Angular characteristics of synchrotron light in magnetic fields with hard focusing

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The motion and magnetobremsstrahlung of an electron in a strongly focusing magnetic field are analyzed. Vertical betatron oscillations of the particle substantially alter the angular distributions of the radiation intensity.

This topic has been studied previously for uniform and weakly focusing magnetic fields, in particular, in Refs. 1 and 2.

An electron is assumed to be moving through a system in which the magnetic field gradients are greater than one and are equal to n_1 and $-n_2$ for one period. If this step function is expanded in a Fourier series, the Lorentz equations for the linear approach become

$$\frac{d^2\rho}{d\tau^2}+\frac{1-n(\tau)}{N^2}\rho=0, \quad \frac{d^2z}{d\tau^2}+\frac{n(\tau)}{N^2}z=0, \tag{1}$$

where

$$\tau=N\varphi,\ n(\tau)=\frac{n_-}{2}+\frac{2n_+}{\pi}\sum_{k=0}^{\infty}\frac{\sin(2k+1)\tau}{2k+1},$$

N is the number of periodicity elements along the entire circular trajectory, $n_+=n_1+n_2$, $n_-=n_1-n_2$, $\rho=r-R$, and R is the radius of the main revolution. We seek particular solutions of these equations in the form

$$\rho=\exp(i\gamma_x\tau)\varphi_x(\tau), \quad z=\exp(i\gamma_z\tau)\varphi_z(\tau).$$

Now expanding φ_x, φ_z, γ_x, and γ_z in these expressions in power series in $1/N$, and eliminating the secular terms (we are discussing only the stable motion), we can write the asymptotic solutions of Eqs. (1) as

$$\rho=A\cos\left(\frac{\nu_x}{N}\tau+\chi\right)(1-S)+\sqrt{4-2n_-}\,A\sin\left(\frac{\nu_x}{N}\tau+\chi\right)C, \tag{2}$$

$$z=B\cos\left(\frac{\nu_z}{N}\tau+\psi\right)(1+S)-\sqrt{2n_-}\,B\sin\left(\frac{\nu_x}{N}\tau+\psi\right)C,$$

where

$$S=\frac{2n_+}{\pi N^2}\sum_{k=0}^{\infty}\frac{\sin(2k+1)\tau}{(2k+1)^3}, \quad C=\frac{2n_+}{\pi N^3}\sum_{k=0}^{\infty}\frac{\cos(2k+1)\tau}{(2k+1)^4}.$$

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Here A,B and χ,ψ are the amplitudes and initial phases of the average radial and axial oscillations; the frequencies are

$$\nu_x\sqrt{1-n_-+\nu_z^2}\,, \quad \nu_z=\sqrt{n_-/2+\pi^2n_+^2/48N^2}\,.$$

To analyze the radiation by an electron we use the "operator method."[3,4] Substituting classical solutions (2) into the expressions for the radiation intensity, we find

$$\frac{dW_\sigma(\nu)}{d\Omega}=\frac{ce^2\nu\nu'}{12\pi^4R^2}\int_0^{2\pi}d\psi\epsilon_1^2K_{2/3}^2\left(\frac{\nu'}{3}\epsilon_1^{3/2}\right),$$
$$\frac{dW_\pi(\nu)}{d\Omega}=\frac{ce^2\nu\nu'}{12\pi^4R^2}\int_0^{2\pi}d\psi\epsilon_1\epsilon_2K_{1/3}^2\left(\frac{\nu'}{3}\epsilon_1^{3/2}\right), \tag{3}$$

where

$$\epsilon_1=\epsilon_0+\epsilon_2,\quad \epsilon_0=1-\beta^2,\quad \beta=\frac{v}{c},\quad \epsilon_2=\left[\cos\theta-\nu_{\text{ver}}\frac{B}{R}\cos(\psi+\psi_0)\right]^2,$$

$$\cos\psi_0=\pi n_+/4N\nu_{\text{ver}},\quad \nu_{\text{ver}}=\sqrt{n_-/2+\pi^2n_+^2/12N^2},$$

$$\nu'=\nu(1+h\omega/E),\quad \omega=\nu\omega_0,\quad \omega_0=eeH/E.$$

The subscripts σ and π specify two orthogonal polarization components of the radiation (for the first component, the electric field is in the plane of the orbit).

If we integrate (3) over the angles θ and φ, the spectral expressions found as a result will be the same, within terms on the order of A^2/R^2 and B^2/R^2, as the corresponding characteristics for a uniform magnetic field.

We find another case by summing over the spectrum in (3):

$$\frac{dW_\sigma}{d\Omega}=\frac{7ce^2}{64\pi^2R^2}\int_0^{2\pi}d\psi\left(\frac{1}{\epsilon_1^{5/2}}-\frac{320}{7\sqrt{3}\pi}\frac{1}{\epsilon_1^4}\frac{h\omega_0}{E}\right),$$
$$\frac{dW_\pi}{d\Omega}=\frac{5ce^2}{64\pi^2R^2}\int_0^{2\pi}d\psi\left[\left(\cos\theta-\nu_{\text{ver}}\frac{B}{R}\cos(\psi+\psi_0)\right)^2\left(\frac{1}{\epsilon_1^{7/2}}-\frac{256}{5\sqrt{3}\pi}\frac{1}{\epsilon_1^5}\frac{h\omega_0}{E}\right)\right]. \tag{4}$$

If we now integrate over θ and ψ, we find expressions for the total intensity:

$$W_\sigma=\frac{7ce^2}{12R^2\epsilon_0^2}\left(1-\frac{25\sqrt{3}}{7\epsilon_0^{3/2}}\frac{h\omega_0}{E}\right),\quad W_\pi=\frac{ce^2}{12R^2\epsilon_0^2}\left(1-\frac{5\sqrt{3}}{2\epsilon_0^{3/2}}\frac{h\omega_0}{E}\right).$$

Methods for integrating the first terms in (4) over ψ were studied elsewhere by Zhukovskiĭ and the present author.[2] We introduce

$$\epsilon=\epsilon_0+\cos^2\theta,\quad q=\nu_{\text{ver}}B/R,\quad p=q^2/\epsilon,\quad g=\cos^2\theta/\epsilon,\quad p_1=q^2/\epsilon_0,$$

$$g_1=\cos^2\theta/\epsilon_0,\quad f=\epsilon_0/\epsilon,\quad \Delta=(1+p)^2-4pg,\quad 2r^2=1-(1-p)/\Delta^{1/2}.$$

As a result, the classical part of the angular distributions of the radiation intensity, in the ultrarelativistic case, can be written

$$\frac{dW_\sigma}{d\Omega}=\frac{14W}{3\pi\epsilon^{5/2}\Delta^{5/4}}\left\{\left(3+p_1+16\frac{pg}{\Delta}-\frac{2p}{\Delta^{1/2}r^2}G_1\right)K+\frac{2\Delta^{1/2}}{f}G_1E\right\},$$

$$\frac{dW_\pi}{d\Omega}=\frac{2W}{3\pi\epsilon^{5/2}\Delta^{5/4}}\left\{\left(G_2-\frac{p}{\Delta^{1/2}r^2}G_3\right)K+\frac{\Delta^{1/2}}{f}G_3E\right\}, \tag{5}$$

where $K(r)$ and $E(r)$ are the complete elliptic integrals,

$$W=ce^2/32\pi R^2,\quad G_1=p_1-g_1+(2/\Delta)[(p+f)^2-g^2],$$

$$G_2=p_1+\frac{1}{\Delta}[8p(p+f)-25pg+15g]+\frac{8pg}{\Delta^2}[9(p-g)^2-7f^2+2f(p+g)],$$

$$G_3=1+2(p_1-g_1)+\frac{1}{\Delta}[4(p^2-f^2)+3g(p-7f)-7g^2]$$

$$+8\frac{gf}{\Delta^2}[7-9p^2+2p(g-f)].$$

In a symmetric model ($n_1=n_2$) the parameter q is $q=(\pi n/2\sqrt{3})B/R$.

We turn now to the quantum corrections in (4). We assume

$$I_k=\int_0^{2\pi}d\psi\epsilon_1^{-k}.$$

In particular, we have

$$I_1=\int_0^{2\pi}\frac{d\psi}{\epsilon_3+(\cos\theta-q\cos\psi)^2}=\frac{\pi\sqrt{2}}{\sqrt{\Delta_1^{1/2}+b}}\left(\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{c}}\right),$$

where

$$\epsilon_3=(1-\beta^2)\sin^2\theta,\quad a=\epsilon_3(\cos\theta+q)^2,\quad \Delta_1=\epsilon^2\Delta,$$

$$b=\epsilon_3+\cos^2\theta-q^2,\quad c=\epsilon_3+(\cos\theta-q)^2.$$

Differentiating I_1 with respect to ϵ_3 three times, we find

$$I_4=\frac{\pi}{8\sqrt{2}}\frac{\sqrt{a}+\sqrt{c}}{\sqrt{\Delta_1^{1/2}+b}}\frac{1}{\Delta_1^3}[5M^3+M^2(6N-9)+M(5N^2-8N-1)$$

$$+5(N^3-N^2-2N+1)],$$

where

$$M=\frac{\sqrt{a}+\sqrt{c}}{2(\Delta_1^{1/2}+b)},\quad N=\sqrt{\frac{a}{c}+\sqrt{\frac{c}{a}}}.$$

The quantum corrections to $dW_\sigma/d\Omega$ and $dW_\pi/d\Omega$ are, respectively,

$$-\frac{5ce^2}{\sqrt{3}\pi^3R^2}I_4\frac{h\omega_0}{E},\quad -\frac{4ce^2}{\sqrt{3}\pi^3R^2}I_5^2\frac{h\omega_0}{E},$$

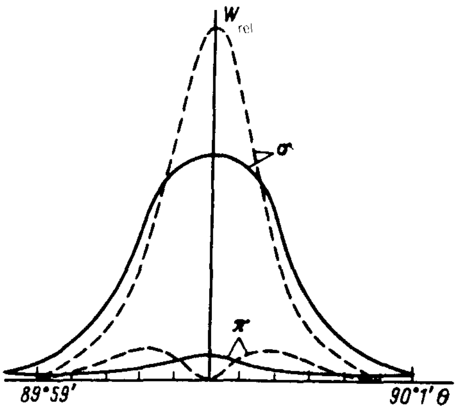

FIG. 1.

where we have

$$I_5^2 = \int_0^{2\pi} d\psi[\cos\theta - q\cos(\psi+\psi_0)]^2/\epsilon_1^5 = \frac{1}{8} I_4 + \frac{1}{48} \frac{\partial^2 I_3}{\partial \cos\theta^2}$$

for the second component of the integral.

If the vertical oscillations are small, then in the case $p \ll 1$ we can expand in powers of p in (5). We find

$$\frac{dW_\sigma}{d\Omega} = \frac{7W}{\epsilon^{5/2}}\left(1 - \frac{5}{4} p + \frac{35}{4} pg\right), \tag{6}$$

$$\frac{dW_\pi}{d\Omega} = \frac{5W}{\epsilon^{5/2}}\left(g + \frac{1}{2} p - \frac{35}{4} pg + \frac{63}{4} pg^2\right).$$

With $p=0$ here, we obtain the equations for the case of a uniform magnetic field.

It appears that no corresponding experiments have been carried out. Nevertheless, we can choose some parameter values close to those of existing apparatus and work from Eqs. (5) and (6) to plot some curves (the quantum increments are small) (Fig. 1). We adopt $E=4$ GeV, $B=0.5$ mm, $R=30$ m, $N=24$, and $n_1=n_2=70$. To find the complete elliptic integrals, we use the tables of Ref. 5. Shown for comparison in Fig. 1 are curves (the dashed curves) for the case of a uniform magnetic field. The behavior of these curves shows that the vertical betatron oscillations, in particular, cause the sharp peaks of the linear-polarization components of the radiation to spread out. Since the π component does not vanish in the orbital plane, the synchrotron light is not completely polarized.

We see that betatron oscillations play such an important role that the π component acquires a maximum, instead of a minimum, at $\theta=\pi/2$. This fact has been established previously for the case of weak focusing (see Fig. 11 in Ref. 6). For the given energy, the corresponding minimum first appears at $B < 0.3$ mm.

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