Effect of superfluid countercurrent on a domain with uniform magnetization precession in ^{3}He–*B*

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A solution is derived for the position and shape of a wall of a uniformly precessing domain in ^{3}He–*B* for the case in which the wall forms in a system in which there is a countercurrent of the superfluid and normal components of the liquid. The behavior of the thickness of the wall as a function of its position agrees qualitatively with results on the increase in the spin-diffusion magnetic relaxation of a uniformly precessing domain during the rotation of ^{3}He–*B*.

A uniform precession of the magnetization of ^{3}He–*B* in a nonuniform magnetic field was discovered in 1984 as the result of an interplay between theoretical and experimental research.[1,2] That effect is now being used extensively to study the quantum rotation of superfluid ^{3}He–*B* in the joint Soviet–French project ROTA. These experiments were stimulated by the 1986 discovery, by Bun'kov and Hakonen, that the characteristics of the uniformly-precessing-domain (UPD) signal are affected strongly both by a countercurrent of the normal and superfluid components and by quantum vortices.[3] Further research revealed that the effect of the countercurrent was linked with an anisotropy of the density of the superfluid component, ^{3}He–*B*, which shifts the position of the UPD wall.[4,5] The study reported below was carried out to determine whether a domain wall of this sort could exist in a steady state, to determine its thickness, and to determine the effect on the experimentally observed increase in the spin-diffusion magnetic relaxation.

A UPD is a unique distribution of the precessing magnetization which is set up by the existence of superfluid spin currents and the characteristic potential of the dipole–dipole interaction in superfluid ^{3}He–*B*. Inside a UPD, the nonuniformity of the Larmor precession frequency is offset by a dipole–dipole frequency shift, which arises when the magnetization is tilted through an angle greater than arccos($-$ 1/4). As a result, the magnetization precesses in a spatially uniform manner. Uniformly precessing domains are observed in a strong nonuniform field in both pulsed and continuous NMR.[6,7] In the case of continuous NMR, the UPD occupies a region in which the magnetic field is weaker than ω_p/γ, where ω_p is the frequency of the rf field. The UPD precession frequency corresponds to the frequency of the rf field in this case, and the phase difference between the rf field and the UPD corresponds to a transfer of energy to the UPD. This transfer of energy tends to make up for the dissipation of Zeeman energy as a result of magnetic relaxation. Near the field ω_p/γ, there is a domain wall, which separates the UPD from a region with a steady-state magnetization. The shape of this domain wall was studied in detail in Refs. 8 and 9. In the present study we focused on the basic characteristics of the UPD in the case in which a countercurrent of the normal and superfluid components of ^{3}He–B is added to the factors considered previously.

For definiteness, we assume that the spatially uniform countercurrent $\vec{v}_s$ is directed across the magnetic-field gradient. This countercurrent gives rise to an orienting effect on the magnetic-field-induced orbital angular momentum $\vec{l} = R(n,\Theta)\vec{s}$, which is characterized by a kinetic-energy density $W_v = -1/2\delta\rho(\vec{l}\vec{v}_s)^2$, where $\delta\rho$ is the difference between the density of the superfluid component ^{3}He–B in a magnetic field in the directions along and across the orbital angular momentum $\vec{l}$. This effect lifts the degeneracy of the energy of the orientation of the order parameter in the plane perpendicular to the magnetic field.

Following Fomin's method,[8,10] we consider the potential of the dipole–dipole and orienting energies for the case of a spatially uniform precession of the magnetization, which is tilted through an angle β. For this purpose we use the expression derived in Ref. 11 for the dipole–dipole energy in the case in which the chamber wall has an orienting effect on the order parameter. After that expression is minimized with respect to the internal degrees of freedom (the angle Φ), it becomes

$$W_D = \chi/\gamma^2\, \Omega_B^2\, \{\mu(1-u)\,(2/5 - 1/4\mu(1-u)) - 1/30\}. \tag{1}$$

Here $u = \cos\beta$, and $\mu = (5/4)\sin^2(\phi)$, where ϕ is the angle between the vector $\vec{n}$, averaged over the precession period, and $\vec{H}$. This angle is directly related to the angle by which $\vec{l}$ deviates from $\vec{H}$, i.e., ψ, by $\sin(\psi)^2 = 2\mu - \mu^2$. The longitudinal NMR frequency Ω_B is a measure of the scale of the dipole energy. The orienting effect of the magnetic field and the countercurrent is described in this notation by

$$\begin{aligned} W_H &= 4/5\,\mu a H^2, \\ W_v &= -1/2\,\delta\rho\, v^2(2\mu - \mu^2). \end{aligned} \tag{2}$$

Figure 1 shows the potential of the dipole–dipole and orienting energies for typical experimental conditions ($P = 21$ atm, $T = 0.625$ of T_c, $H = 284$ Oe, $v_s = 0.7$ cm/s). We see that when the magnetization is tilted 104° by the corresponding UPD, the deep minimum in the dipole energy stabilizes the direction of $\vec{l}$ along $\vec{H}$ (point A in this figure). In the case of a steady-state magnetization, in contrast, the countercurrent rotates $\vec{l}$ along $\vec{v}_s$ (point B in this figure). The corresponding solution for the domain wall between these two states lies on curve AB. Except for the region of small tilts of the magnetization, in which case we have

$$(1-u) < \frac{\gamma^2}{2\chi\Omega_B^2}\left(5\delta\rho\, v_s^2 - 4aH^2\right), \tag{3}$$

the minimum of these energies stabilizes $\vec{l}$ along $\vec{H}$. At smaller magnetization tilt angles, the vector $\vec{l}$ rotates. In this case we have

$$\mu = \frac{10\gamma^2\,\delta\rho\, v_s^2 - 8\gamma^2 aH^2 - 4(1-u)\chi\Omega_B^2}{10\gamma^2\delta\rho\, v_s^2 - 5(1-u)^2\chi\Omega_B^2}. \tag{4}$$

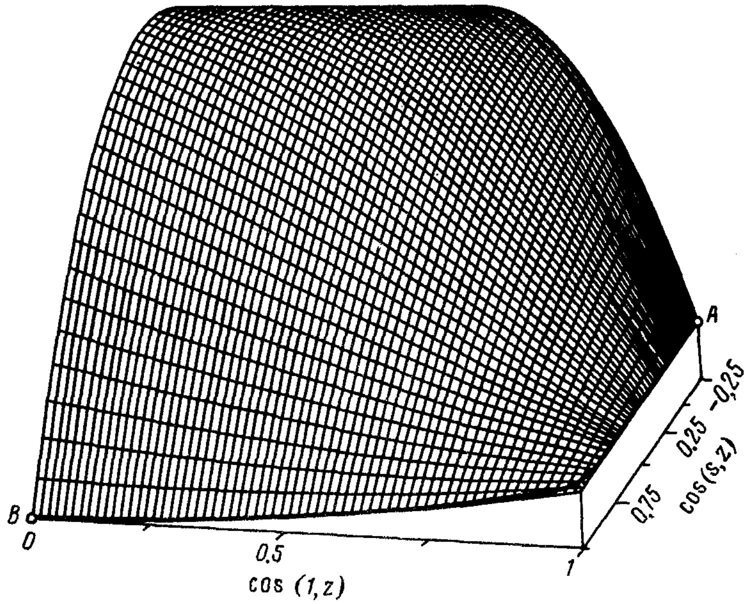

FIG. 1. The dipole–dipole energy and the orientational energy in ^{3}He–B versus the tilt angles of the precessing magnetization and the orbital angular momentum.

To find the position and shape of the domain wall, we assume that the spatially nonuniform solution is a small perturbation and lies at a minimum of the dipole-dipole and orienting energies. The low-frequency dynamics of the ^{3}He–B magnetization against the background of its precession is described by the system of equations[10]

$$\begin{aligned} \frac{\chi}{\gamma^2}\omega_p\frac{\partial\alpha}{\partial t} &= \frac{\partial W}{\partial u} - \frac{\partial}{\partial z}\left(\frac{\partial W}{\partial u'}\right), \\ \frac{\chi}{\gamma^2}\omega_p\frac{\partial u}{\partial t} &= \frac{\partial}{\partial z}\left(\frac{\partial W}{\partial \alpha'}\right), \end{aligned} \tag{5}$$

where α and β are the phase and tilt angle of the magnetization in a coordinate system rotating at the frequency ω_p. The Hamiltonian W consists of the dipole–dipole and orientational energies discussed above and also a term which describes the gradient energy W_∇ and a spectroscopic term which describes the stability of the UPD, W_Ω. They can be written

$$\begin{aligned} W_\nabla &= \frac{\chi}{2\gamma^2}\left[c_\parallel^2\left(2(1-u)\alpha'(\alpha'-\Phi') + \Phi'^2 + \beta'^2\right)\right. \\ &\qquad \left. - 2(c_\parallel^2 - c_\perp^2)\left((1-u)\alpha' - \Phi'\right)^2\right], \\ W_\Omega &= -\frac{\chi}{\gamma}\,u\,\omega_p \nabla H(z-z_0), \end{aligned} \tag{6}$$

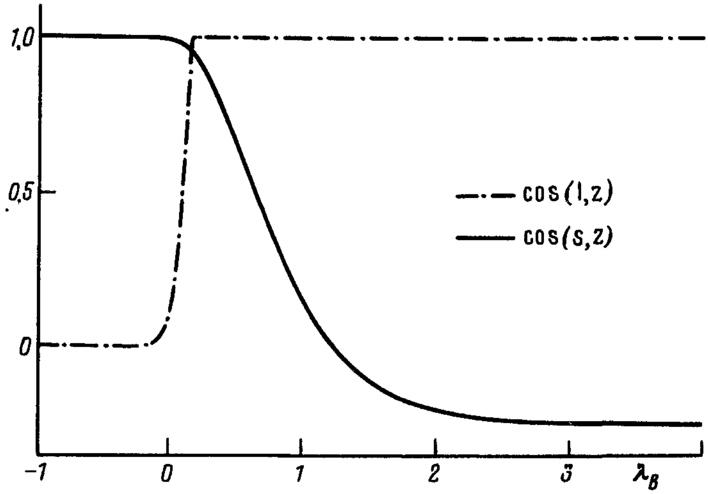

FIG. 2. Spatial variation of the cosines of the tilt angles of the precessing magnetization and the orbital angular momentum versus the direction of the magnetic field for a domain wall under countercurrent conditions.

where Φ is an angle which characterizes the "internal" orientation of the order parameter, $c_{\parallel}$ and $c_{\perp}$ are the velocities of spin waves along and across the magnetic field, z is the coordinate which runs along the gradient of the magnetic field, ∇H, and z_0 corresponds to the point at which the condition $\gamma H = \omega_p$ holds. We seek a steady-state solution of Eqs. (5), as in Ref. 8. The vanishing of the left-hand side of the second equation imposes a relationship between α' and u' which corresponds to the case in which there is no superfluid spin current through the domain wall. We seek a solution in a coordinate system rotating at the frequency of the external rf field, ω_p. The precession frequency $\partial\alpha/\partial t$ is then zero. To find a solution of Eqs. (5) which asymptotically becomes UPD and a domain with a steady-state magnetization as z tends toward $+\infty$ and $-\infty$, respectively, in the presence of a countercurrent, we must resort to numerical methods. Figure 2 shows the spatial variation of the tilt angle of the precessing magnetization and of the orbital angular momentum versus the direction of the magnetic field as found through a numerical solution by the fourth-order Runge-Kutta method, for the same experimental conditions as in Fig. 1.

Let us examine some physical properties of this solution. We first find the position of the domain wall. This position corresponds quite accurately to the equilibrium position in which the difference between the kinetic-energy densities of the countercurrent on the two sides of the wall is offset by a spectroscopic term which characterizes the stability of a domain:

$$\frac{1}{2}\delta\rho\, v_s^2 = \frac{5}{4}\frac{\chi}{\gamma}\omega_p \nabla H \Delta z = \frac{5\chi}{4\gamma^2}\omega_p \Delta\omega. \qquad (7)$$

Here Δz is the shift of the domain wall away from its spectroscopic-equilibrium position. This solution was found by intuition[5] and is supported by the numerical calculation. In the experiments of the ROTA project, a shift of the position of the domain wall upon rotation has been observed. That shift also corresponds well to the solution found.[4,5]

With regard to the shape of the magnetization distribution inside a domain wall, we note that it can be represented by an analytic solution in the following approximate case. We are interested in only that part of the domain wall in which $\vec{l}$ is directed along $\vec{H}$. The requirement that u asymptotically approach $-1/4$ with increasing z and the specification of the position of the wall completely determine the solution. To simplify the problem we consider the isotropic case with $c_{\parallel} = c_{\perp}$. The change of variables $u = 3/8 + 5/8\cos v$ in this case converts the first equation of system (5) into a sine-Gordon equation

$$v'' = -\frac{1}{\lambda_B^2}\sin v, \qquad \lambda_B = \frac{c_{\parallel}}{\sqrt{\omega\Delta\omega}}, \tag{8}$$

where λ_B is a characteristic size of the given wall. There is an analytic solution of this equation which asymptotically approaches $u = -1/4$ as $z \to \infty$; i.e., it converts smoothly into a UPD. This is the one-soliton solution $v = -\pi + 4\arctan[\exp(z/\lambda_B)]$. Correspondingly, for the variable u, it is

$$u = 1 - \frac{5}{4}\tanh^2\frac{z}{\lambda_B}\,. \tag{9}$$

The physical quantity which is determined by the thickness and shape of the domain wall is the spin diffusion across the boundary, which results in a magnetic relaxation and which is found experimentally:[8]

$$\frac{dE}{dt} = -\frac{D\omega^2\sigma}{\lambda}\,. \tag{10}$$

Here σ is a form factor of the wall. For a domain wall in ^{3}He–B at rest, σ has been found to be 1.1. A calculation of the form factor of a domain wall for spin diffusion with the solution discussed above, for a wall thickness λ_B, yields $\sigma = 35/16$. It should be noted that although this solution is not an *a priori* correct solution at small β, i.e., in the region in which the vector l rotates, it gives us a good approximation for the magnitude of spin diffusion across the wall, which is determined primarily by the region of large magnetization tilt angles.

A numerical solution of the wall shape in Fig. 2 was found for the case of anisotropic spin-wave velocities, with $c_{\parallel} = 4/3c_{\perp}$. We assumed $132\omega_p\Delta\omega = \Omega_B^2$. This condition corresponds to the experiments of Ref. 5. The form factor of this solution turns out to be 2.2.

In experiments on the rotation of ^{3}He–B, a value was found for the spin diffusion which corresponds functionally to a wall thickness λ_B with a form factor of 2.29. Our solution thus describes the change in the thickness of a domain wall during rotation not only qualitatively but also quantitatively.

A numerical calculation for the case in which the gradients of the angles α and β

are directed across H shows that the wall has the same shape, and the form factor is $\sigma = 1.76$.

We note in conclusion that the characteristic size of the wall, λ_B, corresponds to an analog of the Ginzburg–Landau length for a spin supercurrent.[10] A domain wall which is shifted from its spectroscopic-equilibrium position z_0 can thus be thought of as a permanent phase disruption.

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