Induced focusing of electromagnetic wave in a wake plasma wave

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Short packets of electromagnetic radiation can be focused or defocused, depending on the phase of their interaction with the wake plasma wave.

The excitation of wake plasma waves by short laser pulses[1,2] or relativistic electron beams[3] is attracting much interest in connection with the acceleration of charged particles[1–5] and the effort to raise the frequency of laser light.[6,7] When the transverse nonuniformity is taken into account in the study of these problems, several new effects, not seen in the one-dimensional approach, arise.[8,9]

Our purpose in the present letter is to examine how a transverse nonuniformity affects the evolution of an electromagnetic pulse as it interacts with a wake plasma wave.

We adopt the approximation that the wavelength of the plasma wave is small in comparison with the length scale of the field variation in the transverse direction. We start from Maxwell's equations and the relativistic hydrodynamic equations for the plasma electrons. We find the following system of equations to describe the self-consistent evolution of an electromagnetic wave packet and the excitation of a wake plasma wave:

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$$2i\frac{\partial \vec{a}}{\partial \tau}+2\alpha\frac{v_g}{c}\frac{\partial^2 \vec{a}}{\partial \tau \partial \xi}+\alpha^2\frac{\partial^2 \vec{a}}{\partial \xi^2}+\Delta_\perp \vec{a}=-\frac{\phi}{1+\phi}\vec{a}, \tag{1}$$

$$\frac{\partial^2 \phi}{\partial \xi^2}=\left(\frac{1+|\vec{a}|^2/2}{(1+\phi)^2}-1\right), \tag{2}$$

where $\phi = e\varphi/m_e c^2$ is the dimensionless electrostatic potential, and $\vec{a}$ is the dimensionless complex amplitude of the vector potential of the electromagnetic pulse. This vector potential is given by

$$e\vec{A}_\perp/m_e c^2=\frac{1}{2}(\vec{a}(\xi,\vec{\rho},\tau)\exp(-i\omega_0 t+ik_0(x-v_g t))+\text{c.c.} \tag{3}$$

Here $\tau = \omega_p^2 t/\omega_0$, $\xi = k_p(x - v_g t)$, and $\vec{\rho} = k_p \vec{r}_\perp$ are the dimensionless time, the dimensionless longitudinal coordinate, and the dimensionless transverse coordinate; $\alpha = \omega_p/\omega_0 \ll 1$; $\omega_p^2 = 4\pi n_0 e^2/m_e$; $\omega_0^2 = k_0^2 c^2 + \omega_p^2$; $k_p = \omega_p/v_g$; and $v_g = c^2 k_0/\omega_0 \simeq c$.

Equations (1) and (2) differ from those derived in Ref. 3 by the last term on the left-hand side of (1), which incorporates a dependence on $\vec{\rho}$. A similar system of equations was written in Ref. 10.

Assuming axial symmetry, we use the paraxial approximation, writing a and ϕ as

$$a(\xi,\rho,\tau)=A(\xi,\tau)\exp(-q(\xi,\tau)\rho^2/2), \tag{4}$$

$$\phi(\xi,\rho,\tau)=\phi_0(\xi,\tau)+\frac{1}{2}G(\xi,\tau)\rho^2+\dots\,. \tag{5}$$

Restricting the analysis to the first terms of the expansion in ρ^2, we find the following system of equations from (1) and (2):

$$2i\frac{\partial A}{\partial \tau}+2\alpha\frac{\partial^2 A}{\partial \tau \partial \xi}+\alpha^2\frac{\partial^2 A}{\partial \xi^2}-2qA=-\frac{\phi_0}{1+\phi_0}A, \tag{6}$$

$$2i\frac{\partial Y}{\partial \tau}+2\alpha\frac{\partial^2 Y}{\partial \tau \partial \xi}+\alpha^2\frac{\partial^2 Y}{\partial \xi^2}-4qY=-\frac{\phi_0}{1+\phi_0}Y+\frac{GA}{(1+\phi_0)^2}, \tag{7}$$

$$\frac{\partial^2 \phi_0}{\partial \xi^2}=\frac{1}{2}\left(\frac{1+|A|^2/2}{(1+\phi_0)^2}-1\right), \tag{8}$$

$$\frac{\partial^2 G}{\partial \xi^2}=-\frac{1+|A|^2/2}{(1+\phi_0)^3}G-\frac{|A|^2}{2(1+\phi_0)^2}\text{Re}q, \tag{9}$$

where $Y \equiv qA$.

We consider the propagation of a short, small-amplitude electromagnetic packet in the field of the wake plasma wave, whose transverse variation is characterized by the quantity G (which therefore also characterizes the transverse variation of the effective dielectric constant

$$\varepsilon_{\text{eff}} = 1 - [\omega_p^2/\omega_0^2(1+\phi)]$$

Directly behind the source of the wake wave, the transverse dimension of the nonuniformity, ϕ, is equal to the transverse dimension of the source. It is easy to see from (9) that G oscillates with distance from the source. As a result, an initially unfocused ($\mathrm{Im} q_1 = 0$) short electromagnetic packet may begin to be focused as it interacts with the wake wave [this happens under the condition

$$G(k_p R_1)^4/2(1+\phi_0)^2 < -1,$$

where $R_1 = k_p^{-1}(\mathrm{Re} q_1)^{-1/2}$ is the initial transverse dimension of the packet]. Alternatively, it may begin to be defocused (if the opposite inequality holds). Working from (6) and (7), and ignoring terms on the order of α and α^2, we find an estimate of the time scale of the focusing ($G<0$):

$$t_f \simeq \pi/2\, \omega_0/2\omega_p^2 [2(1+\phi_0)^2/|G|]^{1/2}.$$

Over this time, the packet shrinks to a minimum size set by diffraction,

$$R_{\min} \simeq 2(c^2/\omega_0 R_1) t_f .$$

In the case of a weak wake plasma wave ($\phi_0 \ll 1$) the optimum region for focusing is that near the maxima of ϕ_0. In this case we have

$$G \simeq -(\phi_0/R_0^2),$$

where R_0 is the radius of the source of the wake wave.

In the case of a highly nonlinear wake wave, the relativistic amplitude dependence of the frequency of the plasma waves accounts for the curvature of the wavefront of the wake wave. As a result, the transverse variation of the potential grows markedly in the region between the first zero and the first minimum of ϕ_0 behind the source. If the wake wave is excited by a relativistically strong ($\int |A|^2 d\xi > 1$), ultrashort ($\Delta\tau \int |A|^2 d\xi < \omega_p^{-1}$) laser pulse, the maximum value of G in this region is on the order of $G \simeq -2(\int |A|^2 d\xi)^2/k_p^2 R_0^2$, as we see from (8) and (9). Here the radius of the source is defined by $R_0 = k_p^{-1}(\mathrm{Re} q_0)^{-1/2}$. The corresponding focusing time is

$$t_f \simeq \frac{2\omega_0}{\omega_p^2} k_p R_0 / \int |A|^2 d\xi. \tag{10}$$

We have solved Eqs. (6)–(9) numerically. We used an intense electromagnetic pulse as the source of the wake plasma wave. In the wake behind the source we had a small-amplitude packet with a width no greater than the radius of the first packet. It was not our purpose in this study to learn about the details of the effect of the transverse nonuniformity on the dynamics of the source itself.

Figure 2 shows the results for the case $\alpha = 0.01$. The parameters of the source here were a maximum amplitude $A_0 = 3$ and $\int |A|^2 d\xi = 3$. The second pulse was of the same length, and its initial amplitude was $A_1 = 1$. The case shown in Fig. 1 corresponds to the two-dimensional model ($q = 0$). The primary effect here, as in Ref. 6, is a rise in the frequency of the guiding packet, which is in that phase of the nonlinear wake wave in which this effect is at a maximum.

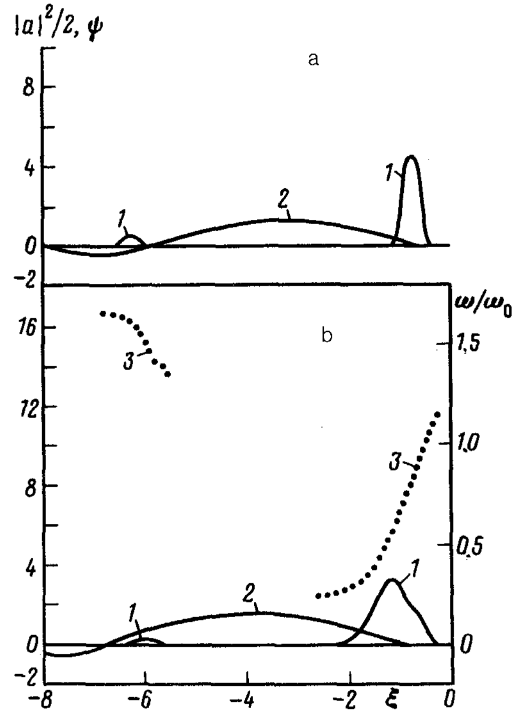

FIG. 1. Excitation of a nonlinear wake plasma wave and increase in the frequency of the guided pulse in the two-dimensional approximation ($q=0$) with $\alpha=0.01$. 1—$|a|^2/2$; 2—ψ; 3—ratio of the local frequency of the electromagnetic radiation, $\omega(\xi, t)$, to ω_0 at the times (a) $t=0$ and (b) $t=150\omega_0/\omega_p^2$.

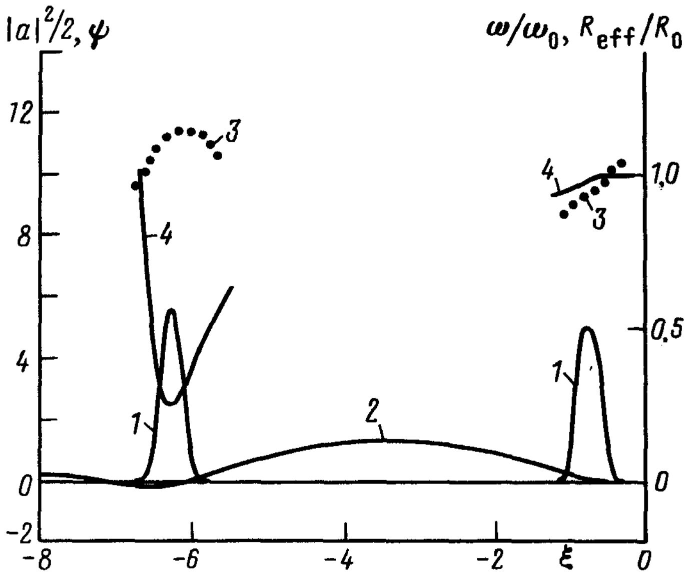

FIG. 2. Induced focusing of the guided pulse with $\alpha=0.01$ and $q_1=q_0=0.001$ at the time $t=27\omega_0/\omega_p^2$. Lines 1–3—The values of $|a|^22$, ψ, and $\omega(\xi, t)/\omega_0$, respectively, at the axis ($\rho=0$); 4—effective values of the local transverse dimension of the electromagnetic packets, $R_{\text{eff}}=(\text{Re}q(\xi, t)^{-1/2}$, normalized to their initial width.

For the parameter values selected, the fastest focusing of the guiding pulse corresponds to the same phase of the interaction. If the transverse-nonuniformity parameter of the source is $q_0 = 0.001$, the induced focusing occurs much more rapidly than the change in the frequency or the growth of the nonlinear distortions of the first pulse. Figure 2 corresponds to the time at which the amplitude of the guiding pulse at the axis has become comparable to the amplitude of the source. The time scale for the focusing of the second pulse (whose initial transverse dimension was assumed to be the same as that of the source: $q_1 = q_0$) agrees fairly well with estimate (10). The compression of the second packet continues to dimensions corresponding to the applicability limit of the quasi-two-dimensional approximation.

In the absence of a source of a wake wave, the effect of the transverse nonuniformity is seen over a time more than an order of magnitude longer than the induced-focusing time for the same parameter values of the pulse (the effect of the nonuniformity is seen in a self-focusing of the rear of the pulse).

A more detailed analysis of the transverse structure of the potential of the wake field near the phase corresponding to the fastest focusing shows that in this example the paraxial approximation is applicable at distances $\rho < \frac{1}{2}(\mathrm{Re} q_0)^{-1/2}$ from the axis, i.e., something on the order of a fourth of the energy of the guiding pulse is drawn into the process of induced focusing at $q_1 = q_0$. Far from the axis, the radiation is dissipated.

For a relativistically strong laser pulse with $\lambda_0 = 1\ \mu\mathrm{m}$, a transverse dimension $R_0 \simeq 0.5$ mm, and an intensity of $10^{19}\ \mathrm{W/cm}^2$, which excites a wake wave in a plasma with a density $n_0 \simeq 10^{17}\ \mathrm{cm}^{-3}$ ($\alpha = 0.01$), the length scale of the induced focusing of the guided packet is a few centimeters.

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