Finite-time heavy quark interaction in QCD

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Heavy $q\bar{q}$ interaction is studied at a finite time T. The nonperturbative background formalism is used to calculate the averages of Wilson loops to lowest order in g^2. The generalized formula of perturbative interaction $V_P(R,T)$ is obtained. It is shown that for the finite time, $T \lesssim R$, $V_P(R,T)$ essentially differs from Coulomb potential. Our analytical results are in good agreement with lattice calculations. © *1995 American Institute of Physics.*

1. Static $q\bar{q}$ interaction $V(R)$ yields important information on nonperturbative (NP) (confining) and perturbative forces between static quarks and their possible interference. The effects of α_s renormalization at small and large distances can also be conveniently formulated in a gauge invariant way in terms of $V(R)$ (Refs. 1 and 2). However, in physical applications the time T of the $q\bar{q}$ interaction is never infinite and one should rather consider finite-time interaction $V(R,T)$ with typical R and T for a given process, e.g., heavy quark $c\bar{c}$ creation inside a nucleus or a hadron. In particular, in lattice QCD one defines $V(R,T)$ in terms of the Wilson loop of size $R \times T$ and the time T on the lattice is kept finite.

For our analytical calculation we use the definition of $V(R,T)$ analogous to the lattice definition[3,4] one

$$V(R,T) = -\ln \frac{W(R,T)}{W(R,T-a)}, \tag{1}$$

where a is a small (lattice) unit of length and $W(R,T) \equiv \langle W(R,T) \rangle$ is the average of the Wilson loop over vacuum fields.

Our purpose here is twofold. First, we shall derive the analytical expression for $V(R,T)$, using NP background formalism,[2] and compare it with the lattice data.[4] Secondly, we carefully study the finite-time corrections to the static Coulomb potential $V(R)$.

2. In the formalism of NP background[2] one represents the total gluonic field A_μ as a sum of the NP background field B_μ and quantum fluctuations a_μ treated perturbatively:

$$A_\mu = B_\mu + a_\mu, \tag{2}$$

where a_μ transforms homogeneously under gauge transformations. To calculate the Wilson loop average, we keep the lowest-order contribution in a_μ

$$\langle W(B+a) \rangle = \langle W(B) \rangle - g^2 \langle W^{(2)}(a) \rangle + 0(g^4). \tag{3}$$

For $V(R,T)$ we have

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$$V(R,T)=-\ln\frac{\langle W(B;R,T)\rangle[1-g^2\langle W^{(2)}(R,T)\rangle/\langle W(B;R,T)\rangle]}{\langle W(B;R,T-a)\rangle[1-g^2\langle W^{(2)}(R,T-a)\rangle/\langle W(B;R,T-a)\rangle]}$$
$$\equiv V_{\rm NP}(R,T)+V_P(R,T), \tag{4}$$

where

$$V_{\rm NP}(R,T)=-\ln\frac{\langle W(B;R,T)\rangle}{\langle W(B;R,T-a)\rangle}, \tag{5}$$

$$V_P(R,T)=g^2\left\{\frac{\langle W^{(2)}(R,T)\rangle}{\langle W(B;R,T)\rangle}-\frac{\langle W^{(2)}(R,T-a)\rangle}{\langle W(B;R,T-a)\rangle}\right\}. \tag{6}$$

The "perturbative term" $\langle W^2(R,T)\rangle$ in fact depends on the NP background, i.e., it contains a gluon propagator in the background field, $G_{\mu\nu}(x,y;B)$, between the points x and y on the contour of the Wilson loop[2]

$$\langle W^{(2)}(R,T)\rangle_B=\left\langle\int\phi_1 G_{\mu\nu}(x,y;B)\phi_2 dx_\mu dy_\nu\right\rangle_B, \tag{7}$$

where ϕ_1 and ϕ_2 represent the left and right parts of the Wilson loop between the points x and y. As was shown in Ref. 5, in the limit of large N_c the gluon line can be replaced by a double fundamental line and then in (7) we obtain the average product of two Wilson loops which factorize in this limit. As a result,[2] for large T, $T>\sigma^{-1/2}$, perturbative and NP contributions in $\langle W^{(2)}\rangle$ also factorize. The first factor of the product is simply $\langle W(B;R,T)\rangle$ and the second—the perturbative term—reduces to the free one-gluon exchange at large T.

From (6) and (7), using the results of Ref. 6, we derive $V_P(R,T)$ (the Feynman background gauge is used for simplicity):

$$V_P(R,T)=+\frac{g^2C_2}{8\pi^2}\int_C\int_C\frac{dx_\mu dy_\mu}{(x-y)^2},\quad T>\sigma^{-1/2}. \tag{8}$$

Here C_2 is the quadratic Casimir operator.

In the range of small times, $T\ll\sigma^{-1/2}$, Eq. (8) is not valid and $V_P(R,T)$ can be found from the hybrid string interaction, as it was done in the lattice calculations in Ref. 7. Here we focus our attention only on the times $T\gtrsim\sigma^{-1/2}$.

3. The perturbative interaction (8) is the one-gluon exchange between heavy quarks inside the Wilson loop which was studied analytically in Ref. 6. It contains the linear divergence [which gives the constant additive term to $V(R,T)$] and also the logarithmic divergences due to the nonanalyticity of the Wilson contour. Both have to be regularized and for this purpose we introduce the minimal cutoff distance ε in the integral (8). Introducing the lattice units, $t=T/a$, $r=R/a$, $V\to aV$, and $e=(4/3)\alpha_s$, we can present expression (4) in the form

$$V(r,t)=V_{\rm NP}(r,t)+V_0+V_P(r,t), \tag{9}$$

where the perturbative part consists of three terms

$$V_P(r,t)=V_s+V_t+V_{\rm reg}. \tag{10}$$

TABLE I. The perturbative part of the interaction, $V_{\rm pert}(r,t)$, as a function of time $t=T/a$ for different fixed distances $r=R/a$ ($e=0.302$, $\delta=0.25$).

$r \backslash t$	1.5	2.0	3.0	4.0	5.0	7.0	10	15	$V_p(\text{asym}) \to -\frac{e}{r}(t\to\infty)$
2	0.255	-0.038	-0.128	-0.144	-0.148	-0.149	-0.1507	0.151	-0.151
4	1.083	0.296	0.018	-0.039	-0.058	-0.070	-0.0746	-0.0751	-0.0755
6	1.892	0.604	0.128	0.024	-0.012	-0.037	-0.046	-0.049	-0.0563
8	2.699	0.908	0.220	0.079	0.024	-0.015	-0.030	-0.035	-0.03775

From (8) for a rectangular contour the regularization term is

$$V_{\rm reg}=\frac{e}{2\pi}\cdot 4\ \ln\left(\frac{t-1-\delta}{t-\delta}\right), \qquad \delta=\frac{\varepsilon}{a} \tag{11}$$

and V_s and V_t in (10) refer to the gluon exchange between space-like links and time-like links, respectively. From (8) we find

$$V_t=-\frac{e}{\pi}\left\{\frac{2}{r}\arctan\frac{t-1}{r}+\frac{2t}{r}\left[\arctan\frac{t}{r}-\arctan\frac{t-1}{r}\right]+\ln\frac{r^2+(t-1)^2}{r^2+t^2}\right\}, \tag{12}$$

$$V_s=-\frac{e}{\pi}\left\{\frac{2r}{t}\arctan\frac{r}{t}-\frac{2r}{t-1}\arctan\frac{r}{t-1}+\ln\frac{[(t-1)^2+r^2]t^2}{(t^2+r^2)(t-1)^2}\right\}. \tag{13}$$

The asymptotic behavior of Eqs. (11)–(13) at large t can be easily found; for $t \gg r \gg 1$ we have

$$V_t \to \frac{e}{2\pi}\left\{-\frac{2\pi}{r}+\frac{4}{t}+\frac{2}{t^2}-\frac{4}{3}\frac{r^2}{t^3}+\dots\right\}, \tag{14}$$

$$V_s \to \frac{e}{2\pi}\frac{4r^2}{t^3}+0\left(\frac{1}{t^4}\right),$$

$$V_{\rm reg} \to -\frac{e}{2\pi}\left[\frac{4}{t}+\frac{2}{t^2}+0\left(\frac{1}{t^3}\right)\right].$$

It is interesting that for $t \gg r \gg 1$ the first two corrections (on the order of 1/t and $1/t^2$) cancel each other inside the perlturbative interaction V_p, so from (10) it follows that

$$V_p \to -\frac{e}{r}+0\left(\frac{r^2}{t^3}\right), \qquad t \gg r \gg 1. \tag{15}$$

This cancellation explains why the asymptotic value $V_p(\text{asymp}) \equiv -e/r$ is approached rather fast: so for $t=r+2$ the difference between exact value $V_p(r,t)$ and $(-e/r)$ is less than 20% but for $t=r+4$ this difference is already less than 5% (see Table I) for any r ($r=2$, 4, 6, 8 were considered).

Nevertheless, the most striking consequence of our calculations is that for values $t \leq r$ (any r) the exact value of $V_p(10)$ differs by several factors from $V_p(\text{asymp})$ and even has another (positive) sign for $t<r$ (see Table I).

As to the NP interaction, it was studied previously,[1] via the cluster expansion and for large r and t it was obtained there:

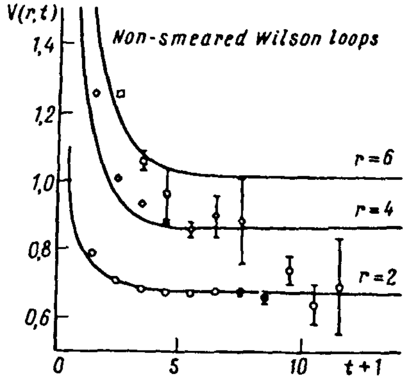

FIG. 1. Comparison of the finite-time interaction $V(r,t)$, given by the analytical expression (9), with the lattice data from Ref. 4. Solid lines represent analytical calculations for different fixed r ($e=0.302$; $\sigma a^2=0.0589$; $aV_0=0.70$; $\delta=0.25$); lattice data are represented by circles for $r=2$, by diamonds for $r=4$, and by squares for $r=6$.

$$V_{\rm NP}(r,t)\rightarrow(\sigma a^2)r+C_0, \quad r>t_g. \tag{16}$$

Here t_g is the characteristic gluonic correlation length which can be found from the quadratic field correlator. The lattice calculations of this correlator in Ref. 8 have shown that t_g is smaller than $\sigma^{-1/2}$. Therefore, in (9) we can use the asymptotic behavior (16) for $V_{\rm NP}(r,t)$.

4. For comparison with the lattice calculations[4] we consider $V(r,t)$ which is defined by Eq. (9) at a fixed r and varying t in the interval $1.5\leqslant t\leqslant 15$. The results are presented in Fig. 1 by the solid curve with the marked values of $r=2, 4, 6...$. The lattice data from Ref. 4 are represented by circles for $r=2$, by diamonds ($r=4$), and by squares ($r=6$). From Fig. 1 we see good agreement between our analytical calculations and the available lattice calculations.

From Eqs. (15) and (16) we obtain asymptotically the expression

$$V(r,t)\rightarrow-\frac{e}{r}+(\sigma a^2)r+V_0 \quad (t\gg r\gg 1). \tag{17}$$

As follows from our calculations, this behavior is already achieved at $t\gtrsim r+4$. Here we would like to stress several points:

1) For rather large r, e.g., $r=6$ and $r=8$, the perturbative term is much smaller than the NP term, so we cannot see the essential difference between $V_{\rm pert}$ (exact) and $(-e/r)$ in the total interaction, V, even for small times ($t<r$) when they have the opposite sign.

Hence, to study pure perturbative effects at finite T on the lattice and, in particular, the problem of freezing of $\alpha_s(R)$ at large distances,[2,9] it is necessary to separate perturbative NP terms with good accuracy.

2) For large r ($r=6$ or 8) the perturbative asymptotic behavior is achieved only for the values of $t\gtrsim r+4$ ($t\gtrsim 10$); meanwhile, on the lattice[4] the fit of parameters (V_0,e) is usually done for smaller times $t=4$ or 5 (where the use of the asymptotic interaction is not valid).

3) In (17) the constant term V_0 is universal for any large t and r ($V_0=0.70$). So for large r (r is fixed), when the Coulomb term in (17) can be ignored, $V(r,t)$ is approaching

the plateau value: V (r is fixed, any t)$\cong[(\sigma a^2)r+V_0]$. This simple asymptotic behavior agrees well with the plateau values in the lattice calculations[4] both for smeared and nonsmeared Wilson loops.

4) Our conclusions about the complex structure of the perturbative interaction at finite T are in agreement with the results of a recent analysis,[10] where the lattice Coulomb contributions $V_s(R,T)$ and $V_t(R,T)$ [analogous to (12) and (13)] were measured and the important role of the "unphysical" term $V_s(r,t)$ was stressed. In addition to this term, there is another "unphysical" term $V_{\rm reg}$ (11), absent in Ref. 10, which is extremely important in achieving the asymptotic regime (15).

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