Finite nature of a 2*d* induced gravity in perturbation theory

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The structure of the divergences of a local 2*d* induced gravity in a generally covariant linear gauge of a fairly general type is analyzed. The effective action is shown to be finite for one particular choice of this gauge. This result agrees completely with the idea that the theory is exactly solvable in the light-cone gauge.

An $SL(2,R)$ Kac–Moody symmetry was recently discovered[1] in a 2*d* induced gravity in the light-cone gauge. The theory was solved exactly on the basis of this symmetry. The situation can be summed up by the following assertion: The effective action of the theory is finite and has the same form as the classical action, except for a finite renormalization of the coupling constant and of the gravitational field $g_{\mu\nu}$ (we are using a nonlocal model[1]):

$$\Gamma = \frac{\gamma}{96\pi}\int R\Box^{-1}R, \quad g_{\mu\nu} = \eta_{\mu\nu} + Zh_{\mu\nu}, \tag{1}$$

$$Z = 1 + \frac{12}{\gamma}, \quad \gamma = \frac{1}{2}[c - 37 \pm \sqrt{(c-25)(c-1)}],$$

The initial coupling constant is $a/16\pi \equiv c/96\pi$, where c is the central charge. The result in (1) was also derived in a conformal gauge[2] in which a Virasoro algebra is realized as the symmetry. Since the results of Refs. 1 and 2 are nonperturbative and were derived in noncovariant gauges, it is exceedingly important to realize how result (1) is manifested in an ordinary perturbative approach and to see the extent to which it depends on the choice of gauge.[3]

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We started from a local model of an induced gravity (this model was discussed in Ref. 2):

$$S = \int d^2x\sqrt{g}\left\{\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + c_1\phi R\right\}. \tag{2}$$

With the choice $c_1 = \frac{1}{2}[(a-1/6)/2\pi]^{1/2}$, the theory is equivalent to the nonlocal model of Ref. 3. We believe that the local form of (2) is more convenient for a perturbation-theory analysis, since it allows us to use the standard methods of local quantum field theory.

Since $[\phi] = 0$, the theory is not multiplicatively renormalizable *a priori*, despite the fact that it is index-renormalizable. The explanation here is that dimensionality and covariance considerations tell us that counterterms of the σ-model type arise:

$$\Delta S = \int d^2x\sqrt{g}\{\frac{1}{2}g^{\mu\nu}Z_1(\phi)\partial_\mu\phi\partial_\nu\phi + c_1 Z_2(\phi)R\phi\}. \tag{3}$$

As we will show below, terms of this sort do not arise if the gauge is chosen in a special way.

In calculating the counterterms, we use the method of a background field along with the standard algorithm for singling out divergences. According to the background-field method, we use $g_{\mu\nu}\to g_{\mu\nu} + h_{\mu\nu}$, $\phi\to\phi+\varphi$, where $h_{\mu\nu}$ and φ are quantum fields.

We choose the action which fixes the coordinate-independent invariance in the form

$$S_{GF} = -\frac{c_1}{2\alpha}\int d^2x\sqrt{g}\chi_\mu\phi\chi^\mu, \tag{4}$$

where the linear gauge χ_μ is

$$\chi_\mu = \nabla_\lambda h^\lambda_\nu - \frac{1}{2}\beta\nabla_\mu h - \frac{\gamma}{\phi}\nabla_\mu\varphi + \varphi X_\mu(\phi) + h_{\rho\sigma}Y^{\rho\sigma}_\mu, \tag{5}$$

and α, β, and γ are parameters of the gauge. The background dimensionalities of the functions X_μ, $Y^{\rho\sigma}_\mu$ are unity; i.e.,

$$X_\mu = X(\phi)\partial_\mu\phi,$$

$$Y^{\rho\sigma}_\mu(\phi) = Y_1(\phi)(\delta^\rho_\mu\nabla^\sigma + \delta^\sigma_\mu\nabla^\rho)\phi + Y_2(\phi)g^{\rho\sigma}\partial_\mu\phi, \tag{6}$$

where $X(\phi)$, $Y_{1,2}(\phi)$ are arbitrary dimensionless functions of ϕ.

Single-loop counterterms in theory (2) were derived in Ref. 3 for the case $\alpha=\beta=1$, $X=Y=0$ (a detailed analysis for a nonlocal model was carried out in Ref. 4). The following results were found:[3]

$$Z_1(\phi) = \frac{1}{\epsilon}\left(\frac{3}{c_1\phi} - \frac{1}{\phi^2}\right),\quad Z_2 = \frac{2}{\epsilon c_1^2}. \tag{7}$$

There counterterms are eliminated through a local nonpolynominal reparametrization of the field $g_{\mu\nu}$ and a multiplicative renormalization of the parameter c_1 (or of the field ϕ, since the β function β_{c_1} is arbitrary).[3]

We now assume $X(\phi)\neq 0$, $\alpha=\gamma$, $\beta=1$, $Y_{1,2}(\phi)=0$ in the gauge condition. In this case the quadratic part of action (2) can be written in the following form, where we are using (4) (and we are breaking $h_{\mu\nu}$ up into a traceless part $\bar{h}_{\mu\nu}$ and a trace h):

$$\mathscr{H}=\begin{pmatrix} c_1\phi & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \hat{H} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1-\frac{c_1\alpha}{\phi} & -\frac{2}{c_1} \\ 0 & -\frac{2}{c_1} & 0 \end{pmatrix}^{-1} (\hat{K}^{\mu\nu}\nabla_\mu\nabla_\nu)^{-1}, \tag{8}$$

where $\hat{K}^{\mu\nu}\equiv\hat{K}^{\mu\nu}(g_{\alpha\beta})$, and the operator $\hat{H}$ has the structure

$$\hat{H}=\hat{1}\Box^2+\hat{\Omega}^{\mu\nu\lambda}\nabla_\mu\nabla_\nu\nabla_\lambda+\hat{V}^{\mu\nu}\nabla_\mu\nabla_\nu+\hat{N}^\mu\nabla_\mu+\hat{U}. \tag{9}$$

Of the four factors in (8), only the operator $\hat{H}$ can contribute to a divergence of the expression Tr ln $\hat{H}$ (a possible contribution from $\hat{K}^{\mu\nu}\nabla_\mu\nabla_\nu$ is proportional to an Euler number).

Working in the standard way, we find the single-loop counterterms in a dimensional regularization:

$$\Delta S=\frac{1}{\epsilon}\int d^2x\sqrt{g}\,\mathrm{tr}\left\{-\frac{3}{32}\hat{\Omega}^{\mu\nu}_{\nu}\hat{\Omega}^{\lambda}_{\mu\lambda}-\frac{1}{16}\hat{\Omega}_{\mu\nu\lambda}\hat{\Omega}^{\mu\nu\lambda}+\frac{1}{2}\hat{V}^{\mu}_{\mu}\right\}+\Delta S_{ghost}. \tag{10}$$

An analysis (which we will not reproduce here) of expression (10) and ΔS_{ghost} shows[1)] that the counterterms which arise have the structure of (3); Z_2 does not depend on ϕ, and Z_1 is a linear combination of the gauge function $X(\phi)$. Choosing $X(\phi)$ and α in a special way, we can then show that the single-loop counterterms vanish.

Going through the usual recurrence procedure [expanding $X(\phi)$ in a series in the loop number parameter], we can show that the counterterms are eliminated in higher orders with the help of gauge condition (4). If a ϕ dependence of Z_2 appears in higher orders, one can cite arguments that this dependence can be eliminated by choosing the functions $Y_{1,2}(\phi)$ appropriately.

We have thus presented arguments for the existence of a covariant gauge in which the effective action and the S matrix are finite, in total agreement with Refs. 1 and 2. In taking this approach, unfortunately, we cannot reproduce the constants of the final renormalization of the effective action, since we looked at only the divergences.

Since there exists a special gauge of the type in (4), in which the effective action is finite, it is natural to suggest that there also exists some new symmetry of action (2), like that in an $SL(2,R)$ current algebra. It would be interesting to derive this symmetry explicitly.

The result in (1) also holds for $2d$ supergravities in the light-cone gauge.[5] It would thus evidently also be possible to construct a gauge of the type in (4), in which the S matrix and the effective action are finite, for a $2d$ supergravity.

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[1)] The details of the (exceedingly tedious) calculations reported below, which required the assistance of a computer, will be published separately.

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