is the mutual repulsion of vortices in the central part of the sample. The size of the region $[-y_0, y_0]$ in the limit $H/i_p \gg 1$ is given by

$$y_0(H, i_p) = 2\theta \exp[-\theta(H/4i_p + \mathrm{Arctanh}\,\theta)]. \tag{11}$$

A corresponding derivation was carried out in Ref. 9 in an analysis of the critical state in a superconducting platelet of thickness $\sim\lambda$.

The field dependence of the magnetic moment per unit length of the film,

$$M = \int_{-1}^{1} y i(y) dy,$$

is given in the weak-pinning case ($q = 4i_p/H_1 \ll 1$) by

$$h(m) = \frac{1+q^2}{m - q\sqrt{1+q^2-m^2}} - q \,\mathrm{Arctanh}\left(\frac{qm + \sqrt{1+q^2-m^2}}{\sqrt{1+q^2}}\right). \tag{12}$$

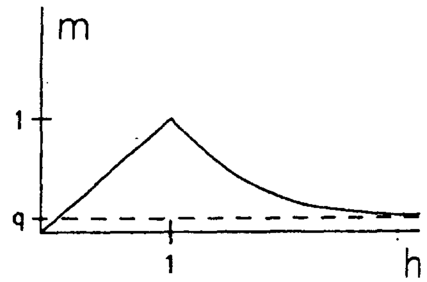

FIG. 3.

Here $h=H/H_1>1$, $m=M/M_1$, and $M_1=H_1W^2/4$ is the value of the magnetic moment in the Meissner state at $H=H_1$. In the region $h-1\ll 1$, the functional dependence $m(h)$ is essentially linear: $m\simeq 2-h$ (Fig. 3). At $H\gg H_1$ we find from (9) that the magnetization of the sample reaches the constant value $M(H)=M_p=W^2i_p$, which corresponds to the Bean critical state.[3]

In summary, we have described the critical state in superconducting thin films with a high edge barrier. We have shown that the structure of this state differs from the distributions of the magnetic field and of the current which arise in films with a suppressed edge barrier. We have derived an analytic expression for the magnetization of the film as a function of the external magnetic field. The model used here can be used to calculate the response of this system to electromagnetic perturbations. It can also be used to calculate dissipative characteristics of superconducting thin films.

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While this paper was being prepared for publication, the authors learned of the contents of Ref. 10, in which a corresponding problem was solved for the case of an edge barrier of a geometric nature, which is important in sufficiently thin platelets, with $d\gg\lambda$.

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