Partial conservation of spin–isospin $SU(4)$ symmetry in nuclei in connection with the rate of $2\nu\beta\beta$ decay

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A new approach to calculating the amplitude for $2\nu\beta\beta$ decay of nuclei with well-developed nucleon pairing is proposed. This approach has been realized in its simplest version. It is based on the use of approximate conservation of spin–isospin $SU(4)_{\sigma\tau}$ symmetry in a description of charge-exchange excitations in the nuclei. The primary cause of the breaking of this symmetry is assumed to be the spin–orbit part of the mean field of the nucleus. In the approach proposed here, this spin–orbit part also determines the rate of $2\nu\beta\beta$ decay. Results calculated for decay half-lives with the help of the standard parameters of the mean field and of the interaction of quasiparticles are compared with existing experimental data. © *1995 American Institute of Physics.*

1. Numerous calculations of the nuclear Gamov–Teller amplitude for $2\nu\beta\beta$ decay, M, with nucleon pairing in the random phase approximation have demonstrated the following (see, for example, Refs. 1 and 2 and the papers cited there): 1) The values found for this amplitude through the use of the standard parameters of the mean field and of the quasiparticle interaction are considerably higher than the corresponding experimental values M^{exp}. 2) The calculated values of the amplitude M are sensitive to variations made in the intensity of the quasiparticle interaction in the particle–particle channel in attempts to reconcile the calculated values of the amplitude with experimental values. The basic conclusion which can be drawn from these calculations is that $2\nu\beta\beta$ decay is suppressed in comparison with the one-particle estimate, and further analysis looks unpromising unless the physical cause and the mechanism for the suppression can be clarified.

In this letter we wish to propose an alternative method for calculating the amplitude for $2\nu\beta\beta$ decay, M. This new method is fairly simple to realize. It is based on partial conservation of the $SU(4)_{\sigma\tau}$ symmetry in nuclei. In the approximation of exact $SU(4)_{\sigma\tau}$ symmetry, $2\nu\beta\beta$ decay is forbidden (we mean the decay to the ground or low-lying excited state of the product nucleus), so the rate of $2\nu\beta\beta$ decay is determined by the mechanism for, and the extent of, the breaking of this symmetry in nuclei. The method we are proposing here is analogous to the approach to the description of Fermi excitations based on the explicit use of an approximate conservation of isospin $SU(2)_\tau$ symmetry in intermediate-weight and heavy nuclei on the basis of a perturbation theory in the variable part of the mean Coulomb field of the nucleus, which breaks isospin symmetry (see, for example, Refs. 3 and 4 and the papers cited there). There are direct experimental indications of a partial conservation of $SU(4)_{\sigma\tau}$ symmetry: 1) The Gamov–Teller resonance and the isobaric analog resonance exhaust the greater part of the

Gamov–Teller and Fermi sum rules; respectively. 2) The energies of the Gamov–Teller resonance and the isobaric analog resonance (E_G and E_I, respectively) are fairly close together. The incorporation of these factors, along with the selection of the spin–orbit part of the mean field of the nucleus as the primary cause of the breaking of $SU(4)_{\sigma\tau}$ symmetry, is a sufficient basis for formulating a new approach to the description of Gamov–Teller excitations.

The possibility of using $SU(4)_{\sigma\tau}$ symmetry to analyze nuclear excitations has been discussed in the literature for a fairly long time. Recently published papers[5,6] are apparently the most recent reports on this topic. From these papers we can conclude (despite the different opinions of the authors) that the use of this symmetry to describe Gamov–Teller excitations can be productive. An attempt to use breaking of $SU(4)_{\sigma\tau}$ symmetry to calculate the amplitudes for $2\nu\beta\beta$ decay, M, was made in Ref. 7. The $SU(4)_{\sigma\tau}$-nonscalar part of the effective two-particle interaction was assumed to be the cause of the breaking of this symmetry (we know that this is not the main cause). That study has not been developed further. The approach which we are proposing has been realized previously in the case of $2\nu\beta\beta$ decay of the magic nucleus ^{48}Ca (Ref. 8). Because of the simple shell structure of this nucleus, the amplitude M can be expressed in terms of observable quantities. In the present letter, a new approach is formulated for nuclei with a pronouned pairing of nucleons, so that it becomes possible to analyze the bulk of the experimental data.

2. We denote by $|0\rangle$ the wave function of the ground state of the mother nucleus (N,Z). Our approach starts from the approximation that the isobaric analog and Gamov–Teller states of the isobar $(N-1,\ Z+1)$ are degenerate and are described by the wave functions

$$|I\rangle=(N-Z)^{-1/2}T^{(-)}|0\rangle, \quad \text{(1a)}$$

$$|G,\mu\rangle=(N-Z)^{-1/2}Y_{\mu}^{(-)}|0\rangle, \quad \text{(1b)} \qquad (10)$$

where $T^{(-)}=\Sigma_a\tau^{(-)}(a)$, $Y_{\mu}^{(-)}=\Sigma_a\sigma_{\mu}(a)\tau^{(-)}(a)$, and σ and τ are spherical Pauli matrices. Each of these states exhausts 100% of the corresponding sum rule. In the shell model, the relation between the isovector parts of the mean field of the nucleus and the particle–hole interaction is a consequence of $SU(2)_{\tau}$ symmetry [with which Eq. (1a) is associated] (see, for example, Ref. 9). If we use Landau–Migdal forces[10] $(F'+G'\sigma_1\sigma_2)\tau_1\tau_2\delta(\mathbf{r}_1-\mathbf{r}_2)$ as the interaction of the quasiparticles in the particle–hole channel, then we have $F'=G'$ in this approximation, and the mean field of the nucleus can be written in the form $U(r)=U_0(r)+U_{SY}(r)$. Here $U_0-SU(4)_{\sigma\tau}$ is the SU-scalar part of the mean field, $U_{SY}(r)=F'\rho(r)\tau^{(3)}$ is the symmetry potential, and $\rho(r)$ is the density of excess neutrons. The quantity U_0 and the parameters F' and G' are phenomenological quantities of the model.

Along this approach, a mixing of Gamov–Teller states with other 1^+ states of the proton/neutron-hole type, which we will refer to below as "anti-Gamov–Teller states," is possible by virtue of a difference between the phenomenological mean field of the nucleus and the potential $U(r)$ and also by virtue of the violation of the equation $F'=G'$. A smooth variation of the field $U_C(r)$ in the volume of the nucleus has the consequence that the mixing of nuclear states in terms of isospin can be ignored at an error of no more than a few percent.[4] Because of the smooth variation of the symmetry potential $U_{SY}(r)$ in the volume of the nucleus, the incorporation of the difference $F'-G'$ leads to only a shift of the energy of the Gamov–Teller states with respect to the energy of the isobaric analog states; it has essentially no effect on the wave function of the Gamov–Teller states.[5] Since the difference $(F'-G')/F'$ is relatively small[10] and is not known accurately, we ignore the contribution of this shift to the calculated energy of the Gamov–Teller states. The intensity of the mixing of Gamov–Teller and anti-Gamov–Teller states, α_μ, like the difference between the excitation energies of Gamov–Teller and isobaric analog states, $\Delta_{SO}=E_G-E_I$, is thus governed primarily by the spin–orbit part of the mean field of the nucleus, $U_{SO}(\mathbf{r})$.

In lowest order in the field $V_{SO}=\Sigma_a U_{SO}(\mathbf{r}_a)$, these quantities are, according to (1),

$$\Delta_{SO}=-\tfrac{4}{3}(N-Z)^{-1}\langle 0|V_{SO}|0\rangle, \tag{2}$$

$$E_I-E_0=(N-Z)^{-1}\int U_C(r)\rho(r)d\mathbf{r}\equiv\Delta_C;$$

$$\alpha_\mu=(N-Z)^{-1/2}\langle A,\mu|\delta V_\mu|0\rangle(E_G-E_I)^{-1}, \tag{3}$$

$$\delta V_\mu\equiv[V_{SO},Y_\mu^{(-)}]-\Delta_{SO}Y_\mu^{(-)},$$

where $|A,\mu\rangle$ and E_A are the wave function and energy, respectively, of the anti-Gamov–Teller state. By virtue of the orthogonality of the wave functions $|A,\mu\rangle$ and $|G,\mu\rangle$, the replacement of the mixing field $V_{SO}\rightarrow V_{SO}-\Delta_{SO}$ does not alter the mixing amplitude α_μ. A modification of this sort is convenient in practical calculations. It is analogous to the replacement $U_C\rightarrow U_C-\Delta_C$,which is used in describing processes associated with a breaking of $SU(2)_\tau$ symmetry.[3,4] The amplitude for the mixing of the ground state of the $(Z+2,N-2)$ isobar, $|0'\rangle$, with an excited state of the same nucleus which is a Gamov–Teller state "constructed" from an anti-Gamov–Teller state is determined in a corresponding way. Using (1), we find

$$\beta_\mu=(N-Z-2)^{-1/2}\langle 0'|\delta V_{-\mu}|A,\mu\rangle(E_G+E_A)^{-1}. \tag{4}$$

Here and below, the energies of the nuclear states are reckoned from the value $(E_0+E_{0'})/2$.

For $0^+\rightarrow 0^+$ transitions, the nuclear Gamov–Teller amplitude for $2\nu\beta\beta$ decay is (Ref. 11, for example)

$$M=\sum_{S,\mu}(-1)^\mu\langle 0'|Y_{-\mu}^{(-)}|S,\mu\rangle\langle S,\mu|Y_\mu^{(-)}|0\rangle E_S^{-1}, \tag{5}$$

where $|S,\mu\rangle$ and E_S are the wave functions and energies, respectively, of the 1^+ states of the $(Z+1,N-1)$ isobar. If we ignore the approximate conservation of $SU(4)_{\sigma T}$ symmetry, the amplitude in (5) can be expressed in the following way in terms of the one-quasiparticle matrix elements and coefficients of a Bogolyubov transformation for a nucleus with a well-expressed pairing, in the one-quasiparticle approximation:[12]

$$M_{sq}=e_q^2\sum_{\pi\lambda}\langle\pi|\lambda\rangle^2\langle\pi\|\sigma\|\lambda\rangle^2 u_\pi v_\pi u_\lambda v_\lambda E_{\pi\lambda}^{-1}. \tag{6}$$

Here $E_{\pi\lambda}$ is the energy of 1^+ two-quasiparticle proton/neutron-hole (π/λ) states, $\langle\pi|\lambda\rangle$ is the overlap integral of the corresponding radial wave functions, and $\langle\pi\|\sigma\|\lambda\rangle$ is a reduced matrix element. Equation (6) incorporates the "effective charge" for Gamov–Teller excitations associated with quenching (Ref. 9, for example). If the neutron subsystem in the initial (final) state is "magic +2 neutrons" (or "magic −2 neutrons"), then we need to replace the product $u_\lambda v_\lambda$ in (6) by c_λ, where c_λ are coefficients which determine the wave function of neutron pairing vibrations and which are normalized by the relation $\Sigma_\lambda(j_\lambda+1/2)c_\lambda^2=1$. For the proton subsystem, the situation is analogous.

Along our approach, in the zeroth approximation in the field δV_μ, which breaks the $SU(4)_{\sigma T}$ symmetry, amplitude (5) is zero according to (1). In lowest order in the mixing field we have, according to (3)–(5),

$$M=e_q^2\sum_{A,\mu}(-1)^\mu\frac{\langle 0'|\delta V_{-\mu}|A,\mu\rangle\langle A,\mu|\delta V_\mu|0\rangle}{E_G^2-i\Gamma E_G-E_A^2}E_A^{-1}. \tag{7}$$

This expression reflects the circumstance that a Gamov–Teller state is actually a resonance with a total width $\Gamma\simeq 4$ MeV. For a nucleus with well-defined pairing we find the following expression, in the one-quasiparticle approximation for amplitude (7):

$$M=e_q^2\sum_{\pi\lambda}\frac{((2l_\pi+1)(j_\pi-j_\lambda)\langle\pi|U_{SO}|\lambda\rangle-\Delta_{SO}\langle\pi|\lambda\rangle)^2}{E_G^2-i\Gamma E_G-E_{\pi\lambda}^2}\langle\pi\|\sigma\|\lambda\rangle^2u_\pi v_\pi u_\lambda v_\lambda E_{\pi\lambda}^{-1}. \tag{8}$$

The summation is dominated by transitions with a change in radial quantum number. The energy of the Gamov–Teller state in (8) is calculated in the one-quasiparticle approximation, in accordance with (2).

Comparison of (6) and (8) yields the following conclusions: 1) Partial conservation of $SU(4)_{\sigma T}$ symmetry leads to a substantial suppression of the rate of $2\nu\beta\beta$ decay. The value of the ratio $|M/M_{sq}|$ determines the extent of this suppression. In the problem at hand, this ratio can be regarded as a parameter of a perturbation theory in the mixing field. 2) The approach proposed here is essentially an approximate method for incorporating in the $SU(4)_{\sigma T}$-symmetry limit those polarization effects which stem from the interaction of quasiparticles in the particle–hole channel and which are responsible for the formation of a Gamov–Teller state which exhausts most of the Gamov–Teller sum rule. In the simple version of the approach under consideration here, we ignore the influence of polarization effects on only the renormalization of the mixing field. This renormalization leads to some redistribution of the Gamov-Teller force among the various anti-Gamov–Teller states, without any significant change in their overall strength. Since expression (8) contains a sum over all the anti-Gamov–Teller states, a redistribution of this sort does not substantially alter the calculated rate of $2\nu\beta\beta$ decay.

3. To calculate the amplitudes M in accordance with (8), we choose the $SU(4)_{\sigma T}$-scalar part of the mean field $U_0(r)$, like the spin–orbit part of the nuclear mean field $U_{SO}(\mathbf{r})$, in the Nemirovskiĭ–Chepurnov parametrization.[13] The symmetry potential $U_{SY}(r)$ is found from the condition that this potential be consistent with the density of excess neutrons. The mean Coulomb field $U_C(r)$ is calculated in the Hartree approximation from the one-particle density of distribution of protons in the nucleus. The intensity

TABLE I. Results of calculations of the decay half-lives of nuclei.

Mother nucleus	$T_{1/2}^{theo}$ $(2\nu\beta\beta)$ yr	$T_{1/2}^{exp.}$ $(2\nu\beta\beta)$ yr	$\lvert M/M_{sq}\rvert$
^{48}Ca	5×10^{20}	$>3.6\times10^{19}$ (Ref. 15)	
^{76}Ge	3×10^{21}	0.92×10^{21} (Ref. 16)	0.21
^{82}Se	3×10^{20}	1.1×10^{20} (Ref. 17)	0.20
^{96}Zr	1×10^{19}		0.31
^{100}Mo	2×10^{19}	1.15×10^{19} (Ref. 18)	0.29
^{116}Cd	1×10^{19}	2.2×10^{19} (Ref. 18)	0.25
^{128}Te	4×10^{24}	7.7×10^{24} (Ref. 19)	0.14
^{130}Te	7×10^{20}	2.7×10^{21} (Ref. 19)	0.12
^{136}Xe	1×10^{21}	$>2.3\times10^{20}$ (Ref. 20)	0.12
^{150}Nd	1×10^{19}	1.7×10^{19} (Ref. 21)	0.16

of the isovector part of the interaction of quasiparticles in the particle–hole channel is chosen to be $F'=0.95\times300$ MeV$\cdot$fm^3, in accordance with Ref. 10. To solve the problem of nucleon pairing in the BCS model, we used the relations and parametrization of the pairing interaction given in Ref. 14. We used a basis of bound and quasibound states up to an energy $\simeq5$ MeV.

The amplitude M in (8) determines the decay half-life in accordance with the relation of Ref. 11: $T_{1/2}=B|M|^{-2}$. The values of the quantity B, which are determined by the window for $2\nu\beta\beta$ decay and by the nuclear charge, were taken from Ref. 11. The value of e_q in (8) was taken to be[9] 0.8. Results calculated on the ratio $|M/M_{sq}|$ are shown in Table I, along with calculated and experimental values of the decay half-lives. For completeness, we also show in this table some results calculated for the nuclei ^{48}Ca by the method of Ref. 8. The results of calculations of the decay half-lives agree with the experimental data within a factor of 2–3. The values of the ratio $|M/M_{sq}|$ are small enough that a perturbation theory in the mixing field is justified.

The results can be refined by 1) taking account of the influence of polarization effects on the strength of the mixing field, 2) taking the pairing of nucleons into account more systematically, and 3) refining the coefficient B by incorporating relativistic effects.

Although the results found here do agree with the experimental data within a factor of 2–3, the approach proposed here seems preferable to other approaches, for the following reasons: 1) The nuclear amplitude for $2\nu\beta\beta$ decay is determined directly by the mechanism for, and the extent of, the breaking of $SU(4)_{\sigma\tau}$ symmetry in nuclei. Partial conservation of this symmetry in nuclei explains the suppression of the strength of 2 $\nu\beta\beta$ transitions. 2) There are no adjustable parameters in this approach. All the standard parameters of the model required for calculations are taken from independent data. 3) In a relatively simple approach, without adjustable parameters, it has been possible to find a satisfactory description of the half-life for $2\nu\beta\beta$ decay for nuclear ground states which are fairly different in structure.

In conclusion we would like to state the following. The method proposed here is based exclusively on the experimental facts that the Gamov–Teller state exhausts the greater part of the corresponding sum rule and that the energies of Gamov–Teller and isobaric analog resonances are approximately equal. From the practical standpoint, there is no need to make explicit use of the concept of the conservation of $SU(4)$ symmetry, which serves only for a theoretical interpretation of these experimental results.

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