# Nonlocal nature of vortex threads in layered superconductors

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In a medium with a 2D superconductivity, the current and the magnetic-field components tangent to the superconducting layers fall off by power laws, $j \propto r^{-3}\tan^2\alpha$ and $B \propto r^{-2}\tan\alpha$, far from a vortex thread. Here $r$ is the distance from the core, and $\alpha$ is the angle between the thread and the perpendicular to the layers. In other words, the 2D magnetic field is a dipole field. The component of the magnetic field normal to the layers remains well-localized. © *1995 American Institute of Physics.*

Vortex threads are fundamental entities in the electrodynamics of type-II superconductors. Their microscopic properties strongly influence macroscopic characteristics of the superconductor.[1,2] In the present letter we wish to call attention to the circumstance that in a layered superconductor structure, in which the conductivity perpendicular to the layers is zero or normal the anisotropy of the medium, causes a qualitative change in the structure of the threads: The slightest deviation of the thread axis from the perpendicular to the superconducting layers causes the current density and the magnetic-field components parallel to the layers to fall off by a power law, rather than an exponential law, far from the vortex core. Furthermore, the amplitude of the current density and the parallel field components remain nonzero as the standard screening parameter, the London depth $\lambda_L = c/\omega_{pe}$, goes to zero. Essentially all the high-$T_c$ ceramics are layered,[3–5] and we do not rule out the possibility that they embody certain properties of the idealized model described here.

The treatment in the present letter is purely classical. We recall that, from the standpoint of classical physics, a vortex thread is a $\delta$-function singularity of the curl of the generalized electron momentum $\mathbf{P} = m\mathbf{v} - eA/c$ frozen in the superconducting electron current.[6] This $\delta$-function is concentrated on a line. Away from this singularity, we have curl $\mathbf{P} = 0$, and the circulation around any contour circling the thread satisfies

$$\oint \mathbf{P} d\mathbf{r} = \Phi_0 = \text{const.} \tag{1}$$

In this approach, quantum-mechanical effects[1,2] merely impart a finite size to the vortex core, where the curl is nonzero. This size is the correlation length $\xi$ (for definiteness, we assume $\lambda_L \gg \xi$, although the effect of interest in the present letter is totally independent of the relation between $\lambda_L$ and $\xi$). Quantum-mechanical effects also cause its amplitude—the circulation in (1)—to become discrete:

  

$$\Phi_0=\frac{Nh}{2},$$

where $h$ is the Planck's constant, and $N$ is an integer.

To demonstrate the effect, we consider the example of an inclined straight thread. We model the layered superconducting structure by a homogeneous, continuous, anisotropic medium in which a superconducting flow of electrons is possible only in planes parallel to the $xy$ plane. In other words we have $\mathbf{j}=(j_x, j_y, 0)$. (The assumption that the medium is continuous is legitimate if the distance between layers satisfies $a \ll \xi$ and/or we are interested in length scales considerably larger than $a$. However, the nonlocal-nature effect is again independent of this assumption, generally speaking.) This particular assumption (and only this one), which is actually equivalent to the assumption that the electrons have an infinite mass in the perpendicular motion (along the $z$ axis), is crucial to the effect. We will discuss the influence of a slight exchange between layers ($m_\perp \neq \infty$) below.

We assume that the core of the thread runs along the straight line $x=0$, $y-z\tan\alpha=0$. It then follows from symmetry considerations that all physical quantities in the problem are functions of only two variables, of the type $f(x,y-z\tan\alpha)$. We will write all the equations below in the $z=0$ plane, and we will assume that the operator $\nabla$ is a 2D operator for the functions $f(x,y)$. In this formulation of the problem, only the $z$ component of the curl of the generalized momentum is a frozen-in quantity:

$$\nabla\times\frac{m\mathbf{j}}{ne}-\frac{\mathbf{e}_z B_z e}{c}=\Phi\mathbf{e}_z=\Phi_0\mathbf{e}_z\delta(x,y). \tag{2}$$

We wish to repeat that the result [Eqs. (7) and (8) below] does not change if we smear the $\delta$-function over some nonzero width. Furthermore, if this width is greater than $\lambda_L$, we can immediately discard the inertial term from (2), and the intermediate calculations in fact simplify. Introducing the stream function $\Psi$ for the divergence-free vector $\mathbf{j}$ in accordance with $\mathbf{j}=(c/4\pi)\nabla\times(\Psi\mathbf{e}_z)$ (with $\alpha=0$ we have $\Psi\equiv B_z$), we can rewrite Eq. (2) as

$$\lambda_L^2\nabla^2\Psi-B_z=\frac{c\Phi}{e}. \tag{3}$$

To reach our goal we need to invert (3), i.e., to express $B_z$ and $\Psi$ in terms of $\Phi$. This can be done without difficulty by taking 2D Fourier transforms ( $\mathbf{k}=(k_x,k_y)$) and supplementing (3) with Maxwell's equations:

$$-\frac{4\pi}{c}\mathbf{j}=\nabla^2\mathbf{A}+\tan^2\alpha\,\frac{\partial^2\mathbf{A}}{\partial y^2},\quad B_z\mathbf{e}_z=\nabla\times\mathbf{A}. \tag{4}$$

(Incidentally, it follows from these equations that in the case of any shape of the thread the quantity $\lambda_L^2\Delta B_z-B_z$ with a 3D Laplacian is a frozen-in quantity; cf. Ref. 2.) From (3) and (4) we then find

$$B_{z\mathbf{k}}=-\frac{c\Phi_0/e}{1+\lambda_L^2(k^2+k_y^2\tan^2\alpha)}. \tag{5}$$

In other words, the $z$ component of the magnetic field is, as before, localized (cf. Refs. 1, 2, 7, and 8):

$$B_z = -\frac{c\Phi_0 \cos\alpha}{2\pi e \lambda_L^2} K_0\left(\frac{\sqrt{x^2+y^2\cos^2\alpha}}{\lambda_L}\right).$$

Here $K_0$ is the modified Bessel function. In the expression for $\Psi$,

$$\Psi_{\mathbf{k}} = B_{z\mathbf{k}}\left(1+\tan^2\alpha\,\frac{k_y^2}{k^2}\right) \tag{6}$$

a completely new term appears along with the local term. Far from the core (at distances much greater than $\lambda_L$) we find the following expression from (6):

$$\Psi \simeq -\frac{c\Phi_0 \tan^2\alpha}{2\pi e}\,\frac{x^2-y^2}{(x^2+y^2)^2}. \tag{7}$$

Equation (7) proves the assertion above. Taking an equally simple form at these distances is the expression for the magnetic-field component parallel to the layers:

$$\mathbf{B} \simeq \frac{1}{\tan\alpha}\nabla\int \Psi\,dy. \tag{8}$$

(In this region, $B_z$ should be set equal to zero, at a power-law accuracy.)

That a nonlocal thread structure is unavoidable can be demonstrated quite simply by assuming the opposite. Specifically, we assume that an inclined filament is local. At a large distance it then looks like a straight solenoid (not at all circular, of course) with an oblique winding. The magnetic field of such a solenoid (which is equivalent to a solenoid with a straight winding plus two conductors carrying opposite currents) is not localized. In other words, we have a contradiction. This effect is therefore a simple consequence of Maxwell's equations and the absence of superconductivity along one direction. In other words, this is essentially a geometric effect, independent of all physical parameters of the superconductor, such as $\lambda_L$, $\xi$, and $a$ [see Eqs. (7) and (8)]. It follows immediately that a magnetic-field decay in proportion to $r^{-2}$ is a simple combination of the obvious proportionality of the magnetic field to the thread amplitude $c\Phi_0/e$ (to the magnetic flux of the $z$ component of the field) and symmetry considerations. The calculations carried out above were necessary only to determine the magnitude of the magnetic dipole moment in (8).

Another simple way to see the nonlocal-nature effect is to note that 2D currents give rise to a zero average field in the $xy$ plane. Since a localized inclined filament has a nonzero average field in this plane, it must have a "halo," specifically, a dipole halo according to symmetry considerations. We also see that if $m_\perp$ is nonzero ($j_z \neq 0$), and all three components of the curl of the generalized momentum are frozen in $\mathbf{j}$, the inclination angle of the "winding" of the solenoid may change, and at distances much greater than $\lambda_L$ an exponential screening may be restored, as is well known.[2,7,8] However, if the exchange of current between layers is weak, then we have $m_\perp \gg m$, and there is a large "window" of length scales in which a power-law decay occurs. We can point out a physical effect in any version; its realization in each specific physical entity requires a separate study.

This nonlocal-nature effect may be manifested in different ways. Here we will mention only the possibility of a restructuring of vortex arrays,[2,4,5] since even at small values of the inclination angle $\alpha$ vortex threads at distances large in comparison with $\lambda_L$ begin to interact fairly strongly with each other. To support this assertion we write the Lamb expression for the energy integral of the system in terms of the frozen-in quantity $\Phi$ (Refs. 9–11). This expression takes a particularly graphic form for parallel vortex formations inclined at a common angle $\alpha$ with respect to the superconducting layers. The energy per unit length along the $z$ direction is then

$$\mathscr{E}=\int\left(\frac{\mathbf{B}^2}{8\pi}+\frac{nm\mathbf{v}^2}{2}\right)d^2\mathbf{r}=-\int\Psi\Phi\,\frac{c}{8\pi e}\,d^2\mathbf{r}$$

$$=\iint\Phi(\mathbf{r}_1)V(\mathbf{r}_1-\mathbf{r}_2)\Phi(\mathbf{r}_2)d^2\mathbf{r}_1d^2\mathbf{r}_2, \qquad (9)$$

where the "interaction potential" $V(\mathbf{r})$ is a function proportional to $\Psi$ from (6), as is easily seen.

The interaction energy of distant inclined filaments in layered superconducting structures is thus a small quantity in a power-law sense rather than an exponential sense. A similar effect has been seen for vortices in a single thin film, in which the effect is more transparent, since the magnetic field outside the film is obviously nonlocal.[12–15]

Note, however, that for a single, smoothly curving thread which penetrates each superconducting plane only once the self-effect remains local, as in isotropic superconductors.[16]

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