Spectrum of gravitational waves in a double-inflation scenario

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The spectrum of relic gravitational waves formed in a double-inflation scenario might have obvious modulations.

Double inflation, i.e., two successive inflationary stages with different inflation velocities, is characteristic of models which contain several scalar fields or which have terms that are quadratic in the curvature along with the scalar fields.[1–5] The double-inflation scenario can lead to fundamentally different values of the perturbations of the energy density at long and short range.[1,2] This property qualifies the double-inflation scenario as a candidate for explaining the probable deviation of the spectrum of density inhomogeneities at large scale from the flat Harrison–Zel'dovich spectrum.[6]

In order to learn about the inflationary model, it is useful to study, along with the shape of the density perturbation spectrum, the spectrum of stochastic gravitational waves which arise in the stage of inflation.[7,8] This wave spectrum differs from the density inhomogeneity spectrum in that it does not depend on the behavior of the effective masses of the scalar fields, being determined exclusively by the history of the expansion, $a(t)$. It is therefore worthwhile to study possible spectra for gravitational waves which would arise in a double-inflation scenario, especially since yet another method was recently proposed for measuring the amplitude of these waves at a scale of $(1\text{–}3000)h^{-1}$ Mpc, through the observation of gravitational lenses.[9]

As a very simple example of the double-inflation scenario, we adopt a model with the action

$$S = -\int d^4x\sqrt{-g}\{\frac{1}{2}\nabla_\mu\varphi\nabla^\mu\varphi + U(\varphi) + \frac{1}{2}\nabla_\mu\chi\nabla^\mu\chi + V(\chi)\}.$$

Here φ is a field which is dominant in the first inflationary stage. If the condition $U(\varphi) \gg V(\chi)$ initially holds, this stage terminates in oscillations in the field φ, which dominate the situation until their energy density falls below $V(\chi)$. In this model there is accordingly a period between the two inflationary stages in which the effective equation of state is $p = 0$.

If the field φ interacts with other fields, its decay products may be predominant in the intermediate stage. Let us consider the fairly general case in which the effective equation of state in this stage is $p = (\gamma - 1)\rho$. The behavior of the Hubble parameter can then be approximated by

$$H = \begin{cases} H_1, & a < a_1 \\ H_1(a_1/a)^{3\gamma/2}, & a_1 < a < a_2 \,. \\ H_2, & a_2 < a \end{cases} \tag{1}$$

There is no such intermediate stage with a $\ddot{a} < 0$ if $U(\varphi)$ is not initially much greater than $V(\chi)$. We consider perturbations of the metric $h_{\mu\nu}$:

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)dx^i dx_i + h_{\mu\nu}dx^\mu dx^\nu,$$

$$\mu,\ \nu = 0,\ 1,\ 2,\ 3. \quad i,\ j = 1,\ 2,\ 3.$$

The part of $h_{\mu\nu}$ for which the conditions $h^i_{j,i} = h^i_i = 0$ hold corresponds to gravitational waves. The corresponding quantum operator in the gauge $h_{0\mu} = 0$ can be written[10]

$$\hat{h}_{ij}(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3}\{m_i m_j \hat{a}_R(\vec{k})\phi(\vec{k}, t)\exp(i\vec{k}\vec{x})$$

$$+ m_i^* m_j^* \hat{a}_L(\vec{k})\phi(\vec{k}, t)\exp(i\vec{k}\vec{x}) + \text{(Herm. adj.)}\},$$

where R and L specify two polarization states, $m_j = e_j^{(1)} + ie_j^{(2)}$, $e_j^{(1,2)}$ are two purely spatial unit vectors which are orthogonal with respect to k_j and orthogonal with respect to each other and $[\hat{a}_R(\vec{k}_1), \hat{a}_R^+(\vec{k}_2)] = \delta(\vec{k}_1 - \vec{k}_2)$ (there is a similar expression in the L case).

The spectrum of the gravitational waves which have formed is determined by the function ϕ, which satisfies the equation

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + \frac{k^2}{a^2}\phi = 0. \tag{2}$$

Corresponding to a vacuum state which is invariant under the de Sitter group is the following behavior of ϕ at $a < a_1$:

$$\phi(\vec{k}, t) = 8\pi^2 G^{1/2} k^{-3/2}\left(H - \frac{ik}{a}\right)\exp\left(\frac{ik}{Ha} - \frac{ik}{Ha_0}\right), \tag{3}$$

where a_0 is an arbitrary constant, $H = \dot{a}/a$, and G is the gravitational constant.

Solving Eq. (2) under conditions (1) and (3), we find the spectrum of gravitational waves at $a \gg a_2$. This spectrum is shown in Fig. 1 for the case $\gamma = 4/3$ (radiation). That spectrum was found analytically. For arbitrary γ at $H_1 \gg H_2$, the asymptotic behavior of the spectrum is

$$|\phi|k^{3/2} = D\begin{cases} H_1/H_2, & k \ll H_2 a_2 \,, \\ F\left(\frac{k}{2H_1 a_1}\right)^{-\alpha}\left|\sin\left[\frac{\alpha k}{H_2 a_2} - \beta\right]\right|, & H_2 a_2 \ll k \ll H_1 a_1 \,, \\ 1, & k \gg H_1 a_1 \,, \end{cases} \tag{4}$$

where

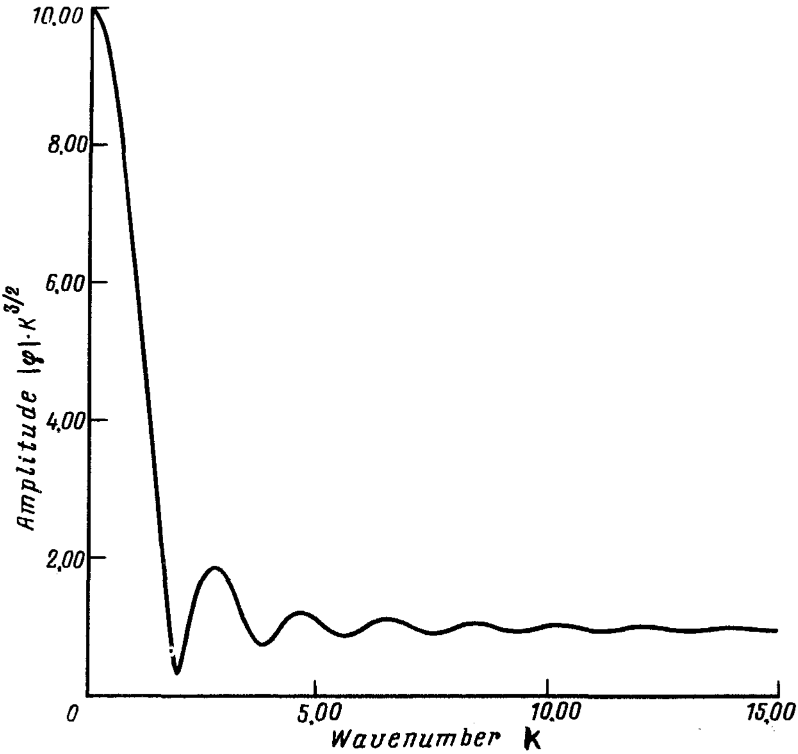

FIG. 1. Spectrum of gravitational waves in the case $\gamma = 4/3$, $H_1/H_2 = 10$. The wave number is expressed in units of H_2a_2. The mean square amplitude of the gravitational waves, $|\phi|k^{3/2}$ is expressed in units of $8\pi^2G^{1/2}H_2$, where G is the gravitational constant.

$$F = (4\pi)^{-1/2}(3\gamma/2 - 1)^{\frac{2}{3\gamma-2}}\Gamma\left(\frac{3\gamma+2}{2(3\gamma-2)}\right),$$

$$D = 8\pi^2H_2G^{1/2}, \quad \alpha = 3\gamma/(3\gamma-2), \quad \beta = \pi/(3\gamma-2).$$

The reason for the modulation of the spectrum is that modes with $H_2a_2 \ll k \ll H_1a_1$ drop below the Hubble horizon a second time and begin to oscillate. Since the initial phases were synchronized by the first inflation stage, and since the oscillation time depends on k, these modes emerge from below the horizon, with different phases.

It has thus been found that a double inflation with an intermediate slowing stage ($\ddot{a} < 0$) generates gravitational waves with a modulated spectrum. The observation of this spectrum might provide unambiguous evidence in favor of field theories which permit such a scenario. Moreover, as we see from (4), a comparison of the heights of the adjacent maxima can provide information about the equation of state of the matter which is predominant in the intermediate stage. Spectra of this type are formed not only in the simple model discussed here but also in any other double-inflation scenario with a sufficiently long intermediate stage of slowing. In most cases, features of the same sort arise in the density perturbation spectrum in these scenarios.[11]

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