SU(2) effective action with the nonanalytic term

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The nonanalytic g^3 term is calculated for the SU(2) effective action at a finite temperature. The status of a gauge field condensation and the problem of degeneracy are briefly discussed.

The gauge models with embedded external fields are a very important subject of modern theoretical physics, because they are rather realistic and adequately describe many physical phenomena. The gauge fields condensation, which can be simulated through a very simple external field, is a typical phenomenon for many unified gauge models and its properties are studied intensively today by using mainly the effective-action technique. However, two-loop effective action calculated with a nonzero external field (initially in Ref. 1 and then for an arbitrary gauge parameter ξ in Ref. 2 for SU(2) group and in Refs. 3 and 4 for SU(3) group) shows that all its minima are physically equivalent, although there is no reason to assume that this degeneracy has been proved. This fact makes the phenomenon (where the gauge field condensate arises spontaneously[5]) unreliable, and it is very important to establish the real meaning of this scenario. Special attention should also be given to a gauge-invariance of the results found in Refs. 4 and 6. In particular, the picture is not clear when higher-order corrections are taken into account. The search for these corrections (at least the g^4-order corrections) is a pressing task, since only they can determine the status of this phenomenon and the role of degeneracy in building the nontrivial vacuum. Of course, the degeneracy which has been found is determined partially by the symmetry of the breaking operator which destroys the initial gauge group. As to the trivial vacuum, this degeneracy is false and results from the imperfection of the calculational scheme based on the lowest-order perturbative graphs.

The quantum SU(2) Lagrangian in the background gauge has the standard form:

$$L=-\frac{1}{4}(G^a_{\mu\nu})^2-\frac{1}{2\xi}[(\bar{D}_\mu V_\mu)^a]^2+\bar{C}\bar{D}_\mu D_\mu C, \tag{1}$$

where the gauge fields V^a_μ are decomposed in the quantum part Q^a_μ and in the classical constant part $\bar{A}^a_\mu$ (here $V^a_\mu=Q^a_\mu+\bar{A}^a_\mu$). The gauge field strength tensor $G^a_{\mu\nu}$ is determined through the new covariant derivative $\bar{D}^{ab}_\mu=\partial_\mu\delta^{ab}+gf^{acb}\bar{A}^c_\mu$, but a term with the ghost fields $\bar{C}$ (and C) in (1) contains [in addition to $\bar{D}(\bar{A})$] the usual derivative D^{ab}_μ, which depends on the total V^a_μ field. The parameter ξ fixes the internal gauge, and the classical field has the form

$$\bar{A}^a_\mu=\delta_{\mu4}\delta^{a3}A^{\rm ext}=\delta_{\mu4}\delta^{a3}\frac{\pi T}{g}x, \tag{2}$$

where x is a new variable. Here T is temperature, and g is the standard coupling constant.

The effective action for this model (including the two-loop graphs) has been calculated by many authors.[1,2] It has a rather simple form

$$W(x)/T^4 = W^{(1)}(x)/T^4 + W^{(2)}(x)/T^4,$$

$$W^{(1)}(x)/T^4 = \frac{2}{3}\pi^2\left[B_4(0) + 2B_4\left(\frac{x}{2}\right)\right], \tag{3}$$

$$W^{(2)}(x)/T^4 = \frac{g^2}{2}\left[B_2^2\left(\frac{x}{2}\right) + 2B_2\left(\frac{x}{2}\right)B_2(0)\right] + \frac{2}{3}g^2(1-\xi)B_3\left(\frac{x}{2}\right)B_1\left(\frac{x}{2}\right),$$

where $B_n(z)$ are the modified Bernoulli polynomials

$$B_1(z) = z - \epsilon(z)/2, \quad B_3(z) = z^3 - 3\epsilon(z)z^2/2 + z/2,$$
$$B_2(z) = z^2 - |z| + 1/6, \quad B_4(z) = z^4 - 2|z|^3 + z^2 - 1/30 \tag{4}$$

with $\epsilon(z) = z/|z|$. Here we assume that $\epsilon(0) = 0$, since we see from direct calculations that (3) is correct.

The action (3) has three extremum points:

$$\bar{x}=0, \quad \bar{x}=1, \quad \bar{x}=2, \tag{5}$$

where two of them ($\bar{x}=0$ and $\bar{x}=2$) are minima of the given action. The effective action which is put on these extremum points is a gauge-independent quantity.[4] Unfortunately, the thermodynamic potential found in this approximation for the trivial vacuum ($\bar{x}=0$) and for the nontrivial vacuum ($\bar{x}=2$) has the same value:

$$\Omega/T^4 = 2\pi^2 B_4(0) + \frac{3g^2}{2}B_2^2(0) = -\frac{\pi^2}{15} + \frac{g^2}{24}. \tag{6}$$

This fact indicates that a degeneracy (which is probably a signal of a real one) takes place in this scenario and the multiloop corrections are essential for resolving the situation. However, direct calculation of a three-loop effective action (the g^4 order) is a hopeless task, and therefore a nonperturbative scheme should be built to define the status of this phenomenon. A simple summation is used below to calculate the leading nonanalytic term in the nonperturbative expansion of $W(x)$ and its gauge dependence is discussed. This term is the g^3 which for many physical phenomena plays a more important role than the g^4 terms.

It is well known (see, e.g., Refs. 7 and 8) that for any non-Abelian gauge theory (despite its more complicated structure) the leading nonanalytic term can be reproduced in terms of the standard formula:

$$\frac{\partial W^{\mathrm{cor}}(x)}{\partial g} = \frac{1}{\beta g}\sum_{k_4}\int\frac{d^3k}{(2\pi)^3}\mathrm{Tr}[D(k)\Pi(k)], \tag{7}$$

where the polarization tensor $\Pi(\bar{k},k_4)$ should be calculated in the lowest order. For this calculation only $\Pi_{44}(|\bar{k}|\to 0,0)$ is used. The final result has the form

$$\Delta W^{(\mathrm{cor})} = -\frac{\Pi_{44}^{3/2}(0)}{12\pi\beta}\,\mathrm{Tr}(I). \tag{8}$$

Here I is the unit matrix in the adjoint representation of the chosen gauge group [for SU(N) we have $\mathrm{Tr}(I) = N^2 - 1$].

Polarization tensor for the broken SU(2) group (when $x \neq 0$) has two components $\Pi^{\parallel}(\bar{k},k_4)$ and $\Pi^{\perp}(\bar{k},k_4)$, which are completely independent in the g^2 approximation. Their calculations are standard and make use of the standard temperature Green's function technique in the imaginary time space. To simplify the following analysis, we omitted all details, and the infrared limits of the $\Pi_{44}(\bar{k},k_4)$ components contain only their leading terms.

The g^2 order infrared limit of $\Pi_{44}^{\parallel}(\bar{k},k_4)$ has the simple form

$$\Pi_{44}^{\parallel}(|\bar{k}| \to 0, \quad k_4 = 0) = 4g^2T^2B_2\left(\frac{x}{2}\right). \tag{9}$$

For its calculation we used the standard prescription (here $k_4 = 0$ and $|\bar{k}| \to 0$). It is important to note that only expression (9) is generated by the effective action (3) in terms of the standard formula

$$m_{\parallel}^2 = \frac{g^2}{\pi^2T^2}\frac{1}{4}\left[\partial^2/\partial\left(\frac{x}{2}\right)^2\right]W\left(\frac{x}{2}\right), \tag{10}$$

and (9) can be improved to the g^4 order terms:

$$m_{\parallel}^2 = 4g^2T^2B_2\left(\frac{x}{2}\right) + \frac{g^4T^2}{\pi^2}\left\{B_1^2\left(\frac{x}{2}\right) + \frac{1}{2}\left[B_2\left(\frac{x}{2}\right) + B_2(0)\right]\right.$$
$$\left. + (1-\xi)\left[B_1^2\left(\frac{x}{2}\right) + B_2\left(\frac{x}{2}\right)\right]\right\}. \tag{11}$$

The infrared limit of $\Pi_{44}^{\perp}(\bar{k},k_4)$ cannot be found in (10) and is calculated directly through the Green's function technique. In addition, there are some peculiarities which complicate a search for this limit when $x \neq 0$, since the initial gauge symmetry is broken. In the transverse sector all gauge bosons acquire a mass (a nonzero damping at the tree level) and the infrared limit of $\Pi_{44}^{\perp}(\bar{k},k_4)$ should be determined near a new mass shell $\hat{k}_4 = 0$ (where $\hat{k}_4 = k_4 + \pi T x$). The calculations are standard and the g^2 order infrared limit of $\Pi_{44}^{\perp}(\bar{k},k_4)$ has the form

$$\Pi_{44}^{\perp}(|\bar{k}| \to 0, \ \hat{k}_4 = 0)2g^2T^2\left[B_2\left(\frac{x}{2}\right) + B_2(0)\right], \tag{12}$$

which is a gauge-invariant quantity for any $x \neq 0$. This is not the case in which all other possible infrared limits of $\Pi_{44}^{\perp}(\bar{k},k_4)$ are studied, and we assume that only expression (12) should be used in Eq. (8).

Collecting all expressions found for the infrared limits of $\Pi_{44}(\bar{k},k_4)$ and using Eq. (8), we obtain the nonanalytic corrections

$$\Delta W^{(\mathrm{cor})}/T^4 = -\frac{2g^3}{3\pi}\left\{ B_2^{3/2}\left(\frac{x}{2}\right) + 2\left[\frac{1}{2}\left(B_2\left(\frac{x}{2}\right) + B_2(0)\right)\right]^{3/2}\right\}, \tag{13}$$

which are gauge-invariant themselves. For the case $x=0$ they coincide with the known results [see, e.g., Refs. 7 and 8 for the SU(32) group].

$$\Delta\Omega^{(\mathrm{cor})}/T^4 = -\frac{g^3}{4\pi}\sqrt{\left(\frac{2}{3}\right)^3}. \tag{14}$$

Unfortunately, only the result (14) has a physical meaning, since all positions of the extremum points (at least for a small g) are the same as in (5).

The corrected points (which should be used for treating the g^4 order effective action) are also known.[4] For a small g these corrections are proportional to the g^2 terms

$$\bar{x}_{1,3} = 1 \pm \left[1 - \frac{g^2}{4\pi^2}\left(1 + \frac{1-\xi}{2}\right)\right]. \tag{15}$$

Upon substitution for the lowest-order effective action (3), these points generate the gauge-dependent g^4 corrections

$$\frac{\Omega}{T^4} = \frac{\Omega^{(1)}}{T^4} + \frac{\Omega^{(2)}}{T^4} = 2\pi^2 B_4(0) + \frac{3g^2}{2} B_2^2(0) + \frac{g^4}{48\pi^2}\left(1 + \frac{1-\xi}{2}\right)^2 - \frac{g^4}{24\pi^2}\left|1 + \frac{1-\xi}{2}\right|\left(1 + \frac{1-\xi}{2}\right) \tag{16}$$

and other corrections which cannot be calculated accurately. The nonanalytic g^3 terms found above are therefore the leading terms in the nonperturbative expansion of $W(x)$. Expressions (3) and (13), used jointly, are the closed result until the g^4 terms vanish.

However, all g^4 terms (or at least some of them) should be calculated exactly to solve the problem of degeneracy and to check, with the help of (15), the gauge-invariance of order g^4 thermodynamic potential. In particular, analyzing (16), we see that we can calculate exactly in the three-loop graphs all g^4 terms which are proportional to $(1-\xi)$ multiplier, and then combine these terms with the analogous terms in (16). Although the terms put on the extremum points should be equal to zero, they are essential for checking the necessary condition of a gauge-invariance for the g^4 order thermodynamic potential. Of course, it is necessary to find all other terms which are proportional to the high powers of $(1-\xi)$ multiplier, but this task seems to be more complicated and it can be carried out subsequently.

The problem of degeneracy should be also solved in g^4 terms. It is clear that a trivial vacuum is split, but some degeneracy seems to be retained because this possibility is immediately embedded by the symmetry of the breaking operator built in accordance with the structure of the chosen external field. However, there is no reason to consider this symmetry proved for the order g^4 thermodynamic potential and a signal for breaking it will be received if one finds at least some of the terms which are not proportional to the truncated Bernoulli polynomials $\widetilde{B}_2(z)$ and $\widetilde{B}_4(z)$. Here

$\widetilde{B}_{2n}=B_{2n}(z)-B_{2n}(0)$ and these functions are periodic under substitution $|z|\to 1-|z|$. Unfortunately, these g^4 terms should be calculated directly, since any nonperturbative summations with use of the g^2 terms retain the found degeneracy at least for a small $g\mathbf{q}$. The g^3 terms obtained here indicate this circumstance, although they are very important themselves for the investigation of any application of the found effective action.

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