Charge density waves in a 2D system at a small Landau-level filling factor

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Charge density waves can form in a 2D system in a strong magnetic field at a small value of the filling factor of a Landau level, ν. A distinctive feature of the condition for the formation of charge density waves is that the region in which the waves exist is an oscillatory function of the filling factor ν and of the temperature.

Papers by Lozovik *et al.*[1] and Fukuyama *et al.*[2] have attracted interest to the behavior of a 2D system in a strong magnetic field with respect to the formation of charge density waves, whose classical analog is a Wigner crystal.[3] Some recent experimental studies (e.g., Ref. 4) have found effects which are consequences of the appearance of a quantum Wigner crystal.

In the present letter we use methods of quantum field theory to analyze the conditions for the formation of charge density waves in a 2D system at a filling factor $\nu \ll 1$ of a Landau level ($\nu = 2\pi\hbar c n_s / eH$, where n_s is the density of 2D electrons). The theory is derived for the case of strong magnetic fields H, with $e^2/\epsilon l \hbar\omega_c \ll 1$, where ω_c is the cyclotron frequency, $l(H)$ is the magnetic length, and ϵ is the dielectric constant.

It was shown in Ref. 5 that an instability with respect to the formation of charge

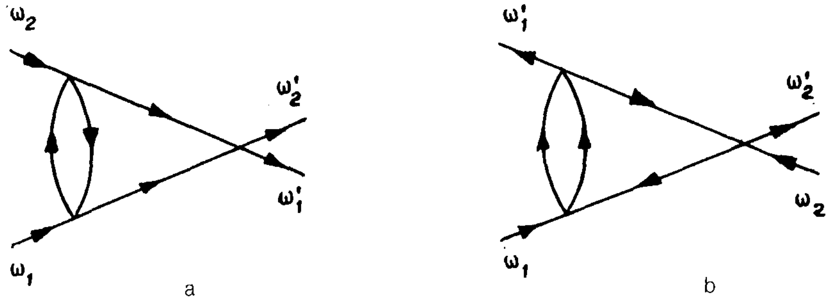

FIG. 1. a—Insertion of a zero-sound loop in a Cooper vertex; b—insertion of a Cooper loop in a zero-sound vertex.

density waves arises from the summation of zero-sound diagrams (and of equivalent diagrams). If only a seed interaction potential is inserted in the zero-sound loops, the condition for the formation of charge density waves which is found agrees completely with the conclusions of Ref. 2. An extremely important aspect of that condition is that it sets a lower limit on the wave vector q for charge density waves; i.e., only waves with $q \geqslant q_0$ exist, where $q_0 l \simeq 0.5$ for a Coulomb potential. In a study of charge density waves with small values of q, it is thus necessary to go beyond the scope of this approximation. All the important diagrams must be carefully selected. The simplest diagrams which go beyond the scope of the ladder approximation are shown in Fig. 1. After a summation over internal frequencies, we find that diagram a is

$$TG(\omega_1)G(\omega_2)\partial f/\partial\xi, \tag{1}$$

while diagram b is

$$\delta_{\omega_1,\omega_1'}(2\nu - 1)G(\omega_1)\partial f/\partial\xi - TG(\omega_1)G(\omega_1')\partial f/\partial\xi. \tag{2}$$

Here

$$G(\omega) = [i\omega - \xi]^{-1}, \qquad f = (I + e^{\xi/T})^{-1}, \qquad \omega = (2n+1)\pi T. \tag{3}$$

The ratio of the second term in (2) [and thus in expression (1)] to the first is $T/\xi = [\ln(1/\nu - 1)]^{-1}$ in order of magnitude. It follows that at $\nu \ll 1$ we can ignore the zero-sound insertions in the Cooper loop, and we can insert in the zero-sound loops, as a seed, a Cooper vertex, retaining only terms with a zero frequency transfer ($\delta_{\omega'\omega'}$). A simple analysis shows that in this approximation the equation for the poles of the zero-sound vertex $\Gamma_H(q,\omega)$ is

$$\Gamma_H(q,\omega) = \frac{T}{\pi}\sum_{\omega'}\Gamma_k(q,\omega+\omega')G^2(\omega')\Gamma_H(q,\omega'). \tag{4}$$

The Cooper vertex calculated in the ladder approximation is

$$\Gamma_k(q,\omega) = 4\pi\sum_m E_m \frac{i\omega - 2\xi}{i\omega - \xi_m}\phi_m(ql), \tag{5}$$

where

$$\xi_m = 2\xi + (1-2\nu)E_m, \qquad \phi_m(x) = e^{-x^2/2}L_m(x^2),$$

E_m are the energy levels of the two-electron problem, $L_m(x)$ is the Laguerre polynomial, and the summation is over odd values of the integer m (Ref. 6).

It is not possible to solve Eq. (4) exactly. Analysis shows, however, that at low temperatures the only terms in expression (5) which are important are those for which the relation $E_m \leqslant \xi \cong T\ln(\nu^{-1})$ holds. As a result, Eq. (4) becomes, approximately,

$$1 = 4\sum_{E_m \leq \xi} E_m\phi_m(ql)\partial f/\partial\xi, \tag{6}$$

and at $\nu \ll 1$ we have $\partial f/\partial\xi \cong -\nu/T$. For a Coulomb potential with $m \gg 1$ we have $E_m \cong e^2/2\epsilon l\sqrt{m}$.

Equation (6) has a solution under the condition

$$lq\epsilon_c \gg \xi, \qquad \epsilon_c = e^2/\epsilon l, \tag{7}$$

in which case its final form is

$$1 = \frac{\nu\epsilon_c}{Tql}J_1[\epsilon_c ql/T\ln(\nu^{-1})], \tag{8}$$

where $J_1(x)$ is the Bessel function.

According to Eq. (8), the region in which charge density waves exist is an oscillatory function of the density and the temperature. Using (7), we find that the conditions for the formation of charge density waves are

$$T_c \leq \epsilon_c\nu^2(ql)^{-3}\ln(\nu^{-1}), \qquad 1 \gg (ql)^2 \gg \nu\ln(\nu^{-1}). \tag{9}$$

The last relation means that these charge density waves do not constitute a Wigner crystal in the full sense of the term; for the latter we would apparently expect the condition $ql = (2\pi\nu)^{1/2}$ to hold.

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