Regge trajectory of a gluon in the two-loop approximation

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The Regge trajectory of a gluon in QCD is constructed in the two-loop approximation. It is found from the amplitude for quark–quark scattering with gluon quantum numbers in the t channel, calculated at a logarithmic ($\log s$) accuracy in the two-loop approximation for asymptotically large energies $\sqrt{s}$ and for a fixed momentum transfer $\sqrt{-t}$.

Perturbative quantum chromodynamics (QCD) has scored a string of successes in describing "hard" processes, in which the validity and power of this theory have been firmly established. Its record is not as bright in the field of "semihard" processes. It is usually assumed that perturbation theory in QCD can be used to calculate parton distributions and the cross sections for these processes if the characteristic virtuality Q^2 is large enough to make the coupling $\alpha_s(Q^2)$ small. At a high energy $\sqrt{s}$ of the colliding particles, however, the logarithm of the ratio $1/x = s/Q^2$ may be so large that terms of the type $\alpha_s^n(\ln(1/x))^m$ must be summed. This problem was solved many years ago[1] in the leading-log approximation, which in the case at hand implies a summation of terms with $n=m$. The results of the leading-log approximation are now widely known and are used to describe experiments. Still, these results suffer from at least two serious shortcomings. First, the constraints of s-channel unitarity for scattering amplitudes do not work in this approximation. As a result, there is a violation of the Froissart theorem $\sigma_{\text{tot}} < c(\ln s)^2$; alternatively, structure functions increase sharply, by a power law, at small values of x. Second, since the dependence of the coupling constant α_s on the virtuality goes beyond the accuracy of the leading-log approximation, the numerical results of this approximation can be varied dramatically by a change in the scale of the virtuality. Different predictions may thus be generated by what would appear to be a single theory. The problem of calculating corrections to the leading-log approximation (i.e., terms with $n=m+1$) is accordingly a very important one.

A key to the solution of this problem may be[2] a property of non-Abelian gauge theories which has been proved in the leading-log approximation:[3] Gauge bosons in these theories are reggeized with a trajectory

$$j(t) = 1 + \omega(t), \tag{1}$$

where, in the single-loop approximation for the gauge group $SU(N)$ ($N=3$ for all QCD)

$$\omega(t) = \omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{3+\epsilon}} \frac{N}{2} \int \frac{d^{2+\epsilon}k_\perp}{k_\perp^2 (q-k)_\perp^2} \,. \tag{2}$$

Here g is the coupling constant of the gauge theory, q is the momentum transfer, and $t=q_\perp^2$. The integration is carried out over momenta perpendicular to the plane of the momenta of the initial particles. A dimensional regularization of the Feynman integrals is used:

$$\frac{d^2k}{(2\pi)^2}\to\frac{d^{2+\epsilon}k}{(2\pi)^{2+\epsilon}},\quad \epsilon=D-4, \tag{3}$$

where D is the dimensionality of the space–time ($D=4$ in the physical world).

The problem of calculating corrections to the leading-log approximation can be reduced to one of calculating corrections to the kernel of a Bethe–Salpeter equation for a t-channel partial amplitude with the quantum numbers of the vacuum.[1] This kernel is expressed in terms of a gluon trajectory and a reggeon–reggeon–gluon vertex. Corrections to the vertex have been calculated,[4,5] so calculating the two-loop correction to the gluon trajectory, $\omega^{(2)}(t)$, is the most urgent problem. The correction can be extracted from the s-channel jump $[(\mathscr{A}_8^{(-)})_{AB}^{A'B'}]_s$ in the elastic-scattering amplitude $(\mathscr{A}_8^{(-)})_{AB}^{A'B'}$(two loop) calculated in the two-loop approximation within a constant. Let us consider the amplitude for the process $A+B\to A'+B'$ with gluon quantum numbers in the t channel and with a negative signature. In factorized form this amplitude is

$$(\mathscr{A}_8^{(-)})_{AB}^{A'B'}=\Gamma_{A'A}^{c}\frac{s}{t}\left[\left(\frac{s}{-t}\right)^{\omega(t)}+\left(\frac{-s}{-t}\right)^{\omega(t)}\right]\Gamma_{B'B}^{c}, \tag{4}$$

where $\Gamma_{A'A}^{c}$ are the vertices of the interaction of a reggeized gluon with particles (PPR vertices). In the single-loop approximation we have

$$\Gamma_{A'A}^{i}=g\langle A'|T^i|A\rangle(\Gamma_{A'A}^{(0)}+\Gamma_{A'A}^{(1)}), \tag{5}$$

where $\langle A'|T^i|A\rangle$ is a matrix element of the color-group generator in the corresponding representation (i.e., in the associated representation for gluons and in the fundamental representation for quarks), and $\Gamma_{A'A}^{(0)}$ and $\Gamma_{A'A}^{(1)}$ are the Born and single-loop contributions. Using this form of the expression, we find the following result from (4) for the two-loop contribution to the jump $[(\mathscr{A}_8^{(-)})_{AB}^{A'B}]_s$:

$$[(\mathscr{A}_8^{(-)})_{AB}^{A'B'}]_s(\text{two loop})=-2\pi i g^2\langle A'|T^i|A\rangle\langle B'|T^i|B\rangle\frac{s}{t}$$

$$\times\left[\Gamma_{A'A}^{(0)}(\omega^{(1)}(t))^2\left(\ln\frac{s}{-t}\right)\Gamma_{B'B}^{(0)}+(\Gamma_{A'A}^{(0)}\Gamma_{B'B}^{(1)}+\Gamma_{A'A}^{(1)}\Gamma_{B'B}^{(0)})\omega^{(1)}(t)\right.$$

$$\left.+\Gamma_{A'A}^{(0)}\omega^{(2)}(t)\Gamma_{B'B}^{(0)}\right]. \tag{6}$$

Since the single-loop corrections $\Gamma_{A'A}^{(1)}$ to the PPR vertices are known,[4,6,7] the only unknown on the right side of (6) is the two-loop contribution to the gluon trajectory, $\omega^{(2)}(t)$. This contribution can thus be found once we have found $[(\mathscr{A}_8^{(-)})_{AB}^{A'B}]_s$(two loop) within a constant.

By definition, the trajectory must not depend on the particles being scattered, so we are free to choose any process to calculate this trajectory. Below we report results of calculations for the jump in the amplitude for quark–quark scattering. The details of the calculations will be published separately.[8] To simplify the equations we assume that the quarks here are massless. In this case the helicity of each of the colliding particles is strictly conserved, so the vertices $\Gamma^{c}_{A'A}$ have a definite spin structure as well as color structure:[1,7]

$$\Gamma^{i}_{Q'Q}=g\langle Q'|T^{i}|Q\rangle\delta_{\lambda_{Q'}\lambda_{Q}}(1+\Gamma^{(+)}_{QQ}). \tag{7}$$

Here $\lambda_{Q'}$ and λ_{Q} are the quark helicities, and $\Gamma^{(+)}_{QQ}$ is the single-loop correction to the vertex, calculated in Ref. 7.

Conservation of the helicity of each particle makes it possible to represent the s-channel jump in the part $\mathscr{A}_8^{(-)}$ of the amplitude with gluon quantum numbers in the t channel and with a negative signature as follows:

$$[(\mathscr{A}_8^{(-)})_{AB}^{A'B'}]_s=g^2\langle A'|T^{i}|A\rangle\langle B'|T^{i}|B\rangle\delta_{\lambda_A\lambda_{A'}}\delta_{\lambda_B\lambda_{B'}}(-2\pi is/t)\Delta_s. \tag{8}$$

In the two-loop approximation, this jump is given by the sum of the contributions of two-particle and three-particle intermediate states in the s channel:

$$\Delta_s=\Delta_s^{(2)}+\Delta_s^{(3)}. \tag{9}$$

For the first contribution we find

$$\Delta_s^{(2)}=\frac{-2g^4N^2t}{(4\pi)^{D/2}}\frac{\Gamma(2-D/2)\Gamma^2(D/2-1)}{\Gamma(D-3)}\int\frac{d^{(D-2)}q_{1\perp}}{(2\pi)^{(D-1)}}\frac{1}{(q_1-q)_\perp^2(-q_{1\perp}^2)^{3-D/2}}$$
$$\times\left[\ln\left(\frac{s}{-q_{1\perp}^2}\right)+c_g+c_q\right], \tag{10}$$

where c_g and c_q are coefficients associated with the gluon and quark contributions, respectively, given by:

$$c_g=\psi\left(3-\frac{D}{2}\right)-2\psi\left(\frac{D}{2}-2\right)+\psi(1)+\frac{1}{(D-3)}\left(\frac{1}{4(D-1)}-\frac{2}{D-4}-\frac{7}{4}\right),$$
$$c_q=\frac{1}{N(D-3)}\left[-\frac{n_f(D-2)}{2(D-1)}-\frac{1}{2N}\left(D-4+\frac{D}{D-4}\right)\right]. \tag{11}$$

Here $\psi(x)$ is the logarithmic derivative of the gamma function:

$$\psi(x)=\Gamma'(x)/\Gamma(x).$$

The contribution $\Delta_s^{(3)}$ is conveniently broken up into two parts:

$$\Delta_s^{(3)}=\Delta_s^{(3a)}+\Delta_s^{(3na)}. \tag{12}$$

The first part is of an Abelian nature; only the first part survives in the case of an Abelian gauge group (in QED). The second part is essentially non-Abelian. Here

$$\Delta_s^{(3a)}=\frac{g^4t}{4}\left(\frac{2}{D-4}-\frac{2}{D-3}+\frac{1}{2}\int\frac{d^{(D-2)}q_1d^{(D-2)}q_2}{(2\pi)^{(2D-2)}q_1^2q_2^2}\right.$$
$$\left.\times\left[\frac{-q^2}{(q_1-q)^2(q_2-q)^2}+\frac{2}{(q_1+q_2-q)^2}\right]\right. \tag{13}$$

and

$$\Delta_s^{(3na)}=\frac{g^4N^2t}{4}\int\frac{d^{(D-2)}q_1d^{(D-2)}q_2}{(2\pi)^{(2D-2)}q_2^2(q_2-q)^2}\left[2\left(\psi(D-3)-\psi(1)+\frac{3}{4(D-3)}\right)\right.$$
$$\left.\times\left(\frac{-q^2}{q_1^2(q_1-q)^2}+\frac{2q_2^2}{q_1^2(q_1-q_2)^2}\right)+\frac{q^2\ln(s/-k^2)}{q_1^2(q_1-q)^2}-\frac{2q_2^2\ln(s/-q_1^2)}{q_1^2(q_1-q^2)^2}\right]. \tag{14}$$

Using these results, we find the following expression from Eq. (6):

$$\omega^{(2)}(t)=\frac{g^4t}{4}\int\frac{d^{(D-2)}q_1d^{(D-2)}q_2}{(2\pi)^{(2D-2)}q_1^2q_2^2}\left[\frac{q^2N^2}{(q_1-q)^2(q_2-q)^2}\ln\left(\frac{q^2}{(q_1-q_2)^2}\right)\right.$$
$$+\frac{2N^2}{(q_1+q_2-q)^2}\ln\left(\frac{q_1^2}{(q_1-q)^2}\right)+\left(\frac{-q^2}{(q_1-q)^2(q_2-q)^2}+\frac{2}{(q_1+q_2-q)^2}\right)$$
$$\times\left(N^2\left(2\psi(D-3)+\psi\left(3-\frac{D}{2}\right)-2\psi\left(\frac{D}{2}-2\right)-\psi(1)\right.\right.$$
$$\left.\left.\left.+\frac{1}{(D-3)}\left(\frac{1}{4(D-1)}-\frac{2}{D-4}-\frac{1}{4}\right)\right)-\frac{n_fN(D-2)}{2(D-1)(D-3)}\right)\right]. \tag{15}$$

As we have already pointed out, the trajectory must not depend on the particles which are being scattered.

The derivation presented above used the amplitude for quark–quark scattering. A calculation of a jump in the amplitude for gluon–gluon scattering[9] confirms the result found here.

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