Determination of the effective electron mass in an optical field from the measured emission spectrum of ultrarelativistic electrons at a laser focus

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By measuring the emission spectrum of an electron beam as it interacts with a focused laser pulse, one can directly observe a frequency shift of the radiation caused by a dependence of the effective mass of the electron on the intensity of the optical field. © *1995 American Institute of Physics.*

The nonlinear Compton scattering of intense laser light by a beam of ultrarelativistic electrons is of interest for testing predictions of nonlinear quantum electrodynamics[1] and also in connection with some laser-synchrotron designs which are being discussed.[2] The fields involved here would have a dimensionless intensity $\eta^2 = e^2\langle E^2\rangle/(2m^2\omega^2)$ approaching one. At a laser frequency $\omega = 1$ eV, the value $\eta = 1$ would be reached at an intensity $\sim 10^{18}$ W/cm^2.

A theory in which the intense laser field is treated as a monochromatic plane wave[3] predicts that the spectral–angular distribution of the scattered radiation would consist of discrete harmonics. In the lab frame of reference, the frequency of the fundamental frequency would depend on the direction of the radiation, and it would be shifted from the frequency of the scattered photon in the ordinary Compton effect. In general, the magnitude of the shift would depend on the expectation value of the 4-momentum of the electron, q_μ, which in turn depends on the wave intensity. Under certain conditions it turns out that the shift depends not on the components q_μ individually but on the effective mass found from the relation $q_\mu^2 = m_*^2$, which has a value $m_*^2 = m^2(1+\eta^2)$.

Fields with η on the order of one can be reached in short, focused laser pulses whose field is nonuniform in space and time. In such fields the expectation value of the momentum and the effective mass vary along the trajectory of the electron, causing a modulation of the frequencies of the radiated harmonics. This modulation substantially changes the spectral–angular distribution: The discrete harmonics are broadened, and, at a sufficiently high intensity $(\eta > 1)$, they overlap, with the result that the spectrum becomes continuous.[4]

There has been no previous direct experiment to observe an intensity-dependent shift of the frequency and the effective mass. In an experiment of this sort, of course, one could study the spectral–angular distribution of the radiation directly. However, such measurements run into serious difficulties, since there must be a good angular spread within the narrow cone in which the emission of a relativistic electron is concentrated.

Our purpose in the present letter is to show that the effective mass can be determined in a simpler experiment, by measuring the emission spectrum, integrated over all directions, which arises in the collision of a fast electron with a focused laser beam. We formulate a model as follows. A free electron with an energy

$$1 \ll \gamma = \epsilon/m \ll m/\omega = 10^5 \tag{1}$$

is incident along the x axis on a focused laser beam of frequency ω, which is propagating along the z direction. The radius of the beam focus is much larger than the wavelength ($R \gg \lambda$), and the maximum intensity at the beam axis satisfies the condition $\eta^2(0) < 1$. This limitation simplifies the calculations substantially, since (on the one hand) the fundamental frequency dominates the spectrum, and (on the other) the condition $\gamma \gg \eta(0)$ is satisfied automatically. As a result of this condition, the average trajectory is essentially rectilinear, and along this trajectory the intensity depends on only one Cartesian coordinate: $\eta^2 = \eta^2(x)$. The inequality on the right in (1) allows us to ignore quantum effects in the course of the emission.

Since $\gamma \gg 1$, $\eta(0)$, an electron entering the field remains ultrarelativistic and radiates into a narrow cone along the average velocity. In the approximation $\theta \ll 1$, where θ is the angle between the emission direction and the x axis, the frequencies radiated within the fundamental harmonic are[4]

$$\omega' = 2\omega\gamma^2/[(m_*(x)/m)^2 + (\gamma\theta)^2]. \tag{2}$$

There is an upper limit $\omega_{max} = 2\omega\gamma^2$ on the frequency. This upper limit is radiated at an angle $\theta = 0$ at the periphery of the focus, where the intensity is low and we have $m_* = m$. This frequency is the same as the frequency of the scattered photon in the ordinary Compton effect at a moving electron. The frequency emitted at the angle $\theta = 0$ at the center of the beam, where the laser intensity and the effective mass are at their highest, is $\omega(0) = \omega_{max}[m/m_*(0)]^2$ and is lower than ω_{max}. In a nonuniform field, a given frequency ω' is radiated from different points of the trajectory at different angles under the condition that the denominator of expression (2) remain constant.

The emission spectrum is found by integrating the spectral–angular distribution found previously.[4] Skipping over the details of the calculations, we show the result in Fig. 1a. The spectral density of photons is found by dividing the spectral density of the emission by the photon energy ω'. Shown for comparison in Fig. 1b is the emission spectrum of the fundamental frequency in the uniform field of a monochromatic wave with an intensity $\eta^2(0)$. Here the upper boundary of the spectrum, $\omega(0)$, is shifted with respect to the Compton frequency, in the red direction, because of the increase in effective mass. The spectrum in the region $\omega' < \omega(0)$ is related to the emission of the fundamental harmonic at angles $\theta \neq 0$. On the side $\omega' > \omega(0)$ there is a contribution from higher harmonics, but at $\eta < 1$. Consequently, there is an abrupt decay of the spectrum at $\omega' = \omega(0)$.

It can be seen from Fig. 1 that a remarkable feature of the spectrum in the focused field is a high-frequency region $\omega(0) < \omega' < \omega_{max}$, which is formed primarily as a result of the variable effective mass under the condition $\eta(0) < 1$. For the parameter values in Fig. 1, the contribution of higher harmonics at the point $\omega' = 0.9\omega_{max}$ does not exceed 10%. With increasing intensity, the width of this region increases, and at $\eta^2(0) = 0.5$ it makes up a third of the entire spectrum of the fundamental harmonic. The abrupt change

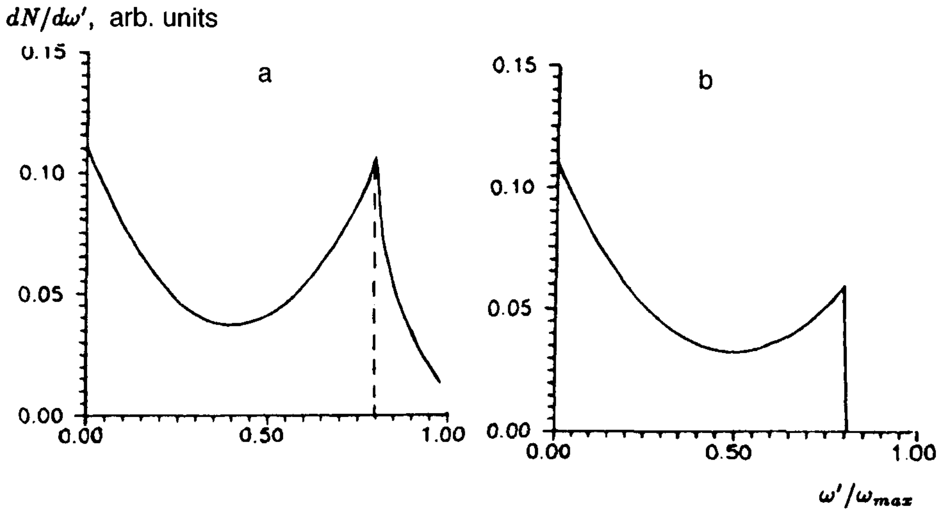

FIG. 1. Spectral density of photons emitted by a fast electron ($\gamma = 10^3$). a—Over a passage through a focused laser beam [$\eta(0) = 0.5$, $kR = 200$]; b—in a uniform optical field ($\eta = 0.5$) over a time $\tau = \sqrt{\pi}R/c$.

predicted by the theory in a uniform field transforms into an asymmetric maximum near $\omega(0)$. The variability of the effective mass is less obvious in the spectrum to the left of the maximum [$\omega' < \omega(0)$].

In summary, measurements of the emission spectrum of fast electrons crossing a focused laser beam whose intensity is not too high, $\eta(0) < 1$, will yield information on the effective mass of an electron. The position of the $\omega(0)$ maximum determines the intensity at the axis of the focus highly accurately: $\omega_{\max} / \omega(0) - 1 = \eta^2(0)$.

From the standpoint of applications and, in particular, from the standpoint of developing a laser synchrotron, a variable mass is an additional cause of a broadening of the emission spectrum. It becomes a principal cause as η approaches one.

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