Current-voltage characteristic of a magnetized 2D electron channel with an approximately integer filling factor

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A method is proposed for calculating the current–voltage characteristic of a 2D electron channel in a high magnetic field at approximately integer values of the filling factor. In this method, most of the Hall current is of edge origin. The bulk component, exponentially small in the linear regime, begins to grow rapidly above a threshold, while the total Hall current increases gradually. At low temperatures this abrupt process corresponds to breakdown of the quantum Hall effect. © *1995 American Institute of Physics.*

The well-known breakdown of the quantum Hall effect is generally observed[1–5] at $eV \gg \hbar\omega_c$, where V is the Hall voltage, and ω_c the cyclotron frequency. In this case the critical current I_c increases linearly with increasing channel width. However, it was recently shown[6] that for sufficiently narrow channels the current–voltage characteristic becomes highly nonlinear as early as $eV \sim \hbar\omega_c$. In these experiments, the critical current was a weak (logarithmic) function of the channel width $2W$. Balaban *et al.*[6] explained their results (primarily the dependence $I_c \propto \ln W$) by using the picture, presented in Ref. 7, of the distribution of Hall currents in the electron channel. However, the theory of Ref. 7 is valid only under the condition $eV \ll \hbar\omega_c$. It would be difficult to justify the application of this theory to current–voltage characteristics under nonlinear conditions.

Our purpose in the present letter is to derive a current–voltage characteristic for an extended 2D electron channel in a high magnetic field, directed normal to the plane of the channel, at filling factors ν which are approximately integers: $\nu = 1, 2, 3$. We use the interpretation of Ref. 8 of magnetoelectric phenomena. The calculations verify that there is a critical current I_c in the problem, and they verify that the scale of this current is that observed in Ref. 6. The dependence of I_c on the channel width in this model is fairly weak. However, the results of the numerical calculations presented below do not indicate that a logarithmic law $I_c \propto \ln W$ must hold.

The information presented below is also useful for refining the conditions and approximations under which the edge interpretation of the quantum Hall effect is valid (Refs. 9 and 10, for example).

We consider an electron channel, extended along the y axis, which has a width $2W$, $-W < x < +W$ and which is immersed in a magnetic field H, directed normal to the plane of the channel. Assuming $\sigma_{xx} \ll \sigma_{xy}$ and using Ohm's law from Ref. 7, we see that the Hall current is basically of edge origin. Introducing an edge component I_e and a bulk component I_b of this current, we have

$$I=I_e+I_b=\nu e^2V/2h, \tag{1}$$

$$I_e=\nu e^2[\mu(W)-\mu(-W)-e\varphi(W)+e\varphi(-W)]/h, \tag{2}$$

$$I_b=\nu e^2[e\varphi(W)-e\varphi(-W)]/h, \tag{3}$$

$$\mu(W)-\mu(-W)=eV. \tag{4}$$

Here $\mu(x)$ and $\varphi(x)$ are the values of the electrochemical and electric potentials along the cross section of the band, and V is the Hall potential difference. The relationship between $\varphi(\pm W)$ and V is determined below; the Hall conductivity σ_{xy} is expressed in units of e^2/h.

The voltage drop along the y axis, V_{yy}, arises from the requirement that current not flow across the faces $x=\pm W$; it is given by

$$\sigma_{xy}V_{yy}=\sigma_{xx}\int_1^2 dy\, d\varphi(\pm W)/dx. \tag{5}$$

Points 1 and 2 are the positions of the measurement contacts.

Using appropriate dimensionless variables

$$v=eV/\hbar\omega_c, \quad \phi=e\varphi/\hbar\omega_c, \quad t=T/\hbar\omega_c, \quad \xi=x/W, \tag{6}$$

we can write the relationship between v and ϕ which was found in Ref. 8:

$$\phi(+1)-\phi(-1)=\gamma j, \tag{7}$$

$$v=\gamma j-t\ \ln\ S_+/S_-, \tag{8}$$

$$j=j_e/j_*, \quad j_*=\nu e^3 n_d/\hbar\kappa, \quad \gamma=2We^2n_d/\kappa\hbar\omega_c, \tag{9}$$

$$2S_\pm=(\nu_\pm^{-1}-1)+\sqrt{(\nu_\pm^{-1}-1)^2+4\epsilon(2\nu_\pm^{-1}-1)}, \tag{10}$$

$$\nu_\pm=\nu\pm j/\sqrt{2\delta}, \quad \delta=l_H/W, \quad l_H^2=\cos\hbar/eH, \quad t=T/\hbar\omega_c, \tag{11}$$

$$\nu=\pi l_H^2 n_s, \quad \epsilon=\exp(t^{-1})\ll 1, \tag{12}$$

where j is the current density, constant over the cross section of the channel, n_s is the equilibrium electron density in the channel, and l_H is the magnetic length.

In the case $\nu=1$, $j<\epsilon$, the expression for $v(j)$ in (8) linearizes:

$$v\cong\gamma j+tj/\sqrt{2\delta\epsilon}. \tag{13}$$

If the parameter ϵ is exponentially small, the second term on the right side of (13) is the leading term, and the bulk current density j is exponentially small for the given v. This result complements well the model of Ref. 9, according to which the quantum Hall effect is of edge origin in the linear regime, $eV\ll\hbar\omega_c$. In the case at hand, the total current I in (1) is basically an edge current if $I_e\gg I_b$. According to (13), (7), (3), and (2), this inequality does indeed hold.

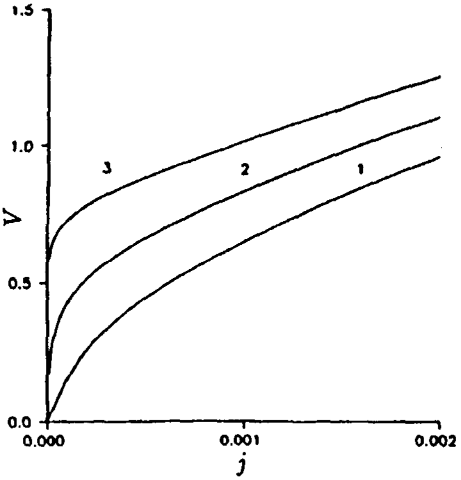

FIG. 1. Relationship between the density of the "bulk" component of the Hall current j and the Hall voltage according to Eq. (8). Curves 1–3 correspond to various temperatures t: 0.075, 0.050, 0.025. The parameter values used to determine the number γ from (8) are $\kappa=10$, $W=10^{-3}$ cm, $n_d=10^{11}$ cm^{-2}, $m_*=0.07m_e$, and $H=10$ T.

In general, expression (8) for $v(j)$ is strongly nonlinear. Figure 1 shows plots of $v(j)$ for $\nu=1$, various values of t, and various values of the other parameters of the model, corresponding to the data of Ref. 6. At $t \ll 1$ the nonlinearity is appreciable at values $v \leq 1$. For the typical values $t \leq 10^{-3}$ of the experiments of Ref. 6, the nonlinearity takes the form of a change in slope. In terms of the numerical value of I_c, however, the experimental result is higher than that calculated from Fig. 1 by a factor of 1.5 to 2.

Figure 2 illustrates the effect of the channel width $2W$ on the current–voltage characteristic. Obviously, there is a dependence here (in practice, the current increases with increasing channel width), and it is considerably weaker than linear.

In summary, the scheme proposed here contains nonlinear effects which are characteristic of the behavior of the current–voltage characteristics of 2D systems in high

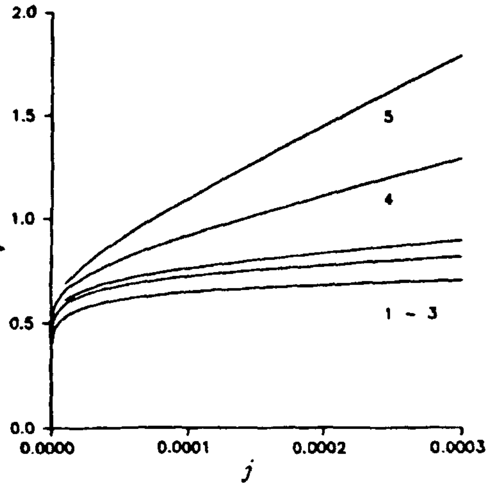

FIG. 2. Effect of the channel width W on the $v(j)$ dependence in (8) at $t=0.025$ and $H=10$ T. Curves 1–5 correspond to the following respective widths: $W=(0.2,2,4,16,32) \times 10^{-3}$ cm.

magnetic fields. This approach gives the correct scale of the critical current I_c and tells us that this current is a weak function of the channel width $2W$. This approach also yields the conditions which are sufficient for realization of the Buttiker scenario[9] in the quantum Hall effect near integer values of ν, specifically, a dissipative nature of the edge currents and the expression $j_i = \sigma_{ik} \partial\varphi / \partial x_k$ for the bulk currents.

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