Nonperturbative IR limit of the gluon polarization tensor at $T \neq 0$

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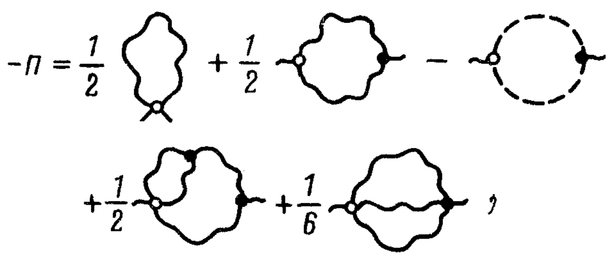
The nonperturbative IR limit of the trace of the transverse components of the gluon polarization tensor is calculated in the Feynman gauge for $T \neq 0$ in $SU(N)$ gluodynamics. The result of the calculation is a logarithmic function multiplied by a power law and is not amenable to a series expansion in g^2.

The problem of summing the IR-divergent diagrams in gluodynamics [in a non-Abelian $SU(3)$ Yang–Mills theory] has recently attracted much interest among many authors. Some of the corresponding papers[1-3] are particularly important and pertinent. The problem is to develop a reliable method for studying the nonperturbative properties of Green's functions (or of other gauge-theory entities), particularly in the IR momentum region, where the existing results (found for the most part through a summation of infinite perturbation-theory series) might serve as helpful guidelines in the development of such a method. The primary subject of this research is the gluon polarization tensor, whose IR behavior found at the single-loop level[1,4] is anomalous and leads to a false IR pole in all the Green's functions which determine the $SU(N)$ gluodynamics. In the IR momentum region (where the coupling constant of the effective interaction is extremely large), however, single-loop calculations are not exhaustive or reliable, although they do point out the nonstandard properties of the theory. It has already been established (Ref. 5, for example) that incorporating two-loop corrections leads to a qualitative change (from a power law to a logarithmic behavior) in the IR asymptotic behavior of the inverse gluon propagator. A logarithmic behavior (see also Ref. 1) corresponds better to the essence of the matter. At any rate, the summation of the leading-log contributions of the IR-divergent diagrams carried out below supports that suggestion. The result found is a closed nonperturbative expression for the IR asymptotic behavior of the gluon polarization tensor. The behavior is described by a logarithmic function multiplied by a power law. ("Nonperturbative" here means that the expression cannot be expanded in a series in the coupling constant g.)

The gluon polarization tensor in relativistic α gauges is determined by a set of five nonperturbative diagrams:



(1)

where the wavy lines are exact gluon propagators, the straight lines are the propagators of ghost fields, and the heavy lines are the exact vertex functions of $SU(N)$ gluodynamics. The various iterations of diagram series (1), aside from the single-loop iteration

$$\Pi_{ij}^{(1)}(k_4=0;|\vec{k}|) = -\left(\delta_{ij} - \frac{\vec{k}_i\vec{k}_j}{\vec{k}^2}\right)\frac{g^2NT|\vec{k}|}{64}(\alpha^2+2\alpha+9), \tag{2}$$

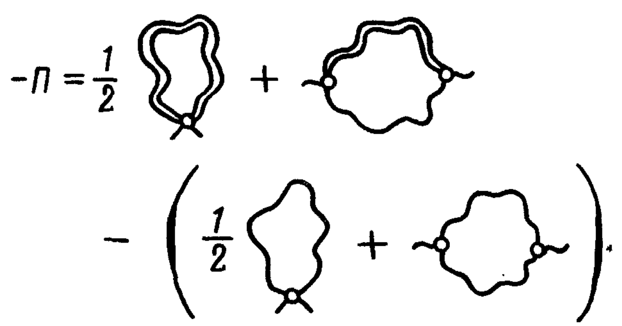
are IR-divergent, and (the more important point) the analytic form of the single-loop result changes when the higher-order loops are taken into account. The two-loop diagram series, for example, diverges logarithmically,

$$\Pi_{ii}^{(2)}(k_4=0;|\vec{k}|) = -\frac{g^4N^2T^2}{(2\pi)^3}\frac{\pi}{2}\ln\left(\frac{\vec{k}^2}{\mu^2}\right), \tag{3}$$

and it is necessary to also introduce an IR cutoff parameter μ (on the order of g^2T) in order to keep the nonperturbative calculations meaningful (Ref. 5, for example).

It has also been established (in Ref. 5) that not all of the two-loop diagrams of series (1) contribute to the leading IR asymptotic behavior of $\Pi_{ij}(k_4=0,|\vec{k}|)$. A large number of perturbative diagrams (which are unimportant from this point of view) can be discarded. In particular, expression (2) is the result of calculations of only a limited subclass of two-loop diagrams which arise from an iteration of the gluon lines of nonperturbative single-loop diagrams (the first two) of series (1), in which all the vertex functions are assumed to be seed functions. This fact (that it is legitimate to ignore a set of the most difficult vertex diagrams in a nonperturbative calculation of the leading IR asymptotic behavior of Π_{ij}) opens up the opportunity to sum a simplified series of ring diagrams and thereby determine the "actual" IR behavior $\Pi_{ij}(k_4=0,|\vec{k}|\to 0)$. It also becomes possible to eliminate a phenomenological parameter from expression (3) and to extend the range of applicability of that expression.

The series of diagrams to be summed is conveniently written as two nonperturbative diagrams as follows:



(4)

The double lines here are "exact" gluon propagators, and the coefficients of series (4) have been chosen to correctly reproduce the results of the two-loop calculation, (3). We have calculated the function $\Pi_{ii}(k_4=0;|\vec{k}|)=2A(\vec{k})$ (in the leading IR approximation, only for the Feynman gauge, with $\alpha = 1$). In the first step of the calculations, we did not specify the form of the gluon propagator:

$$D_{44}(\vec{k}) = [\vec{k}^2 + \Pi_{44}(\vec{k})]^{-1},$$

$$D_{ij}(\vec{k}) = \frac{1}{\vec{k}^2 + A(\vec{k})}\left(\delta_{ij} - \frac{\vec{k}_i\vec{k}_j}{\vec{k}^2}\right) + \frac{\alpha}{\vec{k}^2}\frac{\vec{k}_i\vec{k}_j}{\vec{k}^2}. \tag{5}$$

Using (5) and the standard expressions for the seed vertex functions (Ref. 6, for example), we treated the diagrams of series (4) by the standard methods (using a straightforward algebra and replacing all the sums by a single term with $k_4 = 0$). The result of the calculations for the function $A(\vec{q})$ is an integral equation:

$$A(\vec{q}) = -\frac{g^2N}{2\beta}\int\frac{d^3p}{(2\pi)^3}\left[\frac{4A(\vec{p})}{\vec{p}^2 + A(\vec{p})} + \frac{3\Pi_{44}(\vec{p})}{\vec{p}^2 + \Pi_{44}(\vec{p})}\right]\frac{\vec{p}^2 + 2\overline{\vec{p}\vec{q}} + \vec{q}^2}{\vec{p}^2(\vec{p}+\vec{q})^2}$$

$$+\frac{g^2N}{2\beta}\int\frac{d^3p}{(2\pi)^3}\frac{1}{\vec{p}^2(\vec{p}+\vec{q})^2}\left[\frac{2\vec{p}+\vec{q})^2\Pi_{44}(\vec{p})}{\vec{p}^2 + \Pi_{44}(\vec{p})}\right.$$

$$\left.+(5\vec{p}^2 + 2\vec{p}\vec{q} + 2\vec{q}^2)\frac{2A(\vec{p})}{\vec{p}^2 + A(\vec{p})} + 6\left(\vec{q}^2 - \frac{(\vec{q}\vec{p})^2}{\vec{p}^2}\right)\frac{A(\vec{p})}{\vec{p}^2 + A(\vec{p})}\right]. \tag{6}$$

We wrote this equation in a special diagram-by-diagram form (without identifying the further approximations) and without simplifications. Outside a perturbation theory, Eq. (6) is determined by integrals of a standard type which converge well, but IR divergences (found in the two-loop approximation) arise when (6) is formally expanded in a series in g^2. This situation is not permissible for "exact" integrals.

In a nonperturbative solution of Eq. (6), we use the single-loop values of the functions $A(\vec{p})$ and $\Pi_{44}(\vec{p})$:

$$A(\vec{p}) = -\frac{3g^2N|\vec{p}|}{16\beta};\ \ \Pi_{44}(\vec{p}) = \frac{g^2N}{3\beta^2} - \frac{g^2N|\vec{p}|}{4\beta}. \tag{7}$$

We evaluate the corresponding integrals exactly (without a series expansion in g^2). However, not all the terms in Eq. (6) are identically important in a calculation of the leading IR asymptotic behavior of $A(\vec{q})$, because the functions $A(\vec{p})$ and $\Pi_{44}(\vec{p})$ behave in qualitatively different ways. In particular, all the integrals in (6) which contain the function $\Pi_{44}(\vec{p})$ can be discarded. We have also discarded the terms in (6) which we have singled out, on the basis that they reproduce a "nonleading" asymptotic behavior of the function $A(\vec{q})$. All the other terms in (6) were evaluated with the help of an auxiliary integral which can be approximated in the limit $(\vec{q})\to 0$ by the expression

$$\int\frac{d^3p}{(2\pi)^3}\frac{\vec{p}_i\vec{p}_j}{(\vec{p}+\vec{q})^2(|\vec{p}| - a)\vec{p}^4} = -\frac{\vec{q}_i\vec{q}_j}{32a|\vec{q}|^3} + \delta_{ij}\left[\frac{1}{6\pi^2a^2}\ln\left|\frac{\vec{q}}{a}\right| - \frac{1}{32a|\vec{q}|}\right]. \tag{8}$$

Only the terms which are singular in $|\vec{q}|$ (the most important terms) are shown in (8). Some of these terms correct the single-loop asymptotic behavior of $A(\bar{q})$, but the logarithmic function in (8) is more important. It determines the behavior of $A(\bar{q})$ in the IR momentum region:

$$A(\bar{q}) = -\frac{16}{3\pi^2}\vec{q}^2 \ln\left|\frac{\vec{q}}{a}\right|. \tag{9}$$

The parameter $a(=3g^2N/16\beta)$ serves as a magnetic mass of the gluon. The expression written here is valid under the condition $|\vec{q}| \lesssim a$, in contrast with the two-loop result.[5]

Comparing the result of the nonperturbative calculation, (9), with perturbative expression (3), we find two qualitative features: First, although the logarithmic behavior of the IR limit $\Pi_{ij}(k_4=0; |\vec{k}| \to 0)$ is not reproduced, the function has become more complex, with a power-law factor. Second, the IR cutoff parameter [the parameter a in (9)] has been determined, and the range of applicability of the nonperturbative expression is not the same as that of (3). In particular, this range extends to small values of $|\vec{q}| \lesssim a$. In a sense, expression (3) (which holds at $|\vec{q}| \gtrsim \mu$) and expression (9) complement each other, but the "actual" IR behavior of the polarization tensor is probably determined by a more-complex nonperturbative expression, which cannot be expanded in a finite series in g^2, although it does reproduce (9) near $|\vec{q}| < a$. For example, one might expect that the inverse gluon propagator in the IR momentum region would have the noncanonical behavior

$$D^{-1}(q_4=0; |\vec{q}| \to 0) \sim \vec{q}^2 \exp\left[-\gamma \ln\left|\frac{\vec{q}}{a}\right|\right] = \vec{q}^2 \left|\frac{\vec{q}}{a}\right|^{-\gamma}, \tag{10}$$

and that terms proportional to $|\vec{q}|$ would also be summed in a more complex function. The general trend here is such that the single-loop calculations do not exhaust the essence of the matter, and the false IR pole is apparently not a defect of the theory but a result of the error of the simplest approximation. In (10), $\gamma = 16/3\pi^2$.

[1]R. Jackiw and S. Templeton, Phys. Rev. D **23**, 2291 (1981).
[2]E. Braaten and R. Pisarski, Phys. Rev. Lett. **64**, 1338 (1990); Nucl. Phys. B **337**, 569 (1990), **339**, 310 (1990).
[3]R. Kobes, G. Kunstaller, and A. Rebhan, Preprint CERN-TH. 5937/90.
[4]O. V. Kalashnikov and V. V. Klimov, Yad. Fiz. **33**, 848 (1981)[Sov. J. Nucl. Phys. **33**, 443 (1981)].
[5]O. K. Kalashnikov, V. V. Skalozub, and I. V. Chub, Yad. Fiz. (1991), (in press).
[6]O. K. Kalashnikov, Fortschr. Phys. **32**, 525 (1984).

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