Quantum Abrikosov–Nielsen–Olsen strings

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Quantum string theory is discussed in an Abelian Higgs model. In the limit of infinitely thin strings, this theory becomes the effective theory discussed by Polchinski and Strominger, in which there is no conformal anomaly. The complete string theory constructed here has no tachyons in the spectrum, so it actually exists in 4D space–time.

1. INTRODUCTION

One of the basic tasks in quantum field theory today is to search for an upper vacuum state. Along the phenomenological approach, it is customary to use some vacuum consisting of classical solutions of the instanton type.

In the present letter we are interested in the string vacuum. We consider an Abelian Higgs model in 4D Euclidean space. Guided by the measure and the quantum anomalies, we accordingly distinguish from the functional integral a part which corresponds to topological defects (the string theory will be derived for the case in which the world surfaces have the topology of a sphere). We are actually doing in the continuum limit what was done on a lattice for compact QED in Ref. 1 and for an Abelian Higgs model in Ref. 2. The partition function of the theory for compact fields is factorized into two partition functions: $Z_{\rm com}=Z_{\rm ncom}Z_{\rm top}$, where $Z_{\rm ncom}$ is the partition function for noncompact fields, and $Z_{\rm top}$ is that for topological defects. In the case of compact electrodynamics, topological defects are monopoles; in the case of an Abelian Higgs model, they are Abrikosov–Nielsen–Olsen strings. In a lattice regularization there are no divergences. In the continuum limit the Abelian Higgs model (Ginzburg–Landau theory) can be thought of as an effective theory for a type II superconductor near its critical point. Accordingly, we will not discuss the zero-charge problem, and we will in practice be dealing with theories which actually exist.

In the first papers on strings in an Abelian Higgs model in the continuum limit,[3,4] their quantum properties were studied in the London limit (the Higgs boson has an infinite mass). It was shown that in the strong-coupling limit (long, thin strings) they can be described by a Nambu–Goto action. A more accurate action for strings which follows from the Abelian Higgs models was derived in Refs. 5 and 6. In Ref. 5, a derivation was carried out at the classical level in the London limit, not in the limit of a large photon mass (thin strings). It turned out that a necessary condition for stability of classical string solutions is that the action contain terms which depend on the external-curvature tensor raised to a power higher than the second. In Ref. 6, a correction of the Nambu–Goto action in an expansion in the string thickness was derived at the level of tree diagrams, and a string Θ term was analyzed. We should also mention Ref. 7, which discusses a duality transformation for the Abelian Higgs model.

Those papers, however, did not take up quantum effects for the string theory, so they overlooked such important points as the integration measure and the conformal anomaly (which arises in the limit of infinitely thin strings, in which the theory becomes conformally invariant).

An effective quantum theory for strings in an Abelian Higgs model was proposed in Ref. 8. Polchinski and Strominger proposed a conformal theory which gives an adequate description of infinitely thin strings in an Abelian Higgs model: A term with an arbitrary coefficient was added to the Nambu–Goto action on the basis of general considerations. They then found the value of this coefficient which would be required in order to contract the conformal anomaly in 4D space–time. We show below that a theory of specifically this type arises in a natural way for infinitely thin strings if we consider the Jacobian in the replacement of the integration over field variables by an integration over string variables. In the next order of an expansion in the string thickness, i.e., in the first correction, we find (as in Refs. 5 and 6) a term, with the opposite sign, for the stiffness for a string. This term was first discussed in Refs. 9 and 10.

2. FROM AN ABELIAN HIGGS MODEL TO A QUANTUM STRING THEORY

In the functional integral for the 4D Abelian Higgs model,

$$Z=\int \mathscr{D}A_\mu \mathscr{D}\Phi \exp\left\{-\int d^4x[\beta F_{\mu\nu}^2+\tfrac{1}{2}|D_\mu\Phi|^2+\lambda(|\Phi|^2-\Phi_0^2]^2\right\},$$

$$D_\mu=\partial_\mu+iA_\mu, \tag{1}$$

the measure of the integration over the complex scalar field $\Phi=|\Phi|\exp(i\theta)$ is

$$\int \mathscr{D}\Phi \ldots=\int \mathscr{D}\,\mathrm{Re}\,\Phi\cdot\mathscr{D}\,\mathrm{Im}\,\Phi\ldots=\int [|\Phi|\mathscr{D}|\Phi|]\mathscr{D}\theta\ldots\,. \tag{2}$$

In the last integral, the integration is over all functions which are regular everywhere except on 2D surfaces, since θ is not defined where the conditions $\mathrm{Im}\Phi=\mathrm{Re}\Phi=0$ hold. Specifically, these two conditions in 4D lead to closed 2D singularities, which are world surfaces of the strings. Any closed singularity in $\theta=\tilde{\theta}+\theta^s$ can represented in the form[5,7]

$$\partial_{[\mu}\partial_{\nu]}\theta^s(x,\tilde{x})=2\pi\epsilon_{\mu\nu\alpha\beta}\int_\Sigma d\sigma_{\alpha\beta}\delta^{(4)}[x-\tilde{x}(\sigma)],$$

$$d\sigma_{\alpha\beta}=\epsilon^{ab}\partial_a\tilde{x}_\alpha\partial_b\tilde{x}_\beta d^2\sigma=\sqrt{g}t_{\alpha\beta}d^2\sigma. \tag{3}$$

Here $\theta^s(x,\tilde{x})$ is a function of x and a functional of $\tilde{x}$, where $\tilde{x}=\tilde{x}(\sigma)$ are the coordinates of the 2D singularities, which can be parametrized by the coordinates σ_a, $a=1,2$. In addition, Σ represents all possible positions of singularities (without any loss of generality, we restrict the discussion to a unit magnetic flux within the string):

$$\int_\Sigma \ldots=\sum_{\mathrm{sing}}\int_{\mathrm{sing}}\ldots, \tag{4}$$

where "sing" specifies the position of one singularity; $g=\det\|g_{ab}\|$, $g_{ab}=\partial_a\tilde{x}_\mu\partial_b\tilde{x}_\mu$ and $t_{\mu\nu}=(\epsilon_{ab}/\sqrt{g})\partial_a\tilde{x}_\mu\partial_b\tilde{x}_\nu$ are tensors of the induced metric and the external curvature on Σ have no internal metric in the theory); $t^2_{\mu\nu}=2$; and $\partial_a=\partial/\partial\sigma_a$.

For simplicity we consider the London limit ($\lambda\to\infty$; the theory for the radial part of the field Φ is trivial)[2]:

$$Z=\text{const}\int \mathscr{D}A_\mu\mathscr{D}\theta\exp\left\{-\int d^4x\left[\beta F^2_{\mu\nu}+\frac{\alpha}{2}\left|\partial_\mu\theta+A_\mu\right|^2\right]\right\}, \tag{5}$$

where $\alpha=\langle|\Phi|^2\rangle$. To proceed systematically would require regularizing this theory, by (for example) explicitly introducing Pauli–Willars regulators. As it turns out, however, the following regularization of the δ-functions is sufficient for our purposes:

$$\delta(x-y)=\lim_{M\to\infty}\frac{M}{(2\pi)^{\frac{1}{2}}}\exp\{-M^2|x-y|^2\}, \tag{6}$$

where M is a momentum cutoff. This regularization is to be understood for all the δ-functions, in particular, for that in (3).

We now go over to a second-quantization theory with an integration over string world surfaces. The standard procedure of changing integration variables includes the substitution of unity in the form [see (3)]

$$1=J[\theta^s(x,\tilde{x})]\int \mathscr{D}\tilde{x}_\mu\delta\left\{\partial_{[\mu}\partial_{\nu]}\theta^s(x,\tilde{x})-2\pi\epsilon_{\mu\nu\alpha\beta}\int_\Sigma d^2\sigma\sqrt{g}t_{\alpha\beta}\delta^{(4)}[x-\tilde{x}(\sigma)]\right\} \tag{7}$$

into a path integral. Here $J[\theta^s(x,\tilde{x})]$ is the Jacobian of the transformation from field variables to string variables. Convolving $\mathscr{D}\theta^s$ with the functional δ-function, we find

$$\int \mathscr{D}\theta\ldots=\int \mathscr{D}\tilde{\theta}\mathscr{D}\theta^s\ldots=\text{const}\int \mathscr{D}\tilde{\theta}\mathscr{D}\tilde{x}_\mu\tilde{J}(\tilde{x}_\mu)\ldots,\quad \tilde{J}(\tilde{x})=J[\theta^s(x,\tilde{x})], \tag{8}$$

where $\mathscr{D}\tilde{x}_\mu$ is to be understood as also meaning a summation over the topologies of the world surfaces of the strings.

We now separate collective degrees of freedom from the action in Eq. (5). Here the action is quadratic in A_μ. After the gauge is fixed, $\partial_\mu\tilde{\theta}=0$ ($\tilde{\theta}$ is the regular part of θ), an elementary integration over A_μ yields

$$Z=\int \mathscr{D}\tilde{x}_\mu\tilde{J}(\tilde{x})\exp\left\{-\frac{\alpha\pi^2}{4}\int_\Sigma\int_{\Sigma'}d^2\sigma d^2\sigma'\sqrt{g(\sigma)}t_{\mu\nu}(\sigma)\right.$$
$$\left.\times\mathscr{D}^{(4)}_m(\tilde{x}-\tilde{x}')\sqrt{g(\sigma')}t_{\mu\nu}(\sigma')\right\}Z_{\text{pert}}(\tilde{x}), \tag{9}$$

where $m^2=\alpha/\beta$ is the mass of the gauge boson, $(\Delta+m^2)\mathscr{D}^{(4)}_m(x)=\delta^{(4)}(x)$, and $Z_{\text{pert}}(\tilde{x})$ is a perturbation theory on the string vacuum under consideration. Fixing the reparametrization of the conformal gauge, we find (see the Appendix) $\tilde{J}(\tilde{x}_\mu)$ for the case in which Σ has the topology of a sphere:

$$\tilde{J}(\tilde{x})=\text{const}\exp\left\{\frac{22}{48\pi}\int_\Sigma d^2\sigma[\tfrac{1}{2}(\partial_a\log\sqrt{g})^2+\mu\sqrt{g}]+\frac{\ln M\bar{R}}{\pi}\int_\Sigma d^2\sigma\sqrt{g}(\partial_a t_{\mu\nu})^2\right\}. \tag{10}$$

The string tension coefficient μ, which arises because there is a gradient of the Higgs field, is derived in the Appendix; $\bar{R}$ is the average curvature of the string surfaces. In the limit in which m is larger than the eigenvalues of the second fundamental form of the external curvature, the action found here becomes the action of Ref. 5. However, the Jacobian $\tilde{J}$ was not considered in Ref. 5. The action can be written as an expansion in the quantity $1/m\bar{R}$ or, equivalently, in the string thickness. In first order of the expansion in the thickness, the action is local. If Σ has the topology of a sphere, we can use expression (10) for $\tilde{J}$; we then find

$$S=\mu'\int_\Sigma d^2\sigma\sqrt{g}-\beta'\int_\Sigma d^2\sigma\sqrt{g}(\partial_a t_{\mu\nu})^2-\frac{11}{48\pi}\int_\Sigma d^2\sigma(\partial_a\log\sqrt{g})^2. \tag{11}$$

Here we have $\mu'=4\pi\alpha\ln(M^2/m^2)-\mu$, and $\beta'=\beta-\ln M\bar{R}/\pi$ is the charge of the theory, renormalized for $\tilde{J}(\tilde{x})$. We should point out that this theory has no internal metric (g is an induced metric). This action for a string with a negative stiffness[9,10] is the same, to an accuracy within $\ln\tilde{J}$, as the action derived in Refs. 5 and 6. The term with the stiffness implicitly incorporates a nonzero (although small) thickness of the strings.

We now consider the zeroth order of an expansion in the thickness of the strings; this case corresponds to $\beta'=0$ (the strong-coupling limit). If we fix the induced metric in Eq. (11) in such a way that we have

$$\partial_a\tilde{x}_\mu\partial_b\tilde{x}_\mu=0,\quad a\neq b, \tag{12}$$

through a reparametrization, we find a conformal theory with the action

$$S=\mu'\int_\Sigma d^2\sigma(\partial_a\tilde{x}_\mu)^2=\frac{11}{12\pi}\int_\Sigma d^2\sigma\frac{(\partial_a\partial_b\tilde{x}_\mu\partial_b\tilde{x}_\mu)^2}{(\partial_c\tilde{x}_\nu)^4}. \tag{13}$$

In Ref. 8, the second term in Eq. (13) was added, with an arbitrary coefficient, to a Nambu–Goto action [the first term in (13)]. This term arises in a natural way as a correction to the Nambu–Goto action in an expansion in the reciprocal of the length of the strings. The resulting theory can be quantized in a Hamiltonian formalism.[8] A conformal anomaly arises at the quantum level. If this anomaly is to cancel out when we take account of the contribution of the reparametrized ghosts in gauge (12) in a 4D space–time, then the arbitrary coefficient mentioned above must[8] be the same as in (13).

We also note that the action in partition function (9) contains terms which depend on the external-curvature tensor raised to a power greater than 2. This situation is required[5] for stability of classical string solutions. Consequently, the string theory found [Eq. (9)] is a theory which actually exists in 4D space–time, and which describes Abrikosov–Nielsen–Olsen strings in a type-II superconductor.

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3. APPENDIX

In this appendix we present the calculation of the Jacobian $\tilde{J}$. The definition of this Jacobian, which follows from (7) and (8), is

$$[\tilde{J}(\tilde{x})]^{-1}=\int \mathscr{D}\tilde{y}_\mu \delta\{\partial_{[\mu}\partial_{\nu]}\theta^s(x,\tilde{y})-\partial_{[\mu}\partial_{\nu]}\theta^s(x,\tilde{x})\}, \tag{14}$$

where $\partial_{[\mu}\partial_{\nu]}\theta^s(x,\tilde{x})$ and $\partial_{[\mu}\partial_{\nu]}\theta^s(x,\tilde{y})$ are given by (3). We use the expansion[3)]

$$x_\mu=x_\mu(s)+n^k_\mu(s)\Omega^k, \tag{15}$$

for x_μ, where $x_\mu(s)$ lies on the surface Σ, n^k_μ are two vectors which are orthogonal with respect to Σ at the point s, and Ω^k are the coordinates along these vectors. We then find

$$\delta^{(4)}[x-\tilde{x}(\sigma)]=\frac{\delta^{(2)}(s-\sigma)}{\sqrt{g}}\delta^{(2)}[\Omega^k]. \tag{16}$$

We write the functional δ-function in (14) in the form

$$\delta\{...\}=\prod_x \delta_x\{...\}=\prod_{\Omega^k\neq 0}\delta_x\{0\}\prod_{\Omega^k=0}\delta_x\{...\}, \tag{17}$$

where we have made use of the circumstance that expression (16) [and thus the expression in the functional δ-function in (14)] is zero in the case $\Omega^k \neq 0$. Now regularizing the functional δ-function as in (6), we find

$$\delta\{...\}=\text{const}(M^2)^{\text{const}\cdot M^4\left[V-\frac{1}{M^2}S(\Sigma)\right]}\prod_{\Omega^k=0}\delta_x\{...\}$$

$$=\text{const}\exp\{-\mu S(\Sigma)\}\prod_{\Omega^k=0}\delta_x\{...\},\quad \mu=\text{const}\, M^2\ln M\bar{R}, \tag{18}$$

where V is the volume of the entire space. The region with $\Omega^k=0$ occupies part of the space, with a "volume"

$$S(\Sigma)=\int_\Sigma d^2\sigma\sqrt{h(\sigma)}\tau_{\mu\nu}(\sigma)\int_\Sigma d^2\sigma'\sqrt{h(\sigma')}\tau_{\mu\nu}(\sigma')$$

$$\times\prod_{i=1}^{2}\frac{M}{(2\pi)^{\frac{1}{2}}}\exp\{-M^2|\sigma_i-\sigma_i'|^2\}\lim_{M\to\infty}S(\Sigma)$$

$$=\int_\Sigma d^2\sigma\sqrt{h}. \tag{19}$$

Here $S(\Sigma)$ is a regularized area of all the string world surfaces from Σ in (4) (Ref. 11). The divergent coefficient of the Nambu–Goto action found here is the price paid for taking the London limit. Actually, we could replace M in (18) by a physical Higgs mass, as was pointed out in Ref. 5. Substituting expansion (16) for $\tilde{x}$ and $\tilde{y}$ into (3), and then substituting (3) into the functional δ-function in (14), we find the following expression from (18):

$$\delta\{...\} = \text{const}\,\delta[g-h]\,\delta[t_{\mu\nu}-\tau_{\mu\nu}]\exp\{-\mu S(\Sigma)\}. \tag{20}$$

We now substitute unity in the form

$$1 = \int \mathscr{D}h_{ab}\,\delta(h_{ab}-\partial_a\tilde{y}_\mu\partial_b\tilde{y}_\mu)\mathscr{D}\tau_{\mu\nu}\,\delta\left(\tau_{\mu\nu}-\frac{\epsilon^{ab}}{\sqrt{h}}\partial_a\tilde{y}_\mu\partial_b\tilde{y}_\nu\right) \tag{21}$$

into (14), and we make the replacement

$$\int \mathscr{D}h_{ab}\,\delta(h_{ab}-\partial_a\tilde{y}_\mu\partial_b\tilde{y}_\mu)\mathscr{D}\tau_{\mu\nu}\,\delta\left(\tau_{\mu\nu}-\frac{\epsilon_{ab}}{\sqrt{h}}\partial_a\tilde{y}_\mu\partial_b\tilde{y}_\nu\right)\exp\left\{-\mu\int_\Sigma d^2\sigma\sqrt{h}\right\}...$$

$$= \text{const}\int \mathscr{D}h_{ab}\mathscr{D}\tau_{\mu\nu}\exp\left\{-\frac{\mu}{2}\int_\Sigma d^2\tau\sqrt{h}\,\tau_{\mu\nu}^2-\mu\int_\Sigma d^2\tau\sqrt{h}h^{ab}\right.$$

$$\left.\times\,\partial_a\tilde{y}_\mu\partial_b\tilde{y}_\mu-\mu\int_\Sigma d^2\tau\,\tau_{\mu\nu}\epsilon_{ab}\partial_a\tilde{y}_\mu\partial_b\tilde{y}_\nu\right\}\dots . \tag{22}$$

Here the fields h_{ab} and $\tau_{\mu\nu}$ do not have kinetic terms, so they take on their classical values (see Chap. 9 in Ref. 12 for more details). In the case in which Σ has the topology of a sphere, we fix the conformal gauge[4)] and integrate over $\tilde{y}_\mu$. The first two terms in the exponential function in (22) and the reparametrized ghosts make a standard contribution to the form of the Liouville action with the central charge $(26-D)=22$ (Ref. 12). The third term leads to a term

$$\frac{\ln M\bar{R}}{\pi}\int_\Sigma d^2\tau\sqrt{h}(\partial_a\tau_{\mu\nu})^2$$

in the Jacobian. The integration over h_{ab} and $\tau_{\mu\nu}$ thus leads to expression (10).

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[2)] For arbitrary λ, a theory for the radial part $-|\Phi|$ can be factorized,[7] and all the calculations will be similar to those carried out in the limit $\lambda\to\infty$. In the final expression for the partition function, however, a path integral over $|\Phi|$ remains.

[3)] Whether this expansion is ambiguous was discussed in Refs. 4 and 5.

[4)] Although the calculation method used in this Appendix is the simplest one, it cannot be used to calculate the Jacobian if some other, nonconformal gauge is fixed. It would be more systematic to find the Jacobian by following the zero modes of the determinants for gauge fields (the procedure would be completely analogous to that used to find the instanton measure). In this case one could introduce an internal metric in the theory. With an internal metric, the theory for infinitely thin strings would be analogous to the theory discussed in Refs. 13 and 14. The only difference would be that the induced Liouville field would have a central charge of 23 in our case, not the 1 in Refs. 13 and 14.

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