

# Lasing without a population inversion in the field of an adiabatic coherent pulse

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A new mechanism has been found for lasing without a population inversion in a three-level medium in the field of an adiabatic coherent pulse. This pulse creates an unfilled lower level, putting the system in the form of a standard two-level lasing system.

1. We consider a medium of three-level atoms (Fig. 1) in which levels 1 and 3 are coupled by the field of an adiabatic, intense, coherent pulse with frequency  $\omega$ , detuning  $\Delta = \omega_{31} - \omega \ll \omega$ , and length  $\tau$ . This length is shorter than all the relaxation times of the medium and satisfies the adiabatic condition  $\Delta\tau \gg 1$ . Level 2 is populated by cw incoherent light at a rate  $\Lambda$ . This process leads to a distribution of atoms between levels 1 and 2 with populations  $n_1 = \Gamma/(\Lambda + \Gamma)$  and  $n_2 = \Lambda/(\Lambda + \Gamma)$  ( $\Gamma^{-1}$  is the lifetime of level 2) such that there is no population inversion between these levels:  $\Lambda \ll \Gamma$ . We will show that under these conditions laser light is generated on the  $2 \rightarrow 1$  transition at the frequency

$$\omega_1 = \omega_{21} - (\Delta + \Omega_1)/2, \quad (1)$$

where  $\Omega_1 = (\Delta^2 + 4\Omega^2)^{1/2}$ ,  $\Omega = |V(t)|$ ,  $V(t) = \mu_{31}E(t)/\hbar$ ,  $E(t)$  is the field of the adiabatic pulse, and  $\mu_{ij}$  is the dipole matrix element of the  $i \rightarrow j$  transition. Equation (1) explains the mechanism for lasing without a population inversion.

In terms of the bare states of the atom, the process occurs in two steps. First, the incoherent pump sends atoms from level 1 to level 2. The atoms then go to the empty level 3 through the coherent emission of a photon at the frequency  $\omega_1$  and the absorption of a photon at  $\omega$  from the adiabatic pulse (Fig. 1b). We are assuming that the populations of levels 1 and 2 are not altered by the interaction with the incoherent pump during the application of the adiabatic pulse. The generation process can be described in a particularly simple way in terms of dressed states  $|\Psi_{\pm}\rangle$  (Fig. 1c), with quasienergies  $\lambda_{\pm} = \pm(\Omega_1 \pm \Delta)/2$ . Since the  $|\Psi_{+}\rangle$  state is not filled in the course of the adiabatic interaction, a population inversion arises between levels 2 and  $|\Psi_{+}\rangle$ :  $n_2 - n_{+} = n_2 > 0$ . The system becomes the same as an ordinary lasing system. The emission frequency is given by (1).

To the best of our knowledge, the mechanism proposed here for lasing without a population inversion has not previously been discussed in the literature. It differs from the schemes for lasing without population inversion, which have been discussed in numerous papers,<sup>1–6</sup> in that in all those other papers the lasing mechanism depends on an atomic coherence between two closely spaced lower or upper working levels of the atom<sup>1,2,5–7</sup> or between two dressed states<sup>3,8,9</sup>—not to mention the fact that those other

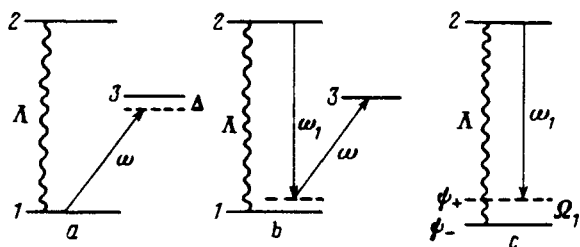


FIG. 1. Schematic diagram of a three-level system for lasing without a population inversion, in the field of an adiabatic pulse with frequency  $\omega$  and detuning  $\Delta$ .

papers dealt primarily with the steady state. In the case of the present letter, a classical two-level laser system with an unfilled lower level  $|\Psi_+\rangle$  is formed as the result of an adiabatic interaction of the atom with a coherent external field.

2. The equation for the amplitude of the field radiated at the frequency  $\omega_1 = \omega_{21} - \Delta_1$  is

$$\partial E_1(z, t) / \partial z = 2i\pi N \omega_1 \mu_{12} c^{-1} \rho_{21}, \quad (2)$$

where  $N$  is the number density of atoms,  $\rho_{ij}$  are the density matrix elements of the medium, which satisfy the conditions

$$\begin{aligned} \dot{\rho}_{21} &= -i\Delta_1 \rho_{21} + iG(\rho_{11} - \rho_{22}) - iV\rho_{23}, \quad G = \mu_{21}E_1/\hbar, \\ \dot{\rho}_{23} &= -i(\Delta - \Delta_1)\rho_{23} + iG\rho_{13} - iV^*\rho_{21}, \\ \dot{\rho}_{31} &= -i\Delta\rho_{31} + iV(\rho_{11} - \rho_{33}) - iG\rho_{32}, \\ \dot{\rho}_{33} &= i(V\rho_{13} - V^*\rho_{31}), \quad \rho_{11} = 1 - \rho_{33} - \rho_{22}, \end{aligned} \quad (3)$$

with the boundary conditions  $\rho_{11}(t \rightarrow -\infty) = n_1$ ,  $\rho_{22}(-\infty) = n_2$ ,  $\rho_{ij}(-\infty) = 0$ ,  $i \neq j$ , and  $E_1(0, t) = \mathcal{E}(t)$ , where  $\mathcal{E}(t)$  is the amplitude of the spontaneous-noise field at the frequency  $\omega_1$ .

We first consider the case  $\Delta_1 \neq \Delta$ . Eliminating  $\rho_{23}$  and  $\rho_{31}$  from Eqs. (3) in an adiabatic fashion, using the solutions  $\rho_{11} = n_1 \lambda_+ / \Omega_1(t)$ ,  $\rho_{33} = -n_1 \lambda_- / \Omega_1(t)$ ,  $\rho_{22} \equiv n_2$  found in zeroth order in  $G$ , and substituting  $\dot{\rho}_{21}$  into (2), we find

$$\frac{\partial^2 E_1(z, t)}{\partial z \partial t} = -i \left[ \Delta_1 - \frac{\Omega^2(t)}{\Delta_1 - \Delta} \right] \frac{\partial E_1}{\partial z} + g[n_2 - F(t)]E_1, \quad (4)$$

where

$$g = 2\pi N |\mu_{21}|^2 / \hbar c, \quad F(t) = n_1 [\lambda_+(t) - \Omega^2(t) / (\Delta_1 - \Delta)] / \Omega_1(t).$$

A solution of (4) is

$$E_1(z, t) = \mathcal{E}(t) \exp[-i\delta(t)] + 2zg \int_{-\infty}^t dt' \mathcal{E}(t') \times [n_2 - F(t')] I_1(\Psi(t', t)) \Psi^{-1}(t', t) \exp[-i\delta(t) + i\delta(t')],$$

$$\Psi(t', t) = 2 \left[ zg \int_{t'}^t dt'' [n_2 - F(t'')] \right]^{1/2}, \quad \delta(t) = \int_{-\infty}^t dt' \left[ \Delta_1 - \frac{\Omega^2(t')}{\Delta_1 - \Delta} \right], \quad (5)$$

where  $I_1(x)$  is the modified Bessel function of the first kind. We note first that emission of radiation at  $\omega_1$  would obviously be possible if the condition  $n_2 - F(t) > 0$  held over a sufficiently large portion of the adiabatic pulse, since otherwise  $I_1(x)$  would be replaced by the ordinary Bessel function  $J_1(x)$ , which vanishes with increasing  $z$ , and  $E_1(z, t)$  would remain at the level of the spontaneous noise. Second, the function  $\delta(t)$  must be a slowly varying function of  $t$  if pronounced oscillations are to be avoided in the integrand in (5).

The results which are physically most transparent are found in the case of an extended adiabatic pulse, whose field amplitude can be assumed constant everywhere [ $E(t) \equiv \text{const}$ ] except in the "turn-on" and "turn-off" regions, in which the adiabatic condition is satisfied. It is easy to see that for such a pulse, with the values  $\Delta_1 = \lambda_{\pm}$ , we have  $\delta(t) = 0$ . However, lasing occurs only in the case  $\Delta_1 = \lambda_+$ , since with  $\Delta = \lambda_-$  we have  $n_2 - F(t) < 0$ . For other values of  $\Delta_1$ , including  $\Delta_1 = \Delta$ , there is again no lasing, because of the pronounced oscillations in  $\exp[-i\delta(t')]$  in (5) or because of absorption:  $n_2 - F(t) < 0$ .

We find the intensity of the emission at frequency (1) at large values of the gain,  $\Psi(t', t) \gg 1$ , by using the asymptotic expression  $I_1(x) = e^x / (2\pi x)^{1/2}$ ,  $x \gg 1$ :

$$J_1(z, t) = \frac{c}{\pi^2} z^2 g^2 n_2^2 \left| \int_{-\infty}^t dt' \mathcal{E}(t') \Psi_0^{-3/2}(t', t) \exp[\Psi_0(t', t)] \right|^2, \quad (6)$$

where  $\Psi_0 = 2[zg(t - t')n_2]^{1/2}$ .

This formula will be analyzed in detail in a separate paper, as will the quantum statistics of the radiation generated. Here we would like to point out that this process is similar to transient stimulated Raman scattering.<sup>10,11</sup> As in the case of Stokes radiation, there are buildup effects in (6), and the lasing reaches a maximum on the trailing edge of the adiabatic pulse.

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