Magnetoplasma excitations of nonuniform 2D electron systems in a strong magnetic field

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The spectrum of magnetoplasmons propagating along a contact between two 2D layers in a strong magnetic field is calculated. It is assumed that the electron density near the interface varies smoothly over a macroscopic distance. It is also assumed that a complex structure of alternating bands of compressible and incompressible fluids can form in quantizing magnetic fields at low temperatures. It is shown that this structure can be studied through spectroscopy of inter-edge magnetoplasmons. © *1995 American Institute of Physics.*

Plasma oscillations of a confined 2D electron gas in strong magnetic fields B have unique properties (see a review[1]). Under the condition $\omega_c\tau \gg 1$, edge magnetoplasmons are strongly confined near the edge of the 2D layer, they have a gapless dispersion $\omega(q_y)$, and they are damped weakly even at very low frequencies ($\omega\tau \ll 1$). Here ω_c is the cyclotron frequency, τ is the momentum relaxation time, and q_y is the wave vector of an edge magnetoplasmon, directed along the edge of the system. The model of a sharp edge of the 2D layer was used in the first theoretical papers on edge magnetoplasmons.[2–4] In the very simple case of a semi-infinite ($z=0$, $x>0$) 2D gas without screening electrodes, the spectrum of edge magnetoplasmons can be described by[3,4]

$$S=S_0(q_y)=\frac{\omega\bar{\kappa}}{2q_y\sigma_{yx}}=\ln\frac{2}{|q_yl|}+1,\quad |q_yl|\ll 1, \tag{1}$$

where the length $l=2\pi\chi_{xx}/\bar{\kappa}$ is determined by the polarizability $\chi_{xx}=i\sigma_{xx}/\omega$ of the 2D gas. In a system with a sharp profile, this length is the distance over which the charge of the edge magnetoplasmons is confined. Here $\sigma_{xx}(\omega)$ and $\sigma_{yx}(\omega)$ are components of the conductivity tensor of the 2D layer, and $\bar{\kappa}$ is the average dielectric constant of the surrounding medium.

In actual structures, the edge of the 2D layer is smeared over some distance h (typically on the order of 1 μm or more[5,6]). In strong fields B, this distance may be significantly greater than the length $|l(\omega)| \propto 1/B^2$ (in typical GaAs–AlGaAs heterostructures with a 2D gas, we would have $|l(\omega)|\approx 40$ nm at $B\approx 5$ T). When the finite width of the transition layer at the boundary of the 2D system is taken into account, the spectrum of the fundamental mode changes (the length $|l(\omega)|$ in (1) is replaced by the width h; Refs. 4 and 7). In addition, some new (multipole) excitations with an acoustic spectrum $S=S_n$, $n=1,2,\ldots$, arise.[3,7,8] Acoustic modes with $n\geq 1$ were recently observed experimentally.[9]

The problem of edge magnetoplasmons at a smeared edge of a 2D system is particularly important because of a problem recently being discussed: that of edge current states under conditions of the quantum Hall effect.It was shown in Ref. 6 that incorporating the screening of the edge potential in the degenerate 2D gas leads to a complex picture of edge states in quantizing magnetic fields at low temperatures. Near the edge, the electron gas breaks up into a system of alternating bands of compressible and incompressible fluids. Within the incompressible bands, the filling factor of the Landau levels is constant, while in the compressible regions the self-consistent electrostatic field is zero. The ratio of the widths of the incompressible and compressible bands is small to the extent that the parameter $(a_B/h)^{1/2}$ is small,[6] where a_B is the Bohr radius. Since edge magnetoplasmons are slightly damped, low-frequency excitations of the system, which are strongly confined near the edge, there is the question of whether a spectroscopy of edge magnetoplasmons might be utilized to study the structure of edge states under the conditions of the quantum Hall effect.

Aleiner and Glazman[7] have recently found an exact solution for the problem of the spectrum of edge magnetoplasmons in strong magnetic fields for a particular profile of the equilibrium charge density: $n_0(x)=(2\bar{n}/\pi)\arctan\sqrt{x/h}$, where $\bar{n}$ is the density of 2D electrons far from the boundary. They showed that the velocity of the fundamental mode, S_0, is given by an expression like (1), with $|l(\omega)|$ replaced by h. They also showed that the spectrum of multipole modes with $n\geqslant 1$ is of the form $S_n=1/n$ in the limit $|q_y|h\rightarrow 0$. Since the incompressible bands make up only a small fraction, $\sim (a_B/h)^{1/2}\ll 1$, of the boundary region, it was concluded that the effect of the complex structure of the edge on the spectrum of edge magnetoplasmons was unimportant.

In the present letter we solve the problem of a spectrum of inter-edge magnetoplasmons,[10] i.e., modes of the edge type which are propagating along a contact between two 2D layers with different densities (n_R at $x\gg h$; n_L at $-x\gg h$). The spectrum of the fundamental mode with $n=0$ was derived in the long-wave limit in Ref. 10 in the model of a sharp jump in the electron density ($h=0$). That spectrum differs from (1) in that σ_{yx} is replaced by $\delta\sigma_{yx}=\sigma_{yx}^R-\sigma_{yx}^L$, and l by $\bar{l}$ = $(l_R+l_L)/2$. The effect of the smeared edge on the spectrum of inter-edge magnetoplasmons in the 2D gas was not discussed. We show below that in this formulation of the problem (in which we use a spectroscopy of inter-edge modes, rather than edge modes) the question posed above can be answered in the affirmative.

We consider a 2D layer with an equilibrium density

$$n_0(x)=\bar{n}+(\delta n/2)\zeta(x), \tag{2}$$

which is in the $z=0$ plane. Here $\bar{n}=(n_R+n_L)/2$; $\delta_n=n_R-n_L$; and the function $\zeta(x)$ describes the transition layer, $\zeta(x)\rightarrow\pm 1$ as $x\rightarrow\pm\infty$. We assume that there are screening metal electrodes in the planes $z=d_1$ and $z=-d_2$, while the regions $0<z<d_1$ and $-d_2<z<0$ are filled with insulators with respective dielectric constants κ_1 and κ_2. Temporal fluctuations of the potential φ and the charge ρ of the inter-edge magnetoplasmons ($\varphi,\rho \propto \exp(-i\omega t)$) are described by the equations

$$\mathrm{div}[\kappa(z)\mathrm{grad}\varphi(\mathbf{r},z)]=-4\pi\rho(\mathbf{r})\delta(z), \tag{3}$$

$$\rho(\mathbf{r})=(i\bar{\sigma}_{xx}/\omega)\Delta_2\varphi+(\delta\sigma_{yx}/2i\omega)(\partial_x\zeta)(\partial_y\varphi)+(i\delta\sigma_{xx}/2\omega)\partial_\alpha[\zeta(x)\partial_\alpha\varphi], \tag{4}$$

where $\mathbf{r}=(x,y)$; Δ_2 is the 2D Laplacian; $\partial_a \equiv \partial/\partial x_\alpha$; α takes on the values $\{x,y\}$; and we are using the identities $\sigma_{xx} \equiv \sigma_{yy}$ and $\sigma_{xy} \equiv -\sigma_{yx}$. In strong fields B ($\omega_c \gg \omega$, $|\bar{l}| \ll h$), we can ignore the term proportional to $\delta\sigma_{xx}$ in (4) (in contrast with Ref. 7, however, we consider the term with $\bar{\sigma}_{xx}$). We then find the following integral equation for the potential $\varphi(x)$ from Eqs. (3) and (4):

$$\varphi(x)=\frac{1}{2S}\int_{-\infty}^{+\infty} dx' \varphi(x')[\partial\zeta(x')/\partial x']L(x-x'). \tag{5}$$

Here $S=\omega\bar{\kappa}/2q_y\delta\sigma_{yx}$ is the dimensionless phase velocity of the magnetoplasmon; $\bar{\kappa}=(\kappa_1+\kappa_2)/2$; and the kernel $L(x=x')$ is given by

$$L(x-x')=\frac{\bar{\kappa}}{2}\int_{-\infty}^{\infty} dq_x \frac{\exp[iq_x(x-x')]}{q\epsilon(q,\omega)}, \quad q=(q_x^2+q_y^2)^{1/2}, \tag{6}$$

$$\epsilon(q,\omega)=[\kappa_1\coth(qd_1)+\kappa_2\coth(qd_2)]/2+2\pi i\bar{\sigma}_{xx}q/\omega. \tag{7}$$

If we ignore the term proportional to $\bar{\sigma}_{xx}$ in (7) and set $d_1=d_2=\infty$, then we have $\epsilon(q,\omega)=\bar{\kappa}$, and the kernel $L(x-x')$ reduces to the modified Bessel function $K_0(|q_y(x-x')|)$. This function describes the potential of a periodically charged filament (the periodic variation is along the y axis) and has a logarithmic divergence as $(x-x')\to 0$. The function $\epsilon(q,\omega)$ describes the screening of the charge in the system by both the metal electrodes [the first two terms in (7)] and by the 2D gas itself (the third term). Incorporating the last term (proportional to $\bar{\sigma}_{xx}$) in the function $\epsilon(q,\omega)$ leads to a cutoff of the logarithmic divergence of the kernel $L(x-x')$ over a distance $|\bar{l}|=|2\pi i\bar{\sigma}_{xx}/\omega\bar{\kappa}|$.

To solve integral equation (5), we use the method of replacing the exact kernel $L(x-x')$ by the approximation $\tilde{L}(x-x')=A\exp(-q_0|x-x'|)$. A similar method (with a different choice of parameters in the approximate kernel) was used in some early papers on edge magnetoplasmons.[2,11] We choose $A(q_y)$ and $q_0(q_y)$ in such way that the integral $\int[L(x)-\tilde{L}(x)]^2dx$ reaches a minimum value under the condition $\int[L(x)-\tilde{L}(x)]dx=0$. From this condition we find equations for determining A and q_0:

$$q_0\int_0^{\infty}\frac{q_x^2dq_x}{q\epsilon(q,\omega)(q_x^2+q_0^2)^2}=\frac{\pi}{16|q_y|\epsilon(|q_y|,\omega)}, \quad A=\frac{\pi\bar{\kappa}}{2|q_y|\epsilon(|q_y|,\omega)}q_0. \tag{8}$$

Replacing the exact kernel $L(x-x')$ in Eq. (5) by the approximation $\tilde{L}(x-x')$, and applying the operator $\partial_x^2-q_0^2$ to the resulting equation, we find a differential equation of the Schrödinger type:

$$-\frac{\partial^2\varphi(x)}{\partial x^2}-\frac{Aq_0}{S}\frac{\partial\zeta(x)}{\partial x}\varphi(x)=-q_0^2\varphi(x). \tag{9}$$

The problem of the spectrum of inter-edge magnetoplasmons and, in a particular case, that of edge magnetoplasmons, propagating along an interface in a 2D layer with density (2) reduces to the problem of the energy levels of a quantum particle in a potential $U(x)\propto-(1/S)\partial\zeta/\partial x$, which is determined by the derivative of the profile function $\zeta(x)$ (Fig. 1). The role of the "wave function" in this Schrödinger problem is played by

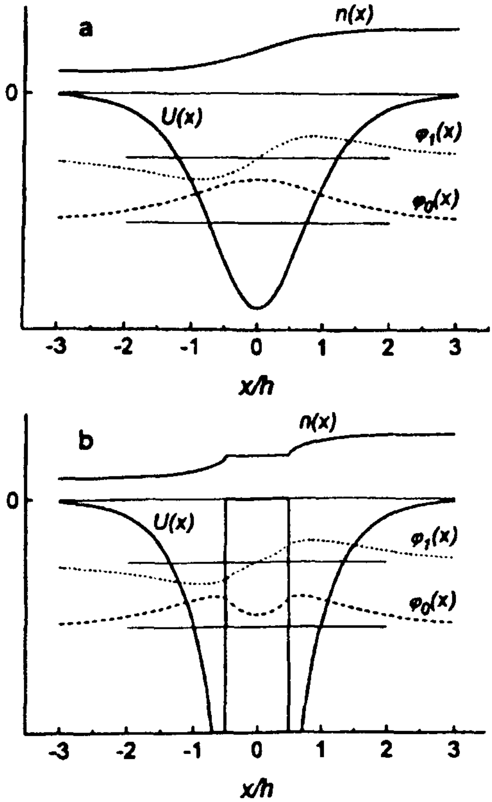

FIG. 1. Sketch of the equilibrium electron density $n(x)$, the effective potential energy $U(x)$ in "Schrödinger equation" (9), and the potential of inter-edge magnetoplasmons for modes with $n=0$ and $n=1$. a—For a classical compressible electron fluid; b—quantum liquid with an incompressible band at the interface.

the potential of the magnetoplasmon, $\varphi(x)$. The fundamental inter-edge-magnetoplasmon mode with the maximum velocity $S=S_0(q_y)$ and with a nonzero eigenfunction $\varphi_0(x)$ corresponds to the ground energy level of the particle in the well. It follows immediately from Eq. (9) that an edge excitation exists in the system for only one sign of the phase velocity ($S>0$ under the condition $\partial\zeta/\partial x>0$). In the opposite case, the function $U(x)$ describes a potential barrier. It also follows from (9) that the solution of the edge-mode problem is unique in the case of a sharp profile $[U(x) \propto -\ \delta(x)]$.

As an example we consider a particular case which can be realized in a natural way experimentally. We assume that the upper screening electrode in the structure discussed above consists of two sections ($z=d_1$, $x>0$ and $z=d_1$,$x<0$), and we assume that a nonuniform distribution of the electron density, $\zeta(x)$, is set up by applying a voltage between these sections. Solving the Laplace equation in the band $0<z<d_1$ under the boundary conditions $\varphi|_{z=0}=0$, $\varphi|_{z=d_1}=V_1$ sign(x), we find the following expression for the profile function:

$$\zeta(x)=\tanh(x/h),\quad h=2d_1/\pi. \tag{10}$$

Using the known analytic solution of the Schrödinger equation in the potential $U(x)=-U_0/\cosh^2(x/h)$ (Ref. 12, for example), we find the following expression for the spectrum of inter-edge magnetoplasmons:

$$S_n=\frac{\omega\bar{\kappa}}{2q_y\delta\sigma_{yx}}=\frac{Aq_0h}{(n+1+q_0h)(n+q_0h)}, \quad n=0,1,\dots . \tag{11}$$

The potential of the two lowest inter-edge-magnetoplasmon modes is (Fig. 1a)

$$\varphi_0(x)=1/\cosh(x/h), \quad \varphi_1(x)=\tanh(x/h)/\cosh(x/h).$$

Along with (8), Eq. (11) gives us the solution of the problem of inter-edge magnetoplasmons for arbitrary values of d_1 and d_2. In the case $\kappa_1\approx\kappa_2=\kappa, d_2=\infty$, which is a typical one for GaAs–AlGaAs heterostructures, and in the long-wave limit, $|q_y|d_1\ll 1$, $|q_y|\bar{l}\ll 1$, we find

$$q_0d_1=F(\bar{l}/d_1), \quad A=\pi F(\bar{l}/d_1), \tag{12}$$

where the function $F(z)$ is given by the equation

$$F\int_0^\infty \frac{xdx}{(x^2+F^2)^2[(\coth x+1)/2+zx}=\frac{\pi}{8}. \tag{13}$$

It has the asymptotic forms

$$F(z)\approx\begin{cases} 1.022-2.437z, & z\ll 1 \\ 1/\sqrt{2z}, & z\gg 1 \end{cases}. \tag{14}$$

Equations (10)–(12) lead to the following final expression for the spectrum of inter-edge magnetoplasmons:

$$S_n=\frac{\omega\bar{\kappa}}{2q_y\delta\sigma_{yx}}=\frac{2F^2(\bar{l}/d_1)}{[n+1+(2/\pi)F(\bar{l}/d_1)][n+(2/\pi)F(\bar{l}/d_1)]}, \quad n=0,1,\dots . \tag{15}$$

Under the condition $\omega\gg 2\pi\bar{\sigma}'_{xx}/\bar{\kappa}d_1$ ($|\bar{l}|/d_1\ll 1$), the damping of the inter-edge magnetoplasmons is slight ($\omega''\ll\omega'$), and the dimensionless phase velocity S_n is independent of the parameters of the problem.

Up to this point, we have treated the 2D system as a classical compressible fluid. Let us analyze at a qualitative level the spectrum of inter-edge magnetoplasmons in quantizing magnetic fields at low temperatures, such that bands of an incompressible fluid form in the system.[6] We assume that in the example which we are discussing here the average equilibrium electron density $\bar{n}$ is such that an integer number of Landau levels are filled in the system in the case $V_1=0$. As the potential difference between the two sectors of the upper gate, $2V_1$, is increased in the region $|x|<a/2$, a band of an incompressible fluid arises. In this case the potential energy $U(x)\propto-(1/S)\partial\zeta/\partial x$ in "Schrödinger equation" (9) describes two coupled quantum wells (Fig. 1b). A wave of oscillations in the width of the incompressible band corresponds to the fundamental mode of inter-edge magnetoplasmons, S_0; a wave of flexural vibrations of the band corresponds to the mode S_1, which is propagating at a lower velocity.

The coupling between the quantum wells in Fig. 1b depends on the parameter $q_0a \approx a/d_1$. As was shown in Ref. 6, we have $a/d_1 \sim a/h \ll 1$ in the case of edge excitations. The incompressible band therefore has a negligible effect on the spectrum of edge magnetoplasmons. In the case of inter-edge modes, the position and width of the bands of incompressible fluid can be varied over a wide range by varying the magnetic field and/or the voltage across the metal electrodes. Spectroscopy of inter-edge magnetoplasmons can thus be utilized to study the structure of inter-edge states under the conditions of the quantum Hall effect.

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