Transverse conductivity of semiconductors in an ultraquantum magnetic field in the case of Boltzmann statistics

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The transverse conductivity of a semiconductor which arises in an ultraquantum magnetic field as a result of the interaction of electrons with an ionized impurity is calculated for the case of Boltzmann statistics. The conductivity is dominated by low-energy electrons undergoing a random walk in the impurity potential. The field dependence (up to 11 T) and the temperature dependence (over the range 20–160 K) of the resistance have been measured for n-InSb samples with electron densities n ranging from 9×10^{12} to 1.2×10^{16} cm^{-3}. The experimental results are described well by an expression derived here.

Our understanding of the kinetics of electrons in strong magnetic fields in which only the lower Landau subband is filled, and ionized impurity centers are responsible for the scattering, is presently undergoing reexamination. The results found by Adams and Holstein,[1] which were regarded as classic for a long time, have proved to be incorrect in this case. Some new expressions for the transverse conductivity of a Fermi gas were derived in Ref. 2; correlations in the scattering were taken into account. In the present letter we examine the case of Boltzmann statistics. These cases are fundamentally different, as we will see below. Since the magnetic length is much shorter than the screening length in the ultraquantum limit, the motion of the electrons across the magnetic field is treated as a drift in crossed fields, i.e., the external magnetic field and the electric field of the impurity centers.[3]

THEORY

To find the conductivity of a Boltzmann gas, we need to take an average over the energy:[4]

$$\sigma_{xx} = -\int e^2 D_{xx}(\epsilon) g(\epsilon) \partial f/\partial \epsilon \; d\epsilon, \tag{1}$$

where $f = A(T)\exp(-\epsilon/kT)$ is the equilibrium electron distribution function, $D_{xx}(\epsilon)$ is the diffusion coefficient for electrons with an energy ϵ in the direction across the magnetic field ($H \| z$), and $g(\epsilon)$ is the density of states. Since we have $g(\epsilon) \propto \epsilon^{-1/2}$ and

$$\int f(\epsilon) g(\epsilon) d\epsilon = n, \tag{2}$$

in the ultraquantum limit, we find from (1)

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$$\sigma_{xx} \sim \int \frac{n \exp(-\epsilon/kT)}{(kT)^{3/2}\epsilon^{1/2}} e^2 D_{xx}(\epsilon)\, d\epsilon. \tag{3}$$

Since the diffusion coefficient in the direction across the magnetic field is[2]

$$D_{xx}(\epsilon) \propto D_{xx}^{-1/3}(\epsilon) \propto \epsilon^{-5/6}, \tag{4}$$

the integral in (3) diverges as[1)] $\epsilon \to 0$. The divergence also occurred in Ref. 1. Adams and Holstein introduced a cutoff at an energy found from the condition

$$\hbar/\tau(\epsilon) \sim \epsilon, \tag{5}$$

where $\tau(\epsilon)$ is the relaxation time of the momentum of an electron along the magnetic field. However, structural features in the density of state lead to large-scale fluctuations of the impurity potential in addition to this smearing. The characteristic width of this smearing region, as in the absence of a magnetic field,[5] is on the order of the scale of these fluctuations, multiplied by the electron charge:[6]

$$U_T \sim eQ/\kappa r_D \sim e^2(Nr_D^3)^{1/2}/(\kappa r_D) \sim U_0^{3/4}(kT)^{1/4}. \tag{6}$$

Here $Q \sim e(Nr^3)^{1/2}$ is the deviation from the average charge of the impurity centers in a sphere of radius r, N is the density of impurity centers $U_0 \equiv e^2 N^{2/3}/\kappa n^{1/3}$, and κ is the dielectric constant of the lattice. The maximum amplitude U_T is exhibited by fluctuations with dimensions on the order of the screening length $r_D = (4\pi e^2 n/\kappa kT)^{-1/2}$. We assume $kT \gg U_0$. The classical description of the impurity potential, on the basis of which expression (6) was derived, is valid when the wavelength λ of an electron with an energy $\sim U_T$ is much smaller than the screening length: $\lambda \ll r_D$. We thus have

$$kT \gg U_0 \left(\frac{U_0}{\epsilon_B}\frac{n^2}{N^2}\right)^{4/5}, \tag{7}$$

where $\epsilon_B = me^4/2\kappa^2\hbar^2$ is the Bohr energy. The energy found from condition (5) is much smaller than U_T.

The cutoff should therefore be made at energies on the order of U_T. Here, however, we are ignoring the contribution to the conductivity from electrons with low energies $\epsilon_c < \epsilon < U_T$ (ϵ_c is the mobility threshold). The motion of such an electron along the magnetic field **H** is obstructed by hills in the impurity potential (Fig. 1a), but an electron can circumvent these hills as the result of a drift across the magnetic field[7] (Fig. 1b). This electron is far more mobile across **H** than is an electron with a higher energy, since, being boxed in between two potential hills, it drifts in essentially one direction until it has traveled a distance on the order of r_D and has escaped from one of the hills. The diffusion coefficient for such electrons is

$$D_{xx} \sim r_D V_D \sim r_D cE/H \sim cU_T/eH, \tag{8}$$

where $E \sim U_T/er_D$ is a characteristic electric field, and $V_D = cE/H$ is the electron drift velocity. In contrast, the drift direction of an electron with a higher energy $\epsilon_2 > U_T$, free to move along **H**, continuously changes (Fig. 1b). The time it takes to

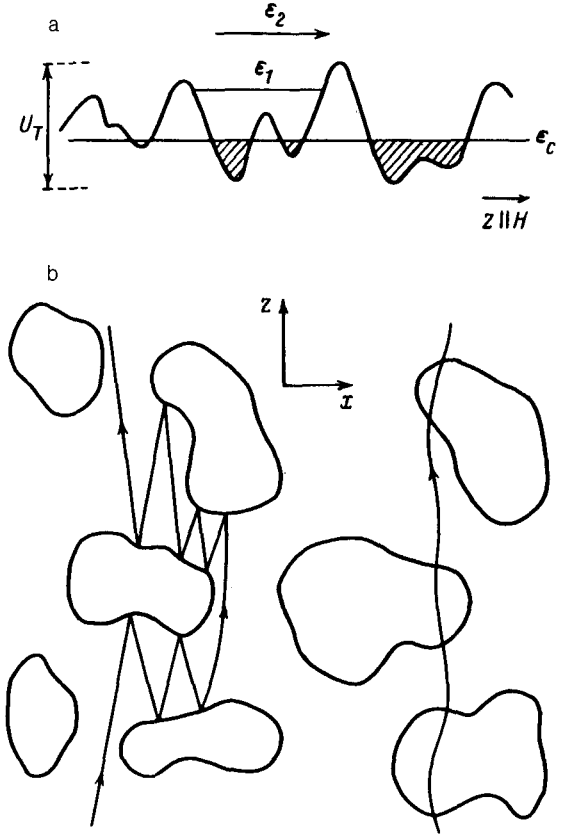

FIG. 1. a: Curvature of the bottom of the conduction band. The z axis runs horizontally, parallel to the magnetic field. The hatched regions lie below the percolation threshold ϵ_c. An electron with an energy ϵ_2 is free to move along the magnetic field. An electron with an energy ϵ_1, which is boxed in between two potential maxima along the z direction, drifts across z. b: Motion of electrons with an energy ϵ_1 (at the left) and with an energy ϵ_2 (at the right) in the xz plane. The closed curves are equipotentials which bound regions that are inaccessible to the electron with an energy ϵ_1.

move a distance $\sim r_D$ across **H** is thus far longer than the corresponding time for an electron with an energy ϵ_1.

To find the contribution of low-energy electrons to the conductivity, we substitute $\epsilon \sim U_T$ into the integrand in (3) and replace the integration by a multiplication by U_T. We then find

$$\sigma_{xx} \sim \frac{nec}{H}\left(\frac{U_T}{kT}\right)^{3/2}. \tag{9}$$

Substituting the value of U_T, and using $nec/H = \sigma_{xy}$ (the Hall conductivity), we find

$$\sigma_{xx} = \alpha\sigma_{xy}\left(\frac{U_0}{kT}\right)^{9/8} = \alpha\frac{e^{13/4}cn^{5/8}N^{3/4}}{\kappa^{9/8}H(kT)^{9/8}}, \tag{10}$$

where α is a numerical coefficient. Using $\sigma_{xx} \ll \sigma_{xy}$, we find the following result for the transverse resistivity:

$$\rho_{xx} = \alpha\rho_{xy}\left(\frac{U_0}{kT}\right)^{9/8} = \alpha\frac{e^{5/4}N^{3/4}H}{\kappa^{9/8}cn^{11/8}(kT)^{9/8}}, \tag{11}$$

where $\rho_{xy} = 1/\sigma_{xy}$. According to the theory of Adams and Holstein, in contrast, this resistivity is given by

$$\rho_{xx} \propto H^0 nN/T^{3/2}. \tag{12}$$

The relative size of the conductivity component from electrons with energies $\epsilon > U_T$ is

$$\frac{(e^2n^{1/3}/\kappa)^{5/8}}{(\hbar\omega)^{1/3}\epsilon_B^{1/6}(kT)^{1/8}}\left(\frac{N}{n}\right)^{1/12} = \left(\frac{U_0}{kT}\right)^{1/8}\left(\frac{\epsilon_B}{\hbar\omega}\right)^{1/12}\left(\frac{\epsilon_{F0}}{\hbar\omega}\right)^{1/4} \ll 1, \tag{13}$$

i.e., negligible. Here $\omega = eH/mc$ is the cyclotron frequency, and $\epsilon_{F0} = \hbar^2n^{2/3}/2m$. Expressions (10) and (11) remain valid in the case in which all the magnetic subbands, with the different spin orientations, are filled, provided that there is no spin–flip scattering.

EXPERIMENTAL PROCEDURE AND RESULTS

To test expression (11), we measured ρ_{xx} on n-InSb samples with various electron densities n and with various concentrations of an ionized impurity, N (Table I). The samples were 1 cm long, about 1 mm wide, and about 2 mm thick (the thickness was the dimension along H). Posts for the potential contacts were cut by electron discharge machining. The machining was followed by etching in the etchant SR-4A. The concentration of ionized impurity centers, N, was found from the results of the ρ measurements with the help of the Brooks–Herring formula. For samples *1–4* we used

TABLE I.

Sample	1	2	3	4	5	6	7
n_{50}, 10^{14} cm^{-3}	0,086	0,36	0,91	3,4	10	30	120
$\rho_{4.2}$, mΩ·cm	-	-	-	-	41	15	4,7
ρ_{30}, mΩ·cm	2200	440	73	33	-	-	-
N, 10^{14} cm^{-3}	5,7	6,0	3,1	7,0	19	50	170
U_0, K	31	20	9,5	10,5	11,5	19	27
ρ_{xx}/ρ_{xy} at 50 K and 11 T	0,17	0,12	0,045	0,047	0,067	0,116	0,142

the formula for Boltzmann statistics and the values of ρ at 30 K; for samples *5–7* we used the formula for Fermi statistics and the values of ρ at 4.2 K (Table I). The Bohr energy of *n*-InSb is 7 K. The values of U_0 for our samples lie in the interval 10–30 K (Table I).

A magnetic field of 11 T was produced by a superconducting solenoid. To reach the temperatures 20–160 K, we used a Dewar, positioned upside down in liquid helium and equipped with a heater.

Most of the measurements were carried out above 20 K, where essentially all the electrons from the shallow donors were scattered into the conduction band, as could be verified on the basis of the Hall effect. The Hall constants changed by less than 20% as the temperature was varied from 70 to 20 K. In the absence of a magnetic field, some of the samples exhibited a Fermi energy ϵ_F greater than 20 K. However, since ϵ_F falls off with increasing field in the ultraquantum limit, the electron gas becomes a Boltzmann gas at $T > 20$ K, even in the sample with $n = 1.2 \times 10^{16}$ cm^{-3} in a field of 11 T. At $T > 20$ K, the ratio ρ_{xx}/ρ_{xy} is much less than unity for all the samples; this result can serve as a test of the applicability of the theory. The experimental values of ρ_{xx}/ρ_{xy} in a field of $H = 11$ T at a temperature 50 K are listed in Table I.

The curves of the field dependence of ρ_{xx} for samples *1–5* are nearly the same, and they have the shape of the curves shown in Fig. 2. In strong fields they are approximately linear. Above 3 T, the 30-K curve can be described by a power law $H^{0.9}$, while the 70-K curve can be described by $H^{0.8}$. For sample *6* the curves of $\rho_{xx}(H)$ are even more nearly linear. For sample *7*, the condition for nondegeneracy of the electron gas begins to be violated as H is lowered from 11 T, so ρ_{xx} varies more rapidly than linearly with the field.

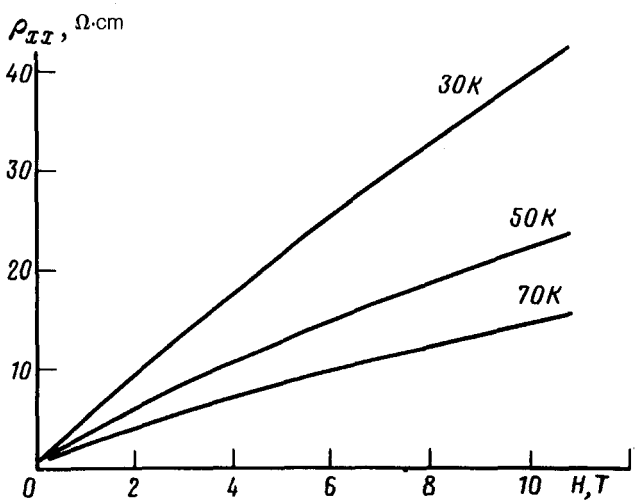

FIG. 2. Magnetic-field dependence of the transverse resistivity ρ_{xx} for sample *2* at temperatures of 30, 50, and 70 K.

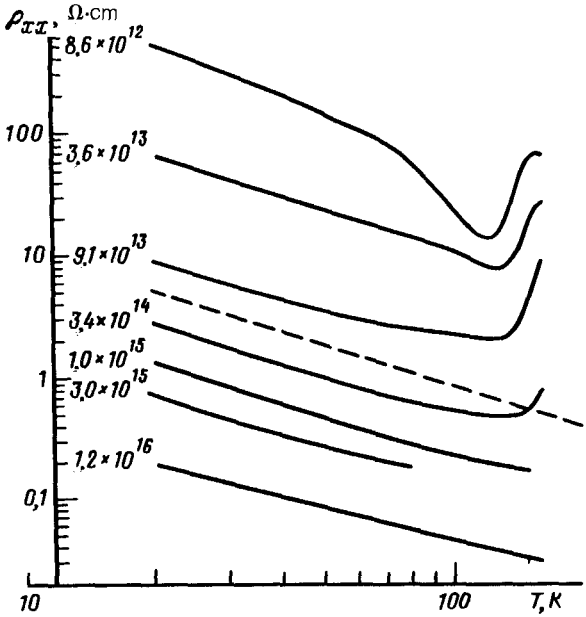

FIG. 3. Temperature dependence of the resistivity ρ_{xx} in a magnetic field of 11 T for various samples. The curves are labeled with the electron density n, in electrons per cubic centimeter. The slope of the dashed straight line corresponds to a $\rho_{xx} \propto T^{-9/8}$ dependence.

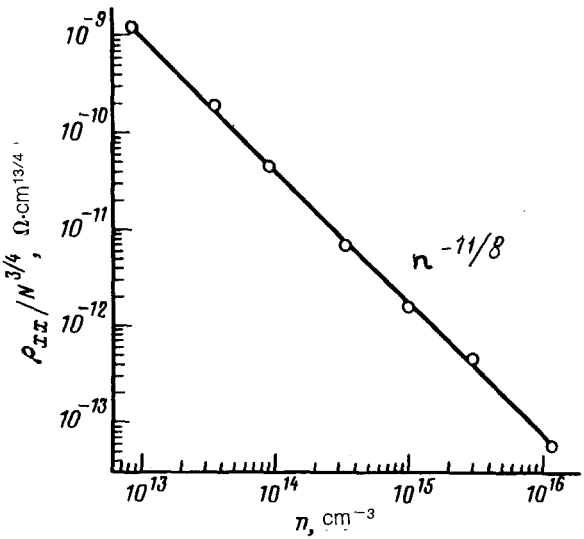

FIG. 4. Plot of the quantity A in a field of 11 T at a temperature of 50 K versus the electron density n, according to the measurements on the various samples. The slope of the straight line corresponds to the theoretical functional dependence $\rho_{xx}/N^{3/4} \propto n^{-11/8}$.

The temperature dependence of ρ_{xx} in a field of 11 T is described approximately by $T^{9/8}$ over the range 20–80 K for all the samples (Fig. 3). With increasing density of electrons and impurity centers, this dependence stretches out toward progressively stronger fields. The faster decay of ρ_{xx} on the two upper curves above 80 K, accompanied by a decrease in the Hall constant, is due to an excitation of electrons from deep donors ($\simeq$60 meV), whose concentration is $\simeq 10^{13}$ cm^{-3}. The subsequent increase in ρ_{xx} is due to the appearance of intrinsic carriers.

To test the dependence of ρ_{xx} on n and N, we plot $\rho_{xx}/N^{3/4}$ versus n in Fig. 4. The experimental points conform well to a straight line, whose slope corresponds to the predicted $n^{-11/8}$ dependence. From the experimental results we can find the value of the numerical coefficient α in expressions (10) and (11):

$$\alpha \simeq 0.3. \tag{1}$$

In summary, the experimental results are described well by expression (11), while they do not agree quantitatively or qualitatively with the results of other existing theories.

We wish to thank B. A. Aronzon for furnishing some of the samples used in this study.

[1)] The derivation of the functional dependence $D_{xx} \propto \varepsilon^{5/2}$ used the assumption that the Born approximation is valid and that localization effects[2,5] are suppressed in a Boltzmann gas. Actually, these assumptions are not particularly important; the only important point is that integral (3) diverges.

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