Nucleon wave function and hard contribution to the nucleon structure function in the limit $x \to 1$

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The hard QCD contribution to the nucleon structure function in the limit $x \to 1$, expressed in terms of the V–A projection, is derived in the quark-parton model. Comparison with experiment shows that the data cannot be described by a hard QCD contribution with a realistic wave function.

In hard processes in quantum chromodynamics (QCD), the contributions from long and short range are known to separate. Studying such processes and determining how well they agree with experimental results are thus important not so much for testing the QCD predictions as for determining the structure of nonperturbative interactions and their space and time scales.

On the other hand, several pieces of experimental and theoretical evidence indicate that in a number of processes this separation of contributions has not yet occurred at fairly large values of the momentum transfer.[1–5] The meaning may be that the conventional ideas regarding the space and time scales of nonperturbative effects should be altered slightly and that they may play an important role at sufficiently short range, at which it has been customary to assume that hard QCD mechanisms are dominant. In the present letter we report some further arguments in favor of that point of view. We show that the behavior of nucleon structure functions in the limit $x \to 1$ is not described by a hard QCD contribution with any plausible nucleon wave function.

The QCD asymptotic form of the nucleon structure functions $F_2(x)$ in the limit $x \to 1$ (the hard contribution) is known to be determined by quark diagrams containing exchanges of two hard gluons (Fig. 1) and is described by

$$F_2(x) - (1-x)^3. \qquad (1)$$

Below we derive exact equations which express $F_2(x)$ as integrals of nucleon wave functions. We are then in a position not only to establish the power-law behavior in (1) but also to find numerical estimates and to compare our predictions with experiment.

It is convenient to conduct the analysis in old perturbation theory, in an infinite-momentum system with $P_z \to \infty$, $\mathbf{p}_\perp = 0$, and $q_0 = -q_z = -q_\perp^2/4P$, where P is the nucleon momentum, and q_μ is the 4-momentum of the virtual photon.[7] It is convenient to select the leading terms as $x \to 1$ on the basis of the properties of the energy denominators and the vertices in a gauge in which there is no zeroth component of the gluon field.[8] In this case the structure function is determined in the limit $x \to 1$ by diagrams of old perturbation theory as in Fig. 1a. The contribution of the diagram in Fig. 1b to

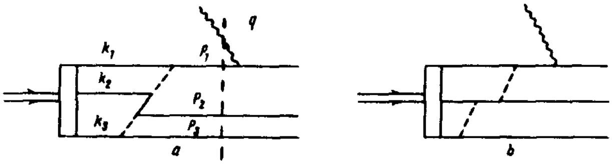

FIG. 1.

$F_2(x)$ is suppressed as $(1-x)^2$. We introduce the standard parametrization of momenta:

$$\mathbf{k}_i=\lambda_i\mathbf{P}+\mathbf{k}_{i\perp}\ ,\quad \mathbf{k}_{i\perp}\,\mathbf{P}=0,\quad \mathbf{p}_i=x_i\mathbf{P}+\mathbf{p}_{i\perp}\ ,\quad \mathbf{p}_{i\perp}\,\mathbf{P}=0. \tag{2}$$

The contribution of the diagram in Fig. 1a (and that of other diagrams, differing in interchanges of quark lines) to the structure function is

$$F_2^{(0)}(x)=3\int \varphi^{s+}_{s_i'}(\lambda_i')Q_1^2B_{s_i',s_i}(\lambda_i,\lambda_i';x_i,p_{i\perp})\varphi_{s_i}^{\ s}(\lambda_i)\delta(x-x_1)d\Gamma, \tag{3}$$

$$d\Gamma=\frac{dx_2dx_3d^2p_2d^2p_3}{(2\pi)^\sigma 4x_1x_2x_3}.$$

Here s and s_i are the z projections of the spins of the nucleon and the quarks, and Q_1 is the charge of the first quark with which the virtual photon interacts. The nucleon wave function $\varphi_{s_i}^{\ s}(\lambda_i)$ is expressed as follows in terms of the vertex representing the conversion of the nucleon into quarks, $V_{s_i}^{\ s}(\lambda_i,\,\mathbf{k}_{i\perp})$:

$$\varphi_{s_i}^{\ s}(\lambda_i)=\frac{1}{(2\pi)^\sigma}\int\frac{d^2k_{2\perp}\,d^2k_{3\perp}}{4\sqrt{\lambda_1\lambda_2\lambda_3}}V_{s_i}^{\ s}(\lambda_i,k_{i\perp})\frac{1}{2P(E-\epsilon_1-\epsilon_2-\epsilon_3)}, \tag{4}$$

where $E=(P^2+M^2)^{1/2}$, $\epsilon_i=(\mathbf{k}_i^2+m_i^2)^{1/2}$, M is the mass of a nucleon, and m is the mass of the ith quark. The "Born" term B_{s_i',s_i} is

$$B_{s_i',s_i}(\lambda_i,\lambda_i';x_i\mathbf{p}_{i\perp})=\frac{256(4\pi\alpha_s)^4x_2^2\,x_3^2\,\delta_{s_1s_1'}\delta_{s_3s_3'}\delta_{s_3s_3'}\,\delta_{s_2-s_3}}{81(1-\lambda_1)^2(1-\lambda_1')^2m_{2\perp}^2\,m_{3\perp}^2\,(m_{2\perp}^2\,x_3+m_{3\perp}^2\,x_2)^2}$$

$$\times\Bigg\{(1-\lambda_1)(1-\lambda_1')+(\lambda_2-\lambda_3)(\lambda_2'-\lambda_3')$$

$$-\frac{4x_2x_3m_{2\perp}^2\,m_{3\perp}^2\,(\lambda_2-\lambda_3)(\lambda_2'-\lambda_3')}{(m_{2\perp}^2\,x_3+m_{3\perp}^2\,x_2)^2}\Bigg\}\frac{1}{\lambda_2\lambda_3\lambda_2'\lambda_3'}. \tag{5}$$

Here $m_{i\perp}^2=m_i^2+p_i^2$ and $\alpha=\alpha[m^2/(1-x)]$. Since the integrals diverge at the lower limit in the integration in (3) over transverse momenta, we retain in (5) the quark mass. This mass is understood to be an effective parameter which is determined by nonperturbative interactions. We write the wave function $\varphi_{s_i}(\lambda_i)$ in the form[5]

$$\varphi^{1/2}_{s_1 s_2 s_3}(\lambda_i)=\frac{1}{24}\{2T(\lambda_i)u\uparrow u\uparrow d\downarrow-(V-A)u\uparrow u\downarrow d\uparrow-(V+A)u\downarrow u\uparrow d\uparrow$$

$$+2\leftrightarrow 3+1\leftrightarrow 4\}_{s_1,s_2,s_3}, \tag{6}$$

where the vector projection $V(\lambda_i)$, the axial projection $A(\lambda_i)$, and the tensor projection $T(\lambda_i)$ of the wave function are given as matrix elements of the corresponding nonlocal operators (Ref. 9, for example). Using the known relations between them,[9] we can write the structure function in (3) in the final form

$$F_2^0(x)=\frac{128}{243}\left(\frac{\alpha_s}{m_\perp}\right)^4(1-x)^3\left\{2Q_u^2\left[A(A+B)+\frac{C(C+D)}{5}\right]+(Q_u^2+Q_d^2)\left(B^2+\frac{D^2}{5}\right)\right\}, \tag{7}$$

where

$$A=\frac{1}{24}\int\frac{d\lambda_2 d\lambda_3}{\lambda_2\lambda_3(1-\lambda_1)}\varphi_N(\lambda_1,\lambda_2,\lambda_3),\quad C=\frac{1}{24}\int\frac{d\lambda_2 d\lambda_3(\lambda_2-\lambda_3)}{\lambda_2\lambda_3(1-\lambda_1)^2}\varphi_N(\lambda_1,\lambda_2,\lambda_3), \tag{8}$$

$$B=\frac{1}{24}\int\frac{d\lambda_2 d\lambda_3}{\lambda_1\lambda_2(1-\lambda_3)}\varphi_N(\lambda_1,\lambda_2,\lambda_3),\quad D=\frac{1}{24}\int\frac{d\lambda_2 d\lambda_3(\lambda_2-\lambda_3)}{\lambda_1\lambda_2(1-\lambda_3)^2}\varphi_N(\lambda_1,\lambda_2,\lambda_3),$$

$$\varphi N=V(\lambda_1,\lambda_2,\lambda_3)-A(\lambda_1,\lambda_2,\lambda_3).$$

Expression (7) is written for the case of the proton; for the neutron we need to make the interchange $Q_u\leftrightarrow Q_d$. Here $\tilde{m}_\perp^2=m^2+\tilde{p}_\perp^2$, where $\tilde{p}_\perp$ is the lower limit on the integration over transverse momenta in (3). Unfortunately, we do not know the value of m; however, we can find a kinematic limitation on $\tilde{m}_\perp$ from the requirement that the process be hard and that the equations derived be applicable.

Let us examine the energy denominator corresponding to the dashed line in Fig. 1a:

$$2P(E-\epsilon_1-\epsilon_2-\epsilon_3)=M^2-\sum\frac{m_{i\perp}^2}{x_i}{}_{s\mp 1}M^2-\frac{4\tilde{m}_\perp^2}{1-x}. \tag{9}$$

If the equations derived are to be applicable, we must have

$$\tilde{m}_\perp^2\gg\frac{M^2(1-x)}{4}\ (\text{as } x\to 1). \tag{10}$$

This condition holds automatically at large values of m; elsewhere it can be satisfied by cutting off the lower limit of the integration over transverse momenta. The limitations which follow from other energy denominators are less stringent.

The QCD evolution of the structure function associated with the emission of gluons can be taken into account by using the results of Ref. 10:

$$F_2(x)=\int_x^1 F_2^0(u)V\left(\frac{x}{u}\right)\frac{du}{u}, \tag{11}$$

TABLE I.

x	$\tilde{m}_\perp$ (GeV)	$F_2^{\rm exp}(x)$	$F_2(x)$
0.9	0.18	$(4.6\pm0.5)\times10^{-4}$	2.5×10^{-8}
0.95	0.12	$(3.6\pm0.6)\times10^{-5}$	1.0×10^{-8}

where $F_2^0(u)$ are structure functions without the emission of gluons, and $V(z)$ is the probability for observing a quark with a fraction z of the momentum of the original quark in the emission of gluons. We can ignore the probability for the production of quark-antiquark pairs as $x\to1$.

Analysis of the terms discarded in the derivation of (5) shows that this expression can be used at $x\geqslant0.9$. To make a comparison with experiment, we thus need to extrapolate the experimental data into this region. With this goal in mind, we have parametrized the SLAC data,[11] the SLAC-MIT data,[12] and the EMC data,[13] eliminating from this parametrization the power-law terms in Q^2, which make a very large contribution, e.g., 30% at $x=0.9$ and $Q^2=100$ GeV^2. We extrapolated the resulting behavior into the region of interest.

Table I shows the results of an extrapolation and of our predictions for the proton structure function (at $Q^2=30$ GeV^2) in the case of the asymptotic wave function[14]

$$\varphi_N^0=120f_0x_1x_2x_3, \tag{12}$$

where we have $f_0=(5\pm0.3)\times10^{-3}$ GeV^2 from Ref. 9.

The theoretical predictions are negligibly small in comparison with the experimental data. The situation is reminiscent of the case of electromagnetic form factors, which again are not described by a hard contribution in the case of function (12).[15] In making the estimates we set $m_\perp^2\geqslant M^2(1-x)/2$ for definiteness. This is a very weak limitation, and the estimates given for $F_2(x)$ are apparently well on the high side. It can be seen from (8) that we can raise the theoretical predictions substantially by considering asymmetric functions which have a sharp peak at $\lambda\sim1$. A function of this type was proposed in Ref. 9 on the basis of the QCD dispersion sum rules:

$$\tilde{\varphi}_N=\varphi_N^0(23.814x_1^2+12.978x_2^2+6.174x_3^2+5.88x_3-7.098). \tag{13}$$

Such a function describes the asymptotic behavior of the nucleon form factors, but it is difficult to reconcile with the low-energy parameters of hadrons.[5] The use of function (13) in our case increases the theoretical predictions by a factor of only 60, and the hard contribution to the structure function remains negligible. The use of functions which are more asymmetric than (13) is pointless, since they lead to contradictions with experiment, not only for the low-energy parameters but also for the asymptotic behavior of the form factors.

In summary, at values of x which are attainable experimentally, the description of the hadron structure function remains an open question. It is necessary to invoke a new physical mechanism.

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