Evolution of a bubble in a theory with a degenerate vacuum in Friedmann and de Sitter spaces

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Equations describing the evolution of a bubble radius in a theory with a degenerate vacuum in Friedmann and de Sitter external gravitational fields are constructed. A bubble does not collapse in de Sitter space. © *1995 American Institute of Physics.*

The properties of bubbles were first determined for a theory with a degenerate vacuum[1] with the Lagrangian

$$\mathscr{L}(\phi)=1/2(\partial\phi)^2-\lambda^2(\phi^2-\eta^2)^2 \tag{1}$$

(λ and η are constants) and were later studied in a theory with a metastable vacuum.[2] They were studied in detail in Refs. 3 for Lagrangian (1). The effect of the gravitational self-field on the dynamics of bubbles has been studied in several papers.[4] The question of how gravitation affects the evolution of bubbles can be approached from a different direction. It is quite possible to imagine a situation in which the effect of the gravitational self-field of a bubble can be ignored against the background of a strong external field.

In this letter we examine the behavior of a bubble in a theory described by Lagrangian (1) against the background of an external metric. We consider two cases. a) an expanding Friedmann space[5] (with a cosmological constant $\Lambda=0$ and a matter pressure $p=0$):

$$ds^2=a^2(\eta)\{d\eta^2-d\chi^2-\sinh^2\chi(d\theta^2+\sin^2\theta d\phi^2)\}, \tag{2}$$

where $a(n)=a_0(\cosh\eta-1)$. b) A de Sitter space[2)] (Ref. 6; $\Lambda=3H^2$, where H is the Hubble constant):

$$ds^2=dt^2-a^2(t)[dr^2+r^2(d\theta^2+\sin^2\theta d\phi^2)], \tag{3}$$

where $a(t)=a_0\exp(Ht)$. The action for scalar field (1) ($\lambda=1/\sqrt{2},\ \eta=1$) with minimal coupling in a gravitational field is of the standard form:

$$S=\int dx\sqrt{-g}[g^{ik}\phi_{,i}\phi_{,k}-(\phi^2-1)^2]. \tag{4}$$

We thus find the following expressions for the equation of motion of a scalar field in metrics (2) and (3):

$$\phi_{\eta\eta}+2a_\eta a^{-1}\phi_\eta-\phi_{\chi\chi}-2\cosh\chi/\sinh\chi\,\phi_\chi+2a^2(\eta)\phi(\phi^2-1)=0. \tag{5}$$

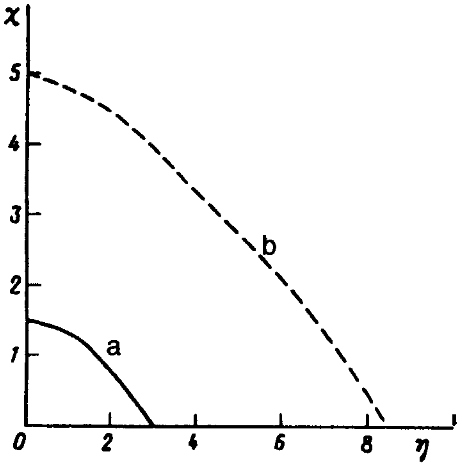

FIG. 1. The bubble radius χ versus the conformal time η for the following intitial conditions: a—$\chi(0)=1.5$; b—$\chi(0)=5.0$, for a Friedmann metric.

We adopt the initial conditions $\phi(\eta_0)=\tanh[a(\eta_0)(\chi-\chi_0)]$, $\phi_\eta(\eta_0)=0$. At large values of χ_0 they correspond to a bubble which is initially at rest (η_0). The corresponding equation in the de Sitter space [Eq. (3)] is

$$\phi_{tt}+3a_t a^{-1}\phi_t-2r^{-1}a^{-2}\phi_r-a^{-2}\phi_{rr}+2\phi(\phi^2-1)=0 \tag{6}$$

with the initial conditions $\phi(t_0)=\tanh[a(t_0)(r-r_0)]$, $\phi_t(t_0)=0$.

An equation describing the evolution of the bubble radius has been constructed by two methods. First, there is the "convolute action" method, first proposed in Ref. 2. Following this method, one can construct an action for the radius of a bubble in a Friedmann space from expression (4),

$$S=\int d\eta a^3(\eta)\sinh^2\chi\sqrt{1-\chi_\eta^2} \tag{7}$$

and a corresponding equation of motion,

$$\chi_{\eta\eta}+2\cosh\chi/\sinh\chi(1-\chi_\eta^2)+3a_\eta/a\chi_\eta(1-\chi_\eta^2)=0 \tag{8}$$

(here $\chi(\eta)$ is the bubble radius). Correspondingly, for a de Sitter space [Eq. (3)] we find

$$S=\int dta^2(t)R^2\sqrt{1-a^2\dot{R}^2} \tag{9}$$

and

$$\ddot{R}+2a^{-2}R^{-1}(1-a^2\dot{R}^2)+3\dot{a}a^{-1}\dot{R}(1-a^2\dot{R}^2)+\dot{a}a^{-1}\dot{R}=0, \tag{10}$$

where $R=R(t)$ is the bubble radius.

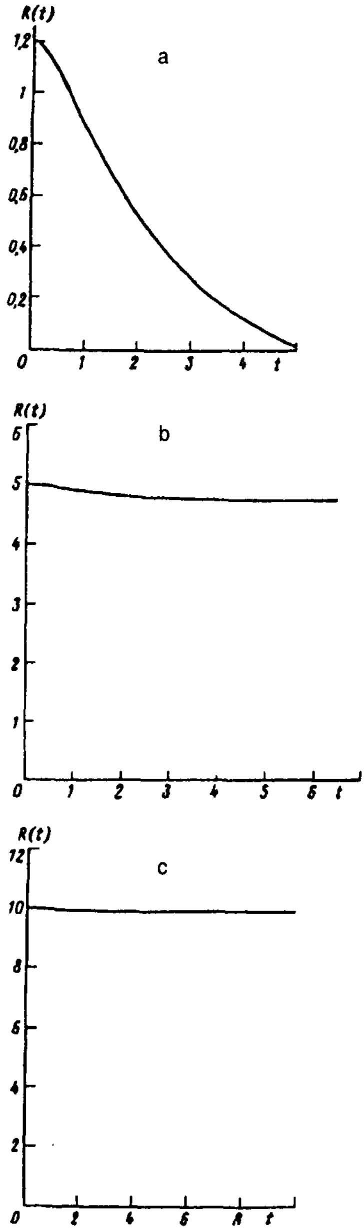

FIG. 2. The bubble radius R versus the time t under the following initial conditions: a—$R_0 = 1.2$ (collapse); b—$R_0 = 5.0$; c—$R_0 = 10$, for a de Sitter metric; $H = 0.5$.

The second method can be used to construct an equation of motion for the bubble radius directly from equations of motion (5) and (6). This method was developed in Ref. 8 for planar space. We will discuss this process briefly as it applies to curved spaces (2) and (3). We seek a solution of the equation

$$1/\sqrt{-g}(\sqrt{-g}g^{ik}\phi_i)_k+2\phi(\phi^2-1)=0$$

in the form $\phi=\tanh\alpha$, where α is a new variable. When we take the limit of an infinitely thin wall, this exact equation decomposes into two equations:

$$g^{ik}\alpha_i\alpha_k+1=\alpha^2 f,$$
$$1/\sqrt{-g}(\sqrt{-g}g^{ik}\alpha_i)_k=\alpha f, \tag{11}$$

where f is a function which does not affect the form of the equation of motion of the bubble radius. Further calculations depend on the particular form of the background metrics. For space (2) we introduce a parametrization which solves the first equation in (11):

$$\alpha_\chi=a(1-\alpha^2 f)^{1/2}(1-v^2)^{-1/2},$$
$$\alpha_\eta=-va(1-\alpha^2 f)^{1/2}(1-v^2)^{-1/2}. \tag{12}$$

The function v is the velocity of a surface $\alpha=\text{const}$: $v=-\alpha_\eta/\alpha_\chi$. For the radius of the $\alpha=0$ surface, which is also the bubble radius, we find from the second equation in (11) an equation for the radius which is the same as expression (8). This method makes it possible to extract, in addition to Eq. (8), some additional information on the behavior of the bubble, by virtue of the integrability condition $\alpha_{\eta\chi}=\alpha_{\chi\eta}$. A corresponding construction can be carried out for de Sitter space:

$$\alpha_r=a(1-\alpha^2 f)^{1/2}(1-v^2a^2)^{-1/2},$$
$$\alpha_t=-va(1-\alpha^2 f)^{1/2}(1-v^2a^2)^{-1/2}. \tag{13}$$

The equation for the bubble radius R (the $\alpha=0$ surface) found as a result is the same as expression (10).

Equations (8) and (10) have been solved numerically by the Runge–Kutta method for initial values corresponding to a bubble which is initially (at t_0) at rest: $\chi(0)=R_0$ and $\dot{\chi}(0)=0$. Figure 1 shows the results of numerical calculations for Eq. (8). For arbitrary initial values of R_0, the bubble collapses. A more complex behavior is observed for Eq. (10). For $R_0<R_{\text{cr}}\approx 1.22$, the bubble is a able to collapse. For initial radii R_0 greater than a critical radius R_{cr}, on the other hand, the bubble does not collapse (Fig. 2). After a slight initial contraction, the bubble gradually approaches a constant asymptotic value R_{as}. Figure 3 shows the asymptotic radius R_{as} versus the initial radius. In the range of applicability of Eq. (10), i.e., for a bubble radius $R\gg l\sim 1$, where l is the wall thickness, the bubble thus never collapses. We also note that Eq. (10) has the exact solution

$$R=C_0\pm a_0^{-1}H^{-1}\exp(-Ht)$$

(C_0 is an arbitrary constant). Unfortunately, this solution does not satisfy the initial condition $\dot{R}(0)=0$. To find a solution which does satisfy this condition, we must expand the function R in a power series in $\exp(-2Ht)$:

$$R=\sum_{n=0}^{\infty} C_n\exp(-2nHt).$$

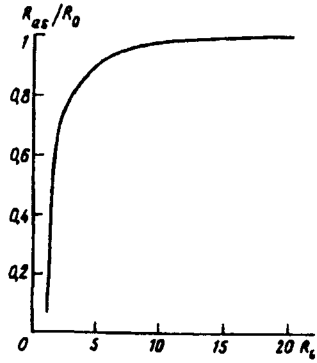

FIG. 3. Asymptotic value of the bubble radius as $t \to \infty$ versus the initial radius for a de Sitter metric; $H = 0.5$

The conclusion that a bubble does not collapse in de Sitter space is supported by a numerical calculation for exact equation (6). The results of this calculation [along with a numerical study of Eq. (5)] will be published later.

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[2)]The behavior of physical fields against the background of a de Sitter metric as a vacuum has been studied in many papers (see, for example, Ref. 7 and the papers cited there).

[1]Ya. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun', Zh. Éksp. Teor. Fiz. **67**, 3 (1974) [Sov. Phys. JETP **40**, 1 (1975)].
[2]M. B. Voloshin, I. Yu. Kobzarev, and L. B. Okun', Yad. Fiz. **20**, 1229 (1974) [Sov. J. Nucl. Phys. **20**, 644 (1974)].
[3]T. I. Belova, N. A. Voronov, I. Yu. Kobzarev, and N. B. Konyukhova, Zh. Éksp. Teor. Fiz. **73**, 1611 (1977) [Sov. Phys. JETP **46**, 846 (1977)]; V. G. Makhankov, Phys. Rep. **35**, 1 (1978); V. G. Makhankov, *Soliton Phenomenology* (Kluver, Dordrecht), 1990; T. I. Belova, Yad. Fiz. **56**(2), 234 (1993) [Sov. J. Nucl. Phys. **56**, 274 (1993)].
[4]S. Coleman and F. de Luccia, Phys. Rev. D **21**, 3305 (1980); S. K. Blau, E. I. Guendelman, and A. H. Guth, Phys. Rev. D **35**, 1747 (1987); C. Barrabes, B. Boisseau, and M. Sakellariadon, Phys. Rev. D **49**, 2734 (1994).
[5]L. D. Landau and E. M. Lifshtiz, *The Classical Theory of Fields* (Pergamon, New York, 1976).
[6]A. D. Dolgov, Yu. B. Zel'dovich, and M. V. Sazhin, *Cosmology of the Early Universe* [in Russian] (IMGU, Moscow, 1988), p. 80.
[7]A. Starobinsky and J. Yokoyama, Phys. Rev. D. **50**, 6357 (1994); B. Basu and A. Vilenkin, Phys. Rev. D **50**, 7150 (1994).
[8]I. Yu. Kobsarev and N. A. Voronov, Preprint ITEP-102, Moscow, 1977.

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