Contribution of the quark-gluon operator to the effective $\Delta S = 1$ nonleptonic Hamiltonian with $m_t \sim M_W$

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The contribution of the quark-gluon operator to $H_{\Delta S=1}$ is calculated in a theory with a heavy t quark (whose mass is on the order of that of the W boson). The effect of this operator on the characteristics of nonleptonic decays of K mesons is evaluated.

Incorporating hard gluons is recognized as being extremely important for finding a description of nonleptonic decays of K mesons.[1] Calculations in the leading-log approximation in a six-quark theory yield the following expression for the effective low-energy Hamiltonian:[2]

$$H_{\Delta S=1} = -\frac{G_F}{\sqrt{2}} \sum_{i=1, i\neq 4}^{6} (\xi_c C_i^c + \xi_t C_i^t) O_i, \tag{1}$$

where $\xi_q = V_{qd} V_{qs}^*$, V is the quark mixing matrix, and O_i is a set of four-quark operators for which we do not need explicit expressions.

Expression (1) was derived by sequentially splitting off the W boson and the heavy quarks from the light sector of the theory under the assumption $m_q \ll M_W$, where m_q is the mass of any of the six quarks. Analysis of the experimental data available shows, however, that the mass of the t quark is at least comparable to that of the W boson.[3] In the standard model, the current limitation on the mass of the t quark is[4] $m_t > 89$ GeV. As a result, the model of Refs. 1 and 2 is not satisfactory for constructing an effective Hamiltonian for nonleptonic decays of K mesons. An effective Hamiltonian was constructed under the assumption $m_t > M_W$ by Paschos *et al.*[5] However, they did not consider the quark-gluon operator of leading dimensionality: $T = m_s \bar{s}_R G_{\mu\nu} \sigma^{\mu\nu} d_L$, where m_s is the mass of the s quark, and $G_{\mu\nu}$ is the stress tensor of the gluon field. Actually, this operator does not contribute to $H_{\Delta S=1}$ in the single-loop approximation if the theory contains only light quarks ($m_q \ll M_W$), by virtue of the unitarity of the mixing matrix. It turns out that in the case $m_q > M_W$ the operator T appears in $H_{\Delta S=1}$ even at the single-loop level, in an annihilation diagram. The corresponding auxiliary contribution ΔH is

$$\Delta H = -\frac{G_F}{\sqrt{2}} \frac{1}{16\pi^2} \xi_t F_t T, \tag{2}$$

$$F_t(x_t) = \frac{1}{3} \frac{1}{(x_t - 1)^4} \left(\frac{3}{2} x_t^4 - 9x_t^3 + \frac{9}{2} x_t^2 + 3x_t + 9x_t^2 \ln x_t\right),$$

where $x_t = m_t^2 / M_W^2$. The function $F_t(x_t)$ is zero at $x_t = 0$ and also has the values $F_t(1) = 1/4$ and $F_t(\infty) = 1/2$.

The calculations were carried out in the R_ξ gauge.

Let us find ΔH at low energies, at which the hadronic matrix elements must be evaluated. We will use the renormalization group in the leading-log approximation. The matrix of anomalous dimensionalities for the operator T and the four-quark operators O_i is block-diagonal. The operator T is not mixed with the operators O_i in the leading-log approximation. It satisfies the renormalization-group equation

$$(\mu^2 \frac{d}{d\mu^2} - \gamma)T = 0. \tag{3}$$

When the normalization point is changed, the operator T, as the solution of renormalization-group equation (3), transforms in accordance with

$$T(M_W) = \eta(M_W,\mu)T(\mu), \quad \eta = (\frac{\bar{\alpha}_s(m_b)}{\bar{\alpha}_s(M_W)})^{\gamma/b_5}(\frac{\bar{\alpha}_s(m_b)}{\bar{\alpha}_s(m_b)})^{\gamma/b_4}(\frac{\bar{\alpha}_s(\mu))}{\bar{\alpha}_s(m_c)})^{\gamma/b_3}, \tag{4}$$

where $\mu < m_c$ and $b_{N_F} = 11 - \frac{2}{3}N_F$. The anomalous dimensionality of the operator T was calculated in Ref. 6; it is $\gamma = -14/3$. The final expression for ΔH is

$$\Delta H = -\frac{G_F}{\sqrt{2}}\xi_t C_T i m_s \bar{s}_R G_{\mu\nu}\sigma^{\mu\nu} d_L, \tag{5}$$

where $C_T = \eta F_t/16\pi^2$.

How does ΔH affect the characteristics of nonleptonic decays of K mesons?

The operator T appears in $H_{\Delta S=1}$ with the coefficient $\xi_t = s_1 c_1 c_3 s_2^2 + s_1 s_2 s_3 c_2 e^{-i\delta}$. Numerical estimates of the mixing angles $s_2 < 0.1$ and $s_3 < 0.041$ indicate that the contribution of the operator T to the real part of the amplitude for the decay $K^0 \to \pi\pi$ is suppressed in comparison with the contribution from four-quark operators by a factor of 10^{-2} and can be ignored. In a description of the deviation from the superweak mechanism for the breaking of CP invariance, however, mixing with the t quark might play a perceptible role. Let us compare the effect of the operator T with that of the four-quark operators O_i on the ratio ϵ'/ϵ, of the characteristics of the CP breaking. Among the various operators O_i, it is[7] O_6 which dominates the ratio ϵ'/ϵ. Let us examine the relative sizes of the contributions of T and O_i to the expression for ϵ'/ϵ:

$$< 2\pi, \ I = 0|\mathrm{Im}H_{\Delta S=1}|K^0 > |_{O_6}$$

$$= -\frac{G_F}{\sqrt{2}}\mathrm{Im}\xi_t(C_6^t - C_6^c) < 2\pi, \ I = 0|O_6|K^0 >= -\frac{G_F}{\sqrt{2}}\mathrm{Im}\xi_t(0.05 - 0.16 \ \mathrm{GeV}^3).$$

The matrix element $\langle 2\pi, I=0|T|K^0\rangle$ can be reduced through a systematic reduction of the meson fields to

$$< 2\pi, I = 0|T|K^0 >= \sqrt{3}(\frac{\sqrt{2}+1}{\sqrt{2}-1})\frac{m_s m_\pi^2}{f_K(m_u+m_d)}m_0^2. \tag{6}$$

Substituting in the numerical estimates $m_s = 130$–200 MeV, $m_0^2(1\ \text{GeV}) = 0.8 \pm 0.2$ GeV, and $m_u + m_d = 10$–15 MeV, we find

$$< 2\pi,\ \ I = 0|\text{Im}H_{\Delta S=1}|K^0 > |_T$$

$$= -\frac{G_F}{\sqrt{2}}\text{Im}\xi_t C_T < 2\pi,\quad I = 0|T|K^0 >= -\frac{G_F}{\sqrt{2}}\text{Im}\xi_t(0.003 - 0.01\ \text{GeV}^3).$$

For convenience in making a comparison with the contribution of the operator O_6, we take the values of the quantities in the expression for ϵ'/ϵ at the point $\Lambda^2_{\text{QCD}} = (0.1\ \text{GeV})^2$, $\overline{\alpha}_S(\mu) = 1$, $m_t = 100$ GeV (Ref. 5).

The matrix element in (6) has been evaluated in the chiral limit, which is not a good approximation for the K meson. Corrections for the nonzero mass of the s quark might, in general, be important. We thus need to use a better low-energy model for evaluating the matrix element. We also have a second comment, which concerns the choice of the point for the normalization $\overline{\alpha}_s(\mu) = 1$ (Ref. 5). The perturbative corrections to the coefficient functions of local operators become poorly controllable at this point, and incorporating them may change the relation between the contributions of O_6 and T by a factor of the order of 2.

It is clear from these estimates that the effect of the quark-gluon operator on the characteristics of kaon decays may be quite large. Accordingly, if the theory contains a heavy t quark, we would need to use the complete set of operators of the leading dimensionality in order to describe processes which involve a breaking of CP invariance.

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