Delayed self-interference of a photon

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A delayed self-interference of a photon in a sample containing resonant two-level atoms is analyzed under conditions such that the difference between two optical path lengths is greater than the "length" of a photon. An electromagnetic readout pulse can excite a stimulated echo response.

The purpose of this letter is to call attention to a delayed self-interference of a photon, which is manifested, in particular, in photon echo signals in which phase information is incorporated in a single photon.

Figure 1 shows a typical sequence of echo signals during an observation of a photon echo.[1] Two "short" pulses of a resonant electromagnetic field, separated by a "long" time interval τ, generate a primary echo signal ϵ_π at a time τ after the second pulse. A third pulse applied a time interval T after the second pulse causes a stimulated echo signal ϵ_c. The same time interval τ separates the signal ϵ_c from the third pulse. In practice, therefore, the third pulse reads out information introduced by the first two pulses. These signals have the following properties: 1) The amplitude dependence is $\epsilon_\pi \propto A_1A_2^2$, $\epsilon_c \propto A_1A_2A_3$; 2) the wave vectors of the electromagnetic fields of the echo responses are $\mathbf{k}_\pi=2\mathbf{k}_2-\mathbf{k}_1$, $\mathbf{k}_c=\mathbf{k}_3+\mathbf{k}_2-\mathbf{k}_1$; 3) the maxima of the wave packets of the echo responses occur at $t_\pi=2\tau$, $t_c=T+\tau$, where A_i and $\mathbf{k}_i$ are the amplitude and wave vector of the electric field of the first pulse. It is important to note that the echo signals ϵ_π and ϵ_c are absent if the amplitude of even one of the preceding pulses is zero. Figure 1 shows a typical layout of an experiment for observing photon echoes. The light from source S can reach sample O along two paths: directly through beam splitter A, in a time $\tau_A=(SA+AO)/c$, and after reflection from two mirrors, in a time $\tau_{AB}=(SA+AB+BO)/c$. In addition, there is another pulsed light source, S^*, of high intensity. The carrier frequencies of the two sources, ck_1 and ck_3, coincide with the resonant frequency ω_0 of the atoms of the sample.

We assume that the intensity of source S is so low that the average time between the emission of two photons satisfies $\tau_0 \gg \tau_{ph}+\tau_{AB}$, where τ_{ph} is the temporal length of one photon. Under these conditions, there can be no more than one photon in the experimental apparatus at any instant. We also assume $\tau=\tau_{AB}-\tau_A \gg l/c+\tau_{ph}$, where l is the linear dimension of the sample. The part of the "photon density" which has followed path SAO then leaves the sample before the photon density from arm $SABO$ reaches it. These two photon wave packets would serve as two optical pulses in a proposed echo experiment. In this experiment, the sample is exposed to light so faint that there is no more than one photon in the apparatus at any instant.[2,3] In a case in which the echo response generated by a single photon is observed, one can say that there is a delayed self-interference of a photon, since the photon state densities prop-

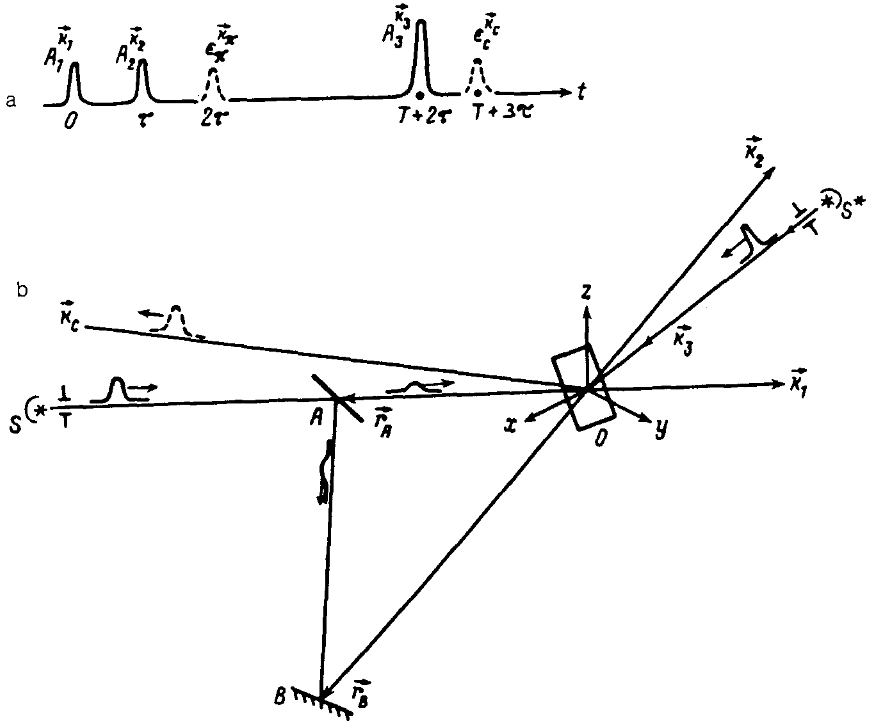

FIG. 1. a—Time sequence of pulses which generate the primary echo signal ($t=2\tau$) and the stimulated echo signal ($t=T+2\tau$); b—schematic diagram of a layout for observing delayed self-interference of a photon.

agating along optical paths $SA+AO$ and $SA+AB+BO$ (Fig. 1b) must not overlap in echo experiments.

We consider a system of two-level atoms to which an electromagnetic field is applied. We take the Hamiltonian of this physical system to be

$$\hat{\mathscr{H}}=\hat{\mathscr{H}}_a+\hat{\mathscr{H}}_{\rm ph}+\hat{V},\quad \hat{\mathscr{H}}_a=\sum_j \omega_j \hat{R}_3^j,\quad \hat{\mathscr{H}}_{\rm ph}=\int_{-\infty}^{\infty} d^3k\omega_{\mathbf{k}}\hat{a}_{\mathbf{k}}^+\hat{a}_{\mathbf{k}},$$

$$\hat{V}=-i\sum_j \int_{-\infty}^{\infty} d^3k[g(k)\hat{a}_{\mathbf{k}}\hat{R}_+^j e^{i\mathbf{kr}_j}-g^*(k)\hat{a}_{\mathbf{k}}^+\hat{R}_-^j e^{i\mathbf{kr}_j}], \qquad (1)$$

where $\omega_k=ck$, $\hat{R}_\pm=\hat{R}_1\pm i\hat{R}_2$ and $\hat{R}_n=1/2\hat{\sigma}_n$, $\hat{\sigma}_n$ are the Pauli matrices, ω_j is the interval between the energy levels of atom j, $\hat{\mathscr{H}}_{\rm ph}$ and $\hat{\mathscr{H}}_a$ are the energy operators of the atoms and the electromagnetic field, $[\hat{a}_{\mathbf{k}'},\hat{a}_{\mathbf{k}}^+]=\delta(\mathbf{k}'-\mathbf{k})$, $g(\mathbf{k})=(\mathrm{e}_{\omega k}\mathrm{d})\times(\omega_0^2/4\pi^2\omega_k)^{1/2}$, d is the transition dipole moment, and $\mathrm{e}_{\omega k}$ is the polarization vector of mode $(\omega,\mathbf{k})$.

We assume that at $t=-\infty$ the physical system is described in the interaction representation by the wave function

$$|\Psi(-\infty)\rangle = \int_{-\infty}^{\infty} d^3k F^{(0)}(\mathbf{k}-\mathbf{k}_1)\hat{a}_{\mathbf{k}}^{+}|OB\rangle,$$

$$|OB\rangle = |O\rangle \otimes |B\rangle, \quad |O\rangle = \prod_j |\chi_j^-\rangle, \tag{2}$$

where $|\chi_j^+\rangle$ and $|\chi_j^-\rangle$ are the upper and lower states of atom j, $|B\rangle$ is the vacuum state of the field, and $F^{(0)}$ $(\mathbf{k}-\mathbf{k}_1)$ is the complex normalized function $\mathbf{k}$, which has a sharp peak at $\mathbf{k}=\mathbf{k}_1$. The function $|\psi_{\rm ph}\rangle$ corresponds to one photon which is propagating near the direction $\mathbf{k}=\mathbf{k}_1$ and which is incident on the first mirror.[4] It can be shown that after two mirrors are passed (Fig. 1b) the function $F^{(0)}$ $(\mathbf{k}-\mathbf{k}_1)$ transforms into $\mathscr{F}^{(0)}(\mathbf{k}|\mathbf{k}_1,\mathbf{k}_2)$:

$$F^{(0)}(\mathbf{k}-\mathbf{k}_1) \rightarrow \mathscr{F}^{(0)}(\mathbf{k}|\mathbf{k}_1,\mathbf{k}_2) = \frac{1}{\sqrt{2}}\{F^{(0)}(\mathbf{k}-\mathbf{k}_1) + F^{(0)}(\mathbf{k}-\mathbf{k}_2)e^{i(\mathbf{RL})}\},$$

$$\mathbf{L} = [2\mathbf{n}_B(\mathbf{n}_A\mathbf{n}_B) - \mathbf{n}_A](\mathbf{n}_A\mathbf{r}_A) - \mathbf{n}_B(\mathbf{n}_B\mathbf{r}_B), \tag{3}$$

where $\mathbf{r}_A$, $\mathbf{r}_B$ are the radius vectors of the centers of mirrors A and B, and $\mathbf{n}_A$, $\mathbf{n}_B$ are the normals to them. The difference between the photon propagation times along the two arms of the interferometer is $\tau = (\mathbf{k}\cdot\mathbf{L})/kc$. Since the field of an individual photon is weak, we can use the first approximation of the wave function in solving the Schrödinger equation:

$$|\psi(t)\rangle \cong \left[1 - i^{-1}\int_{-\infty}^{t} dt\hat{V}(t)\right]\int_{-\infty}^{\infty} d^3\mathbf{k}\, \mathscr{F}^{(0)}(\mathbf{k}|\mathbf{k}_1,\mathbf{k}_2)\hat{a}_{\mathbf{k}}^{+}|OB\rangle. \tag{4}$$

Let us consider how a photon acts on the sample in this experiment as it propagates along the two optical paths. The matrix element $\langle\psi(t)|\hat{R}_{22}^{j}|\Psi(t)\rangle$ of the operator $\hat{R}_{22} = 1/2 + \hat{R}_3$ describes the probability that an atom will be in the upper level at the time t. We can clarify the spatial nature of the excitation of the system of atoms after the photon has passed through the sample:

$$\langle\psi(t)|\hat{R}_{22}^{j}|\Psi(t)\rangle = |\widetilde{F}_1(\omega_j-\omega_0)|^2 + |\widetilde{F}_2(\omega_j-\omega_0)|^2 + |\widetilde{F}_1(\omega_j-\omega_0)$$
$$\times \widetilde{F}_2^{*}(\omega_j-\omega_0)| \cos\left\{\omega_j\tau + \frac{\omega_j}{\omega_0}(\mathbf{k}_2-\mathbf{k}_1)\mathbf{r}_j\right\},$$

$$\widetilde{F}_1(\omega_j-\omega_0) = \sqrt{2\pi}\int_{-\infty}^{\infty} d^3k g(k) F^{(0)}(\mathbf{k}-\mathbf{k}_1)\delta(\omega_j-\omega_k). \tag{5}$$

An interference pattern is thus created in the medium by the passage of one photon along two optical paths of the interferometer. If $\tau=0$ (i.e., if the optical paths are identical), there will be an interference of the photon, as has been detected by classical optical methods.[2,3] If $\tau=\tau_{AB}-\tau_A>\tau_{\rm ph}$, expression (5) means that there is also a delayed interference.

One might expect that the passage of a photon through a sample would generate, in addition to a static grating, an oscillatory polarization of the medium, as in an ordinary echo experiment.[1] The electromagnetic field of the coherent response radiated by this polarization can be found by calculating the expectation values of the field operators $\hat{E}^{\pm}(r,t)$ and the radiation intensity operator $\hat{I}(r,t)$ of system (5) in state (4).

A calculation shows that after the passage of an individual photon the expectation values $\langle\psi(t)|\hat{E}(r,t)|\psi(t)\rangle$ and $\langle\psi(t)|\hat{I}(r,t)|\psi(t)\rangle$ in state (4) contain no terms capable of generating an echo response. One might expect that the primary echo signal would be zero until we get to three-photon initial states. To calculate the stimulated echo, we found the wave function $|\psi(t)\rangle$ after the application of the third pulse $(t>T)$. It turns out that coherent emission arises in the $\mathbf{k}_c$ direction:

$$\langle\hat{E}^{+}_{\mathbf{k}_c}(t)\rangle=\langle\hat{E}^{-}_{\mathbf{k}_c}(t)\rangle^{*}=i\frac{d}{2}\sin\theta_3\Phi(t-t_c)e^{-i\omega_0(t-t_c)}, \tag{6}$$

where

$$\Phi(t-t_{\text{se}})=\int_{-\infty}^{\infty}d\Delta G(\Delta)|\tilde{F}(\Delta)|^2e^{-i\Delta(t-t_c)}$$

is the function $\Phi(t-t_c)$ from echo theory. It has a maximum $\Phi(0)=1$ at $t=t_c=\tau+T$ and falls off rapidly with increasing value of the difference $|t-t_c|$. This circumstance is responsible for a temporal localization of the appearance of the stimulated echo signal: $t^*=t_c-r/c$. Here $\theta_3=dh^{-1}A_3t_3$, where t_3 is the length of the intense third laser pulse.

A surprising aspect of this field is that the coherent part of the expectation value of its intensity is zero. This circumstance distinguishes this field from the familiar quantum fields in Fok, Glauber, and squeezed states.[6] This state of the field has no classical analog. It arises from purely quantum properties of the collective state of the medium which arises under these experimental conditions.

As we know, a stimulated-echo signal can be amplified considerably by a buildup effect: The third pulse would read out the total contribution from many pairs of weak preceding pulses.[7,8] This also happens in the case at hand, if several (M) photons manage to act on the medium before the readout pulse during the lifetime of the atomic excitations, T_1. If the effects of the individual photons on the sample are to be independent, the photons must be emitted by light source S no more rapidly than at time intervals τ_0. Under these conditions, the quantity in (6) is multiplied by M, and the intensity $\langle\hat{I}_{k_c}(r,t)\rangle$ satisfies

$$\langle\psi(t)|\hat{I}_{k_c}(r,t)|\psi(t)\rangle=\frac{M(M-1)\lambda^2}{4S}\frac{h\omega_0}{T_1}|\Phi(t-t_c)|^2, \tag{7}$$

where S is the cross-sectional area of the medium, and λ is the wavelength of the light. If M is large, the amplitude and intensity properties of the field thus approach their classical values. Let us estimate the number (Z) of photons which can be emitted in the stimulated-echo signal from a grating of spatial excitations of atoms formed as the result of a self-interference of individual photons with $\theta=\pi/2$. Assuming $T_2^*\sim T_2/10$, $M\approx T_1/\tau_0$, and $\tau_0\approx T_2$, and assuming the typical values $\lambda\approx0.5\times10^{-4}$ cm, $S\approx10^{-2}$ cm^2, and $T_2\approx10^{-8}$ s, we find the estimate $Z\approx T_1$, where the lifetime T_1 is in seconds. With $T_1>1$ s, the effect would thus be completely observable. For experimental observation of the definitely nonclassical field state which arises at $M=1$, we might propose an optical heterodyning layout. Along with the echo signal, we would need to apply radiation $\hat{E}_{\text{het}}(t)$ in a coherent (Glauber) quantum state with $\langle\hat{E}^{-}_{\text{het}}(t)\rangle = E_{\text{het}}(t)$. The magnitude of the signal to be detected, $I_{\text{sig}}(t) \propto E_{\text{het}}(t)\langle\hat{E}^{+}_{k_c}(t)\rangle$, could be increased by increasing the intensity of the coherent radiation. Significantly, the direct detection of the signal $I_{\text{sig}}(t)$ in the $M=1$ case would make it possible, in the case of a positive outcome, to observe an interference of paths even for a single particle in one experiment. This could be done without having to wait for a necessary buildup, as is usually assumed in quantum mechanics in the case of the conventional methods for detecting microparticles. We might also point out that a simple observation of the time evolution of the intensity would not, according to our calculations, lead to any pulse at the time $t=t_c$.

Dirac offered a pertinent aphorism a long time ago (we are paraphrasing): Each photon interferes only with itself; interference between different photons never occurs.[9] Still, one cannot predict at the outset whether an interference will be observed under the conditions of an echo experiment. The observation of such an interference would apparently make it possible to gain a deeper understanding of the quantum nature of matter, and a new paradox would arise to supplement Dirac's aphorism.

[1] É. A. Manykin and V. V. Samartsev, *Optical Echo Spectroscopy* (Nauka, Moscow, 1984).
[2] R. L. Pfleegor and L. Mandel, Phys. Lett. A**24**, 766 (1967).
[3] A. A. Sanin, A. V. Zharko, V. I. Iveronova *et al.*, Sov. Phys. JETP **29**, 43 (1969).
[4] D. I. Blokhintsev, *Space and Time in the Microworld* (Nauka, Moscow, 1982).
[5] G. S. Agarval, *Quantum Optics*, Vol. 70 (Springer-Verlag, New York, 1974). p. 135.
[6] S. Ya. Kilin, *Quantum Optics: Fields and Their Detection*, Nauka i tekhnika, Minsk, 1990.
[7] H. Hesselink and D. Wiersma, Phys. Rev. Lett. **43**, 1991 (1979).
[8] S. A. Moiseev, N. L. Nevel'skaya, and E. I. Shtyrkov, Opt. Spektrosk, **71**, 510 (1991) [Opt. Spectrosc. (USSR) **71**, 296 (1991)].
[9] P. A. M. Dirac, *Principles of Quantum Mechanics*, Oxford Univ. Press, London, 1958.

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