Excitons in an incompressible fluid: giant polaron effect

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A theory is derived for $2D$ magnetoexcitons under the conditions of the fractional quantum Hall effect. The presence of an incompressible $2D$ fluid leads to a giant suppression of the exciton dispersion. The optical spectrum of the excitons is very sensitive to the geometry of the system. In the quantum limit this spectrum is trivial, being the same as the spectrum of a free exciton in the symmetric model. Under more general conditions, it becomes dependent on the filling factor.

Optical spectroscopy of the incompressible $2D$ fluids[1] responsible for the fractional quantum Hall effect[2] is a very rapidly developing filed.[3–6] There is accordingly a need to develop theoretical models for assistance in interpreting the spectra and for extracting from them the parameters of the interacting electron system. This is not a trivial problem, since in the quantum limit all the competing energies are on the same scale—the scale of the Coulomb energy $\epsilon_C = e^2\kappa l$, where κ is the dielectric constant, and $l = (c\hbar/eH)^{1/2}$ the magnetic length. For this reason, all the Coulomb interactions in the initial and final states are of equal importance, and Auger processes are very intense.[7,8] The spectra may also depend strongly on the spin polarization.[7,9]

We have previously[10] derived a theory for the radiative capture of electrons by neutral centers.[3,6] It was shown that a plot of the position of the emission band versus

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the filling factor ν has "downward cusps" at fractional values of ν. These cusps are related to cusps in the ground-state energy.[11] The gaps can be determined from the strength of the cusps. A subsequent analysis of experimental data[12] revealed that these data agree with the theory. A crucial assumption in the theory of Ref. 10 is that the electronic system in the original state is unperturbed, since the center is electrically neutral. In a recombination of electrons with free holes,[4,5] the situation is completely different: These holes interact strongly with the electrons. In a situation of this sort, the effect of the $2D$ fluid on an exciton becomes a governing factor. In the present letter we examine this question for the first time. We show that the phonons (magnetorotons) of a $2D$ fluid[13] give rise to a strong and extremely specific polaron effect: There is no polaron level shift at a momentum $k=0$, but the exciton dispersion law undergoes a huge renormalization.

The Hamiltonian of the electron–hole interaction in the quantum limit ($\hbar\omega_c \gg \epsilon_C$, where ω_c is the cyclotron frequency, and $\nu<1$) can be written

$$H = \frac{1}{4\pi}\sum_{\vec{q}} \tilde{V}(q)c(\vec{q})c(-\vec{q}), \quad \tilde{V}(q) = V(q)e^{-q^2/2}, \quad V(q) = 2\pi/q, \tag{1}$$

where

$$c(\vec{q}) = (2\pi/A)^{1/2}\sum_{p}\{a^+(p-q_y/2)a(p+q_y/2) \\ -b^+(-p-q_y/2)b(-p+q_y/2)\}\exp(iq_x p), \tag{2}$$

$a(p)$ and $b(p)$ are Fermi operators which annihilate electron states (e) and hole states (h) in the Landau gauge, and A is the area of the normalization region. The energy is expressed in units of ϵ_C, while lengths are expressed in units of l. Equation (1) is valid only if the interaction potentials satisfy the conditions $V \equiv V_{ee} = V_{hh} = -V_{eh}$. We will refer to this model as "symmetric."[14] The operators $c(\mathbf{q}) = c(-\mathbf{q})^+$ obey the commutation relation

$$[c(\vec{q})c(-\vec{q}')] = -2i(2\pi/A)^{1/2}\sin((\vec{q}\times\vec{q}')/2)c(\vec{q}+\vec{q}'), \tag{3}$$

where $(\mathbf{q}\times\mathbf{q}) \equiv q_x q'_y - q_y q'_x$. The operator

$$j^+(\vec{k}) = (2\pi/A)^{1/2}\sum_{p} a^+(p-k_y/2)b^+(-p-k_y/2)\exp(ik_x p), \tag{4}$$

which creates an exciton with a momentum $\mathbf{k}$, satisfies the commutation relation

$$[j^+(\vec{k})c(\vec{q})] = -2i(2\pi/A)^{1/2}j^+(\vec{k}+\vec{q})\sin((\vec{k}\times\vec{q})/2). \tag{5}$$

Several conclusions follow from these relations. Since j^+ $(\mathbf{k}=0)$ commutes with all the operators $c(\mathbf{q})$, it also commutes with the Hamiltonian H. Consequently, corresponding to each state Ψ of the electron–hole system is a state

$$\Psi' = j^+(\vec{k}=0)\Psi \tag{6}$$

of a system which differs from the original system in having an additional electron–hole pair. This state is also an eigenstate of H, and it has an energy

$$E' = E + \epsilon(\vec{k} = 0) \, . \tag{7}$$

Here

$$\epsilon(k) = (2/A) \sum_{q} \tilde{V}(q) \sin^2((\vec{k} \times \vec{q})/2) = (\pi/2)^{1/2}(1 - e^{-k^2/4} I_0(k^2/4)) \tag{8}$$

is the energy of the exciton [some additive constants which are of no importance to the discussion below have been discarded in accordance with our choice of the Hamiltonian in the form in (1)]. Equations (6) and (7) generalize the known result[15,16] that there is no interaction between excitons with a zero momentum. In the configurational representation, Eq. (6) is equivalent to the multiplication of Ψ by the wave function of an exciton with $\mathbf{k} = 0$, so we will call states like that in (6) "multiplicative." Although these states constitute only a small fraction of all the eigenstates of the "large" system, they play an extremely important role. Let us explain. The interaction with light is described by the operators j^+ ($\mathbf{k} = 0$) and j ($\mathbf{k} = 0$). For this reason, transitions to multiplicative states (or from such states) are the only ones allowed in optical spectra, and the frequency of the optical transitions is equal to the frequency of an exciton transition, $\epsilon(\mathbf{k} = 0)$, regardless of the value of ν. This assertion is of course valid only for the ideal system in the symmetric model.

The condition $[Hj^+ (k = 0)] = 0$ reflects the presence of a hidden symmetry in this model. It follows from the discussion above that this symmetry imposes some severe limitations on the energy and optical spectra.

At $k \neq 0$, the excitons interact with phonons; according to (5), this interaction is proportional to k. There is accordingly no polaron effect at $k = 0$, but the $\epsilon(k)$ dependence is renormalized: $\epsilon(k) \to \epsilon^*(k)$. This problem does not have a parameter of the coupling-constant type, so we cannot use a perturbation theory. Moreover, the phonon operators $c(\mathbf{q})$ obey unusual commutation relations (3), reflecting the presence of a Fermi limit $n_0 = 1/2\pi$ on the electron density. We postpone a thorough analysis to a following paper; in the present letter we report numerical calculations carried out in a spherical geometry.[17]

Most of our calculations were carried out for a system consisting of 5 electrons + 1 hole, i.e., for an exciton against the background of a fluid consisting of $N = 4$ electrons. We assumed $\nu = 1/3$, so we have $R^2 = 2S = \nu^{-1}(N - 1) = 9$, where R is the radius of the sphere, S is the Haldane parameter, $k = L/R = L/3$ is the momentum, and L is the angular momentum. The spectrum of the system is shown in Fig. 1. The set of minimum values of the energy with a given L, thought of as a function of L, can be interpreted as the exciton dispersion law $\epsilon^*(k)$. These results are shown in Fig. 2; shown for comparison there are the dispersion laws for a free exciton, $\epsilon(k)$, and for phonons, $\omega_{ph}(k)$.

Several conclusions follow from Fig. 2. The presence of the fluid leads to a giant suppression of the exciton dispersion at $k < 1$: $\epsilon^*(k) \ll \epsilon(k)$. In the absence of a small

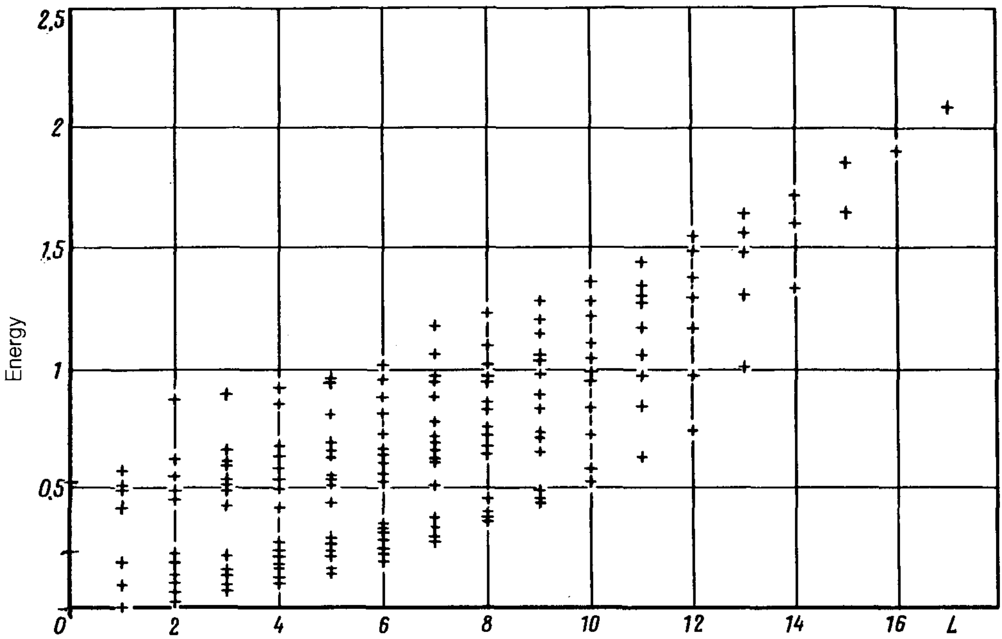

FIG. 1. Energy levels of a system of 5 electrons + 1 hole in spherical geometry versus the angular momentum L at $2S=9$. The origin of the energy scale has been put at the energy of the ground state ($L=0$).

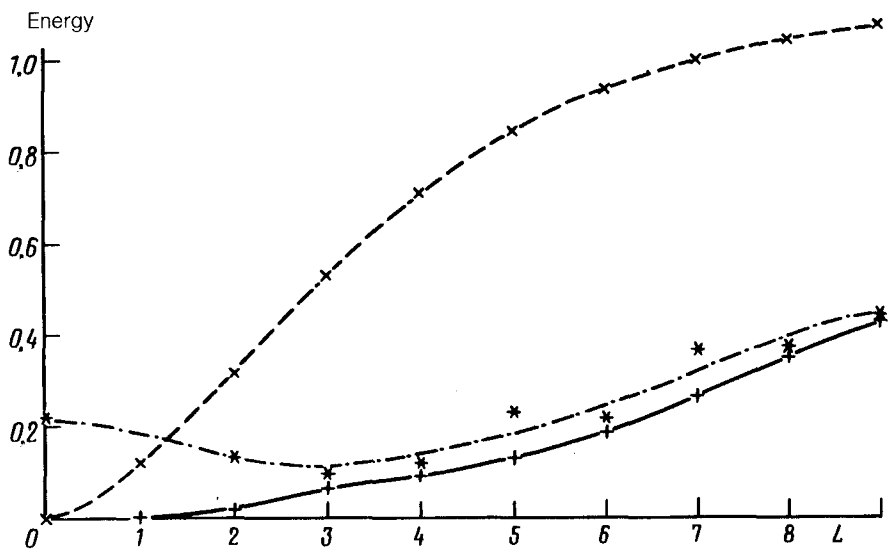

FIG. 2. Dispersion laws. $+$—$\epsilon^*(k)$, for an exciton against the background of an incompressible fluid with $\nu=1/3$; $\times$—$\epsilon(k)$, for a free exciton with $\nu=0$. Shown for comparison (*) is the spectrum of magnetorotons, $\omega_{ph}(k)$, with $\nu=1/3$. The momentum is $k \approx L/3$. The curves are drawn exclusively to aid the eye.

algebraic parameter for this problem, the existence of this strong inequality can be explained only on the basis that $\omega_{ph}(k)$ is numerically small near the roton minimum, $\omega_r \approx 0.08$ (in units of ϵ_C). We have the anomalously small value $\epsilon^*(L=1) \approx 4 \times 10^{-4}$; this value is an order of magnitude smaller than would follow from a parabolic approximation of $\epsilon^*(k)$. Since this fact also holds for $N=3$ and $N=5$, it is not a matter of chance. It is extremely likely that the expansion of $\epsilon^*(k)$ contains no quadratic terms. At $k \approx 1$, the behavior of $\epsilon^*(k)$ changes. In this region we have $\epsilon^*(k) \approx \omega_r$; i.e., the exciton approaches the phonon-emission threshold. At $k > 1$, the curves of $\epsilon^*(k)$ and $\omega_{ph}(k)$ are nearly parallel to each other, but the inequality $\epsilon^*(k), < \omega_{ph}(k)$ holds. In this momentum region the elementary excitations are naturally interpreted as bound states of a phonon with a slow exciton.[18] It can be seen from Fig. 1 that the neighboring states with $L=2$ (and $L=3$) are separated by an energy much smaller than ω_r. This conclusion indicates the appearance of local modes analogous to $1D$ and $3D$ polarons near an exciton.[19]

We get an idea of how to construct the quantum states on the exciton branch $\epsilon^*(k)$ by noting that for an exciton the electron density at the position of the hole is given by

$$\rho_{ex}(k) = \int |\psi_{\vec{k}}^{ex}(\vec{r}_e \vec{r}_h)|^2 \delta(\vec{r}_e - \vec{r}_h) d\vec{r}_e d\vec{r}_h = (1/2\pi)\exp(-k^2/2), \tag{9}$$

i.e., $\rho_{ex}(\mathbf{k}=0) = n_0$, and it falls off rapidly with increasing k. To estimate the characteristic momentum which an exciton brings into the state of the entire system, $\Psi_{\mathbf{k}}$, we introduce the function

$$\Phi_{\vec{k}}(\vec{r}_h) = \int |\Psi_{\vec{k}}(\vec{r}_1 ... \vec{r}_{N+1}|\vec{r}_h)|^2 \sum_{j=1}^{N+1} \delta(\vec{r}_j - \vec{r}_h) d\vec{r}_1 ... d\vec{r}_{N+1}, \tag{10}$$

$$\int \Phi_{\vec{k}}(\vec{r}_h) d\vec{r}_h \le 1/2\pi,$$

and the hole density

$$n_h(\vec{r}_h) = \int |\Psi_{\vec{k}}(\vec{r}_1 ... \vec{r}_{N+1}|\vec{r}_h)|^2 d\vec{r}_1 ... d\vec{r}_{N+1}. \tag{11}$$

The electron density at the hole is then

$$\rho_h(\vec{r}_h) = \Phi_{\vec{k}}(\vec{r}_h)/n_h(\vec{r}_h). \tag{12}$$

It turns out that for all states of the exciton branch $\epsilon^*(k)$ with $0 \leqslant L \leqslant 12$ the relations $0.99 < 2\pi\rho_h(\mathbf{r}_h) \leqslant 1$ hold. In other words, essentially the entire momentum of the system is transported by phonons "dressing" an exciton, and the hole is strongly screened by electrons. The fact that the exciton rapidly loses its individuality with increasing k also follows from two other facts. First, there is a unique multiplicative state $L=0$ on the exciton branch. Second, the scalar products of (on the one hand) the trial functions of exciton states chosen in the form $j^+(\mathbf{k})\Psi_L$, where Ψ_L is the wave function of a Laughlin state, with (on the other) the exact functions fall off rapidly with increasing L (having values of 0.91 at $L=1$, 0.74 at $L=2$, and 0.58 at $L=3$). In other words, trial functions of this sort are quite inaccurate.

Interestingly, we have $\rho_h(\mathbf{r}_h) \approx 1/2\pi$ not only on the exciton branch but for all states which lies below the gap in the spectrum of excited states, which is clearly visible in Fig. 1. It lies between 0.25 and 0.45 at small values of L and stretches out toward larger L. For the states above this gap, ρ_h is smaller by a factor of several units.

The results stated above were derived for a symmetric model. The assertion of a strong renormalization of the dispersion law is of general applicability. Most of the other results are sensitive to a breaking of the hidden symmetry. If this breaking results from a sloping of the quantum well, then pronounced changes occur as soon as the distance between the maxima of the electron and hole densities reaches the value $h \approx 0.5$ (i.e., these changes occur relatively early). A polaron effect arises at $k=0$ here, and there is a significant quadratic contribution to $\epsilon^*(k)$ at $k \ll 1$. The screening of a hole becomes weaker $[\rho_h(\mathbf{r}_h) < 1/2\pi]$. Multiplicative functions (6) cease to be eigenfunctions, and the corresponding states lose their special role, since transitions previously forbidden are now allowed. From this point on, the emission spectrum is dominated by states which lie near the bottom of the energy spectrum, and the magnitude of the transition matrix elements is of only secondary importance. The optical spectrum becomes nontrivial and dependent on ν. This effect is particularly strong when the lower states are states from which transitions are forbidden in the symmetric model. The observed sensitivity of the spectrum to the shape of the well[20] might have a similar origin. Experimental manifestations of a hidden symmetry were observed in Ref. 21.

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