Adiabatic transition of the pump into second optical harmonic

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Mode-type solutions with a z-independent ratio E_{2w}/E_w^2 are found for a medium which supports $\hbar w \leftrightarrow 2\hbar w$ interaction for arbitrary wave vector mismatch $\Delta k = k_2 - 2k_1$. Complete adiabatic energy transition from E_w to E_{2w} is proposed for a medium with gradually z-dependent mismatch $\Delta k(z)$.

The process of second-harmonic generation is described by coupled wave equations for slowly varying envelopes:

$$\frac{\partial E_2}{\partial z} = i\Delta k \times E_2 + i\mu E_1^2 \frac{\partial E_1}{\partial z} = i\mu E_1^* E_2, \tag{1}$$

where $\mu = 2\pi\chi^{(2)} w/cn > 0$ is the nonlinear coupling coefficient. It is usually assumed that most efficient energy transfer $E_w \to E_{2w}$ is attained at zero value of the wave vector mismatch $\Delta k = k_2 - 2k_1$. In particular, for $\Delta k = 0$ there is a well-known solution of (1):

$$E_1 = E_0/\cosh(\mu |E_0| z), \quad E_2 = i \frac{E_0^2}{|E_0|} \tanh(\mu |E_0| z), \tag{2}$$

which corresponds to zero amplitude of second harmonic in the input, $E_{2w}(z=0) = 0$. For $z \to \infty$ that solution describes asymptotically complete energy transfer $E_w \to E_{2w}$. However, the resulting second harmonic field E_{2w} in a medium with $\Delta k = 0$ is unstable relative to the parametric decay down to the waves E_w.

It is suggested here that a medium with gradually changing $\Delta k(z)$ can be used in such a way that an almost pure second harmonic E_{2w} would appear only in the region in which a large $|\Delta k|$ ($|\Delta k| \gg 2\mu |E_0|$) renders the wave E_{2w} stable.

With this goal in mind, we consider first the mode-type solutions of Eq. (1) for a medium with a constant value of Δk. (Mode-type solutions for even more general case of a three-wave interaction $\hbar w_1 + \hbar w_2 = \hbar w_3$, but for zero phase mismatch $\Delta k = 0$, were considered recently by A. E. Kaplan.[1]) By the term "mode" we mean the solution with a dynamic balance of energy transfer $E_w \to E_{2w}$ and back $E_{2w} \to E_w$, i.e., a solution of the type

$$E_1(z) = E_0 e^{i(p/2)z} \cos \psi, \quad E_2(z) = \frac{E_0^2}{|E_0|} e^{ipz} \sin \psi, \tag{3}$$

where $|E_0|^2$ is the conserved value of the total energy flux.

There is one "trivial" mode in Eq. (1),

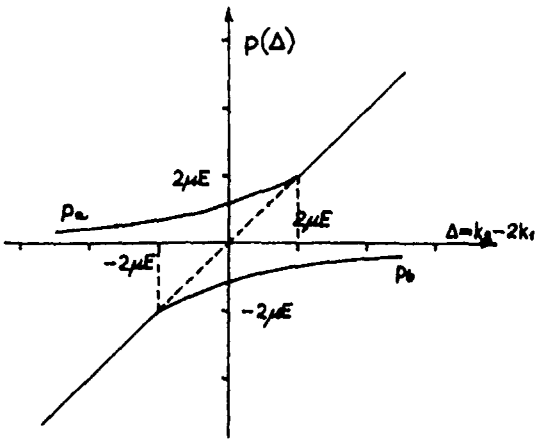

FIG. 1. Dependence of eigenvalues p on the wave vector mismatch $\Delta = k_2 - 2k_1$ for different modes: "a," "b," and the trivial mode $p = \Delta$. Dashed line notes the instability region of the trivial mode.

$$E_1 \equiv 0, \quad E_2 = |E_0| e^{2i\alpha + ipz}, \quad p \equiv \Delta k, \quad \alpha = \text{const}, \tag{4}$$

which corresponds to pure second-harmonic light. That mode is unstable relative to the parametric decay in the interval of the wave vector mismatch $|\Delta k| < 2\mu |E_0|$ and is stable outside this interval (see Fig. 1).

Furthermore, for $-\infty < \Delta k < 2\mu |E_0|$ there is a mode "a" with

$$\cos \psi_a = \sqrt{1 - \frac{p_a^2}{(2\mu |E_0|)^2}}, \quad \sin \psi_a = \frac{p_a}{2\mu |E_0|},$$

$$p_a = \frac{\Delta k}{3} + \sqrt{\left(\frac{\Delta k}{3}\right)^2 + \frac{4}{3} (\mu |E_0|)^2} > 0. \tag{5}$$

In this mode E_w and E_{2w} are mixed with such phases that time-averaged value of the cube of a real optical field is positive, $\langle E_{\text{real}}^3(z,t) \rangle = \frac{3}{4} |E_0|^3 \cos^2 \psi_a \sin \psi_a > 0$ (Ref. 2).

For $\Delta k \to -\infty$ the "a" mode consists predominantly of the pump E_w, with a small admixture of the second harmonic, $E_{2w} \approx \mu E_w^2 / \Delta k$. For $\Delta k \to 2\mu |E_0| - 0$ the "a" mode passes into an almost pure second harmonic E_{2w}, with a small admixture of the fundamental frequency wave E_w. We can therefore assume the trivial E_{2w} mode for $\Delta k > 2\mu |E_0|$ to be the direct continuation of the "a" mode.

In an analogous way, for $-2\mu |E_0| < \Delta k < +\infty$ there is a "b" mode with

$$\cos \psi_b = \sqrt{1 - \frac{p_b^2}{(2\mu |E_0|)^2}}, \qquad \sin \psi_b = \frac{p_b}{2\mu |E_0|},$$

$$p_b = \frac{\Delta k}{3} - \sqrt{\left(\frac{\Delta k}{3}\right)^2 + \frac{4}{3} (\mu |E_0|)^2} < 0. \tag{6}$$

For this mode we have $\langle E^3_{\text{real}}(z,t)\rangle = \frac{3}{4}|E_0|^3 \cos^2 \psi_b \sin \psi_b < 0$. There is a continuous transition of this mode into the almost pure pump for $\Delta k \to +\infty$ and into the "trivial" E_{2w} mode for $\Delta k < -2\mu|E_0|$. For both modes ("a" and "b") the absence of energy exchange between the fundamental frequency wave and the second harmonic is consistent with zero or 180° phase shift between E_{2w} and E^2_w.

The main idea of this letter is to use the medium with a slowly varying $\Delta k(z)$. If the pump E_w enters the medium in the region with $\Delta k(z=0) \ll -2\mu|E_0|$, then an almost pure "$a$" mode is excited. The hypothesis (which we hope to confirm by numerical simulation) is that a slow change of $\Delta k(z)$ from $\Delta k \ll -2\mu|E_0|$ to $\Delta k \gg 2\mu|E_0|$ adiabatically keeps the system in the "a" mode, and hence results in almost 100% transfer of energy of the pump E_w to the second harmonic E_{2w}. Usual adiabaticity satisfies the condition

$$|d\Delta k/dz| \ll (\mu|E_0|)^2. \tag{7}$$

This process is somewhat analogous to adiabatic advance of polarization in smoothly inhomogeneous nematic liquid crystals (see, e.g., Refs. 3–5), adiabatic passage in magnetic and optical resonances,[6] Landau–Zener picture of predissociation of diatomic molecules,[7] etc. In our case, however, the equations are essentially nonlinear, in contrast with the above-mentioned problems. The modes here are not orthogonal in any reasonable sense and the principle of superposition is not valid. The number of modes (three) is even larger than the number of degrees of freedom (two). The results obtained for the above-mentioned adiabatic processes therefore cannot be applied to our problem, and numerical simulation is a necessity. However, the existence of a large splitting of eigenvalues,

$$p_a - p_b = 2\sqrt{\frac{(\Delta k)^2}{9} + \frac{4}{3}(\mu|E_0|)^2}, \tag{8}$$

makes the hypothesis of adiabatic passage in condition (7) look reasonable.

There are several possibilities of the physical realization of a medium with $d\Delta k/dz$. One of them is the gradient of a chemical compound or a temperature gradient in a crystal. Another one is linked with the artificial phase-matching schemes which use periodic domain structures, so that $\Delta k(z) = k_2 - 2k_1 - q(z)$, where $q = 2\pi/\Lambda$, and Λ is the period of the domain's grating. In the latter case one should use the grating with a slowly varying period $\Lambda(z)$.

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