Upper bound on the supersymmetry breaking scale in supersymmetric $SU(5)$ model

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The status of coupling constant unification in the standard supersymmetric $SU(5)$ model and its extensions is discussed. Taking into account the uncertainties associated with the initial coupling constants and threshold corrections at the low and high scales, we find that in the standard supersymmetric $SU(5)$ model the scale of the supersymmetry breaking can be as high as 10^8 GeV. In the extensions of the standard $SU(5)$ model it is possible to increase the supersymmetry breaking scale to 10^{11} GeV.

There has recently been renewed interest[1–12] in grand unification related to the recent LEP data which allow us to measure $\sin^2(\theta_w)$ with unprecedented accuracy. The world averages with the LEP data mean that the standard nonsupersymmetric $SU(5)$ model[13] is ruled out finally and forever [the fact that the standard $SU(5)$ model is in conflict with experiment was well known before the LEP data] but maybe the most striking and impressive lesson from LEP is that the supersymmetric extension of the standard $SU(5)$ model[16–18] predicts the Weinberg angle θ_w, in very good agreement with experiment. The remarkable success of the supersymmetric $SU(5)$ model is considered by many physicists as the first hint in favor of the existence in nature of low-energy broken supersymmetry. Is it possible to invent nonsupersymmetric generalizations of the standard $SU(5)$ model which does not confront the experimental data or to increase the supersymmetry breaking scale significantly? In the $SO(10)$ model the introduction of the intermediate scale, $M_I \sim 10^{11}$ GeV allows us to obtain the Weinberg angle θ_w, in agreement with experiment.[19] In Refs. 20 and 21 it was suggested that the problems of the standard $SU(5)$ model can be overcome by introducing additional split multiplets $5\oplus\bar{5}$ and $10\oplus\overline{10}$ in the minimal $3(\bar{5}\oplus 10)$ of the $SU(5)$ model. The extension of the standard $SU(5)$ model with light scalar colored octets and electroweak triplets was proposed in Ref. 22.

In this paper we discuss the coupling constant unification in the standard supersymmetric $SU(5)$ model and its extensions. Taking into account the uncertainties associated with the initial gauge coupling constants and the threshold corrections at the low and high scales, we conclude that in the standard supersymmetric $SU(5)$ model the scale of the supersymmetry breaking can be as high as 10^8 GeV. We find also that in the extensions of the standard $SU(5)$ supersymmetric model it is possible to increase the supersymmetry breaking scale up to 10^{11} GeV.

The standard supersymmetric $SU(5)$ model[16–18] contains three light supermatter generations and two light super-Higgs doublets. A minimal choice of massive supermultiplets at the high scale is $(\bar{3},2,\frac{5}{2})\oplus$c.c. massive vector supermultiplet with the mass M_v, massive chiral supermultiplets (8,1,0), (1,3,0), and (1,1,0) with the masses m_8, m_3, and m_1 [embedded in a 24 supermultiplet of $SU(5)$], and a $(3,1,-\frac{1}{3})\oplus(-3,1,\frac{1}{3})$ complex Higgs supermultiplet with a mass M_3 embedded in $5\oplus\bar{5}$ of $SU(5)$. In the low-energy spectrum we have squark and slepton multiplets $(\tilde{u},\tilde{d})_L$, $\tilde{u}_L^c$, $\tilde{d}_L^c$, $(\tilde{\nu},\tilde{e})_L$, $\tilde{e}_L^c$ plus the corresponding squarks and sleptons of the second and third supergenerations. In the low-energy spectrum we also have the $SU(3)$ octet of the gluino with a mass $m_{\tilde{g}}$, the triplet of the $SU(2)$ gaugino with a mass $m_{\tilde{w}}$, and the photino with a mass $m_{\tilde{\gamma}}$. For the energies between M_z and M_{GUT} we have the effective $SU(3)\otimes SU(2)\otimes U(1)$ gauge theory. In the one-loop approximation the corresponding solutions of the renormalization-group equations are well known.[18] Instead of the prediction of $\sin^2(\theta_w)$ following Ref. 6, we consider the following one-loop relations between the effective gauge coupling constants, the mass of the vector massive supermultiplet M_v, and the mass of the super-Higgs triplet M_3:

$$A\equiv 2\left(\frac{1}{\alpha_1(m_t)}-\frac{1}{\alpha_3(m_t)}\right)+3\left(\frac{1}{\alpha_1(m_t)}-\frac{1}{\alpha_2(m_t)}\right)=\Delta_A, \tag{1}$$

$$B\equiv 2\left(\frac{1}{\alpha_1(m_t)}-\frac{1}{\alpha_3(m_t)}-3\left(\frac{1}{\alpha_1(m_t)}-\frac{1}{\alpha_2(m_t)}\right)\right)=\Delta_B, \tag{2}$$

where

$$\Delta_A=\left(\frac{1}{2\pi}\right)(\delta_{1A}+\delta_{2A}+\delta_{3A}), \tag{3}$$

$$\Delta_B=\left(\frac{1}{2\pi}\right)(\delta_{1B}+\delta_{2B}+\delta_{3B}), \tag{4}$$

$$\delta_{1A}=44\ln\left(\frac{M_v}{m_t}\right)-4\ln\left(\frac{M_v}{m_{\tilde{g}}}\right)-4\ln\left(\frac{M_v}{m_{\tilde{w}}}\right), \tag{5}$$

$$\delta_{2A}=-12\left[\ln\left(\frac{M_v}{m_8}\right)+\ln\left(\frac{M_v}{m_3}\right)\right], \tag{6}$$

$$\delta_{3A}=6\ln(m_{(\tilde{u},\tilde{d})_L})-3\ln(m_{\tilde{u}_L^c})-3\ln(m_{\tilde{e}_L^c}), \tag{7}$$

$$\delta_{1B}=0.4\ln\left(\frac{M_3}{m_h}\right)+0.4\ln\left(\frac{M_3}{m_H}\right)+1.6\ln\left(\frac{M_3}{m_{sh}}\right), \tag{8}$$

$$\delta_{2B}=4\ln\left(\frac{m_{\tilde{g}}}{m_{\tilde{w}}}\right)+6\ln\left(\frac{m_8}{m_3}\right), \tag{9}$$

$$5\delta_{3B}=-12\ln(m_{(\tilde{u},\tilde{d})_L})+9\ln(m_{\tilde{u}_L^c})+6\ln(m_{\tilde{d}_L^c})-6\ln(m_{(\tilde{\nu},\tilde{e})_L})+3\ln(m_{\tilde{e}_L^c}). \tag{10}$$

Here m_h, m_H, and m_{sh} are the masses of the first light Higgs isodoublet, the second Higgs isodoublet, and the isodoublet of super-Higgses. Relations (1)–(10) are very useful since they allow us to determine separately two key parameters of the high-energy spectrum of the $SU(5)$ model, the mass of the vector supermultiplet M_v, and the mass of the chiral supertriplet M_3. Both the vector supermultiplet and the chiral supertriplet are responsible for the proton decay in the supersymmetric $SU(5)$ model.[18] In the standard nonsupersymmetric $SU(5)$ model the proton lifetime due to the massive vector exchange is determined by the formula[23]

$$\Gamma(p \to e^+ \pi^o)^{-1} = 4 \times 10^{29 \pm 0.7} \left(\frac{M_v}{2 \times 10^{14}\ \text{GeV}} \right)^4 \ \text{yr}. \tag{11}$$

In the supersymmetric $SU(5)$ model the GUT coupling constant is $\alpha_{\text{GUT}} \approx 1/25$ compared to $\alpha_{\text{GUT}} \approx 1/41$ in the standard $SU(5)$ model. We must therefore multiply expression (11) by the factor $(25/41)^2$. From the current experimental limit[24] $\Gamma(p \to e^+ \pi^o)^{-1} \geqslant 9 \times 10^{32}$ yr we conclude that $M_v \geqslant 1.2 \times 10^{15}$ GeV. The corresponding experimental limit on the mass of the super-Higgs triplet M_3 depends on the masses of the gaugino and the squarks.[25,26] In our calculations we use the following values for the initial coupling constants:[24,27]

$$\alpha_3(M_z) = 0.120 \pm 0.07, \tag{12}$$

$$\sin^2_{\overline{MS}}(\theta_w)(M_z) = 0.2319 \pm 0.0005, \tag{13}$$

$$[\alpha_{em,\overline{MS}}(M_z)]^{-1} = 127.79 \pm 0.13. \tag{14}$$

For the top quark mass $m_t = 174$ GeV, after the solution of the corresponding renormalization-group equations in the region $M_z \leqslant E \leqslant m_t$, we find

$$A = 184.45 \pm 0.68 \pm 0.92, \tag{15}$$

$$B = 13.31 \pm 0.24 \pm 0.92. \tag{16}$$

Here the first error is the "electroweak" error and the second error is the "strong-coupling" error. An allowance for the two-loop corrections leads to the appearance of additional factors:

$$\delta_{4A,4B} = 2(\theta_1 - \theta_3) \pm 3(\theta_1 - \theta_3), \tag{17}$$

$$\theta_i = \frac{1}{4\pi} \sum_{j=1}^{3} \ln \left[\frac{\alpha_j(M_v)}{\alpha_j(m_t)} \right]. \tag{18}$$

Here b_{ij} are the two-loop β function coefficients. We start with expression (1) and assume that the masses of the octet supermultiplet and of the triplet supermultiplet are the same as the mass of the vector supermultiplet M_v. We ignore the variation of the low-energy spectrum (we assume that all the squarks and sleptons have the same mass). Numerically, we find

$$M_v = 2.0 \times 10^{16 \pm 0.05 \pm 0.07}\ \text{GeV} \tag{19}$$

for the $M_{SUSY} \equiv (m_{\tilde{g}}\, m_{\tilde{w}})^{1/2} = 174$ GeV. From the lower limit on the value of the mass of the vector bosons which are responsible for the baryon number nonconservation, we find an upper limit on the value of the supersymmetry breaking parameter $M_{SUSY} \leqslant 2 \times 10^8$ GeV. For $M_{SUSY} = 10^8$, 10^7, 10^6, 10^5, 10^4, 10^3, 10^2 GeV we find $M_v = (1.4 \times 10^{15}$, 1.8×10^{15}, 3.0×10^{15}, 5.1×10^{15}, 10^{16}, 1.6×10^{16}, $2.4 \times 10^{16}) \times 10^{\pm 0.05 \pm 0.07}$ GeV. The uncertainty in the masses of the colored octet m_8 and the electroweak triplet m_3 leads to the uncertainty $(m_8 m_3 / M_v^2)^{1/3}$ for the supersymmetry breaking scale M_{SUSY}. The uncertainty due to the difference of squark and slepton masses is small for the realistic spectrum when the difference in the masses is less than 3. We will therefore ignore it.

Let us now consider Eq. (2). For $M_{SUSY}=m_t$ and $m_{\tilde{g}}=[\alpha_3(m_t)/\alpha_2(m_t)]m_{\tilde{w}}$ under the assumption that all squark and slepton masses are the same, that the masses of the colored octet m_8 and the electroweak triplet m_3 are equal to the mass of the vector supermultiplet M_v, and that the masses of the super-Higgs and Higgs isodoublets are equal to M_t, we find

$$M_3=6.6\times10^{14\pm0.27\pm1.05}\ \text{GeV}. \tag{20}$$

It should be noted that, because of the absence of the proton decay, the limit on the mass of the Higgs triplet for $M_{SUSY}=m_t$ is[18] $M_3\geqslant 0(10^{16})$ GeV. When we increase the value of M_{SUSY}, two scenarios are possible. According to the first scenario, only the single Higgs isodoublet, with a mass $O(M_z)$, is light and the masses of the second Higgs isodoublet and the super-Higgses are on the order of M_{SUSY}. In the second scenario the first Higgs isodoublet and the super-Higgses, with the masses $O(M_z)$, are relatively light [or the super-Higgses, with the mass $O(1$ TeV) are slightly heavier] and only the second Higgs isodoublet, with the mass $O(M_{SUSY})$, is relatively heavy. We have investigated two scenarios. In our investigation we have used an upper limit $M_3\geqslant 3M_v$ (Ref. 5) which comes from the requirement of the applicability of the perturbation theory. Taking into account the uncertainties in the determination of the parameter B, we found that in the first scenario we have $M_{SUSY}\leqslant 10^5$ GeV and in the second scenario we have $M_{SUSY}\leqslant 10^8$ GeV. If we assume that the difference between the masses of the colored octets and the colored triplets can be as high as a factor of 3, then for $m_8/m_3=3$ we find that in the first scenario the supersymmetry-breaking scale can be up to 10^7 GeV. It should be noted that the proton lifetime due to the exchange of the Higgs supertriplet is proportional to m_{sq}^2 (here we assume that $m_{sq}\sim M_{SUSY}$), consistent with the nonobservation of the proton decay for big M_{SUSY}.

It is instructive to consider the supersymmetric $SU(5)$ model with relatively light colored octet and triplets. Let us consider the superpotential

$$W=\lambda\sigma(x)[\mathrm{Tr}(\Phi^2(x))-c^2], \tag{21}$$

where $\sigma(x)$ is the $SU(5)$ singlet chiral superfield, and $\Phi(x)$ is the chiral 24-plet in the adjoint representation. For the superpotential (21) the colored octet and the electroweak triplet chiral superfields remain massless after the $SU(5)$ gauge symmetry breaking and they acquire the masses $O(M_{SUSY})$ after the supersymmetry breaking. In this scenario we therefore have additional relatively light fields. Lower limit on the mass of the vector bosons leads to the upper limit on the supersummetry-breaking scale, $M_{SUSY}\leqslant 10^{11}$ GeV. In order to satisfy the second equation for the mass of the Higgs triplets, we introduce in the model two additional super-Higgs 5-plets. If we assume that after the $SU(5)$ gauge symmetry breaking the corresponding Higgs triplets acquire mass $O(M_v)$, the light Higgs isodoublet has a mass $O(M_z)$, and the second Higgs isodoublet and the super-Higgses have masses $O(M_{SUSY})$, then we can satisfy Eq. (2) for $M_{SUSY}\sim 10^{11}$ GeV. According to two other scenarios with relatively light octets and triplets, we can increase the grand unification scale up to $O(10^{18})$ GeV, which is welcome from the point of view of the string-unification scenario. In this case the supersymmetry-breaking scale is $M_{SUSY}\sim 10^8$

GeV. For such a value of the supersymmetry-breaking scale one can satisfy Eq. (2) even without the introduction of the additional super-Higgs 5-plets. It is possible also to have a grand unification scale $M_v = 10^{17}$ GeV and $M_{SUSY} \leq 1$ TeV if the octets and triplets are lighter than the vector supermultiplet by a factor of 100.

Let us formulate the main results. We have found that in the standard supersymmetric $SU(5)$ model with colored octet and triplet masses $O(M_v)$ the nonobservation of the proton decay leads to the upper limit $M_{SUSY} \leq 2\times 10^8$ GeV on the supersymmetry-breaking scale. This limit is consistent with the equation for the super-Higgs triplet mass in the second scenario. In the first scenario it is possible to have the supersymmetry-breaking scale M_{SUSY} up to $O(10^7)$ GeV provided that the difference between the colored octet and the electroweak triplet masses is $m_{\tilde{g}}/m_{\tilde{w}} = O(3)$. For the case in which the octets and triplets have the masses $O(M_{SUSY})$ it is possible to increase the supersymmetry-breaking scale to $O(10^{11})$ GeV. In order to satisfy the equation for the super-Higgs triplet mass we must introduce in this case an additional, relatively light pair of super-Higgs doublets. It is possible to increase the grand-unification scale to $O(10^{18})$ GeV for the case in which the octets and triplets have the masses $O(M_{SUSY})$ and $M_{SUSY} = O(10^8)$ GeV, without the introduction of additional light Higgs superdoublets. It should be noted that estimate of the supersymmetry-breaking scale in the supersymmetric $SU(5)$ model, $M_{SUSY} = 10^{3.0\pm 0.8\pm 0.4}$ GeV, was obtained in Ref. 28 on the assumption that $M_v = M_3 = m_3 = m_8$. Our analysis demonstrates that the value of M_v depends only slightly on the high- and low-energy uncertainties in the determination of the spectrum and the low-energy effective coupling constants. In the extraction of the limit on the value of M_{SUSY} our crucial assumption was the inequality[5] $M_3 \leq 3M_v$. The limit on the M_{SUSY} depends strongly on the particular features of the high-energy spectrum (on the splitting between the octet and triplet masses) and on the initial value of the strong-coupling constant. It should be noted that for $M_{SUSY} \geq O(1)$ Tev we have the fine-tuning problem for the electroweak symmetry-breaking scale.

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