Quantum tunneling in a magnetic vortex in a 2D easy-plane magnetic material

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A quantum tunneling of spins which form an out-of-plane vortex structure is predicted for a 2D easy-plane antiferromagnet.

1. Macroscopic quantum tunneling in magnetic materials has recently been studied on a wide scale, both experimentally and theoretically (see a review[1]). These tunneling effects stem from tunneling transitions between different states of a magnetic material (the states are usually equivalent in terms of energy) which are related in such a way that one state can be obtained from another simply by flipping a large number of spins. The most direct way to observe these effects is to detect resonance transitions between tunneling-split levels corresponding to these "classical" states. A resonance of this type has been observed in ultrasmall antiferromagnetic particles in proteins containing iron.[2] The primary difficulty in experiments of this sort is the need to prepare an ensemble of ultrasmall particles which are sufficiently close in size and shape. An effect of macroscopic quantum tunneling in the domain wall of an antiferromagnet, predicted in Ref. 3, eases the problem only partially, since one parameter (the area of the wall) still has to be monitored in such an experiment.

In this letter we would like to suggest a new macroscopic quantum-tunneling effect: a tunneling-induced change in the out-of-plane structure of a 2D topological soliton: a magnetic vortex in an easy-plane antiferromagnet. We describe the antiferromagnet by means of a normalized antiferromagnetism vector $\mathbf{l}$ ($\mathbf{l}^2=1$; Ref. 4). In terms of the angular variables $l_z=\cos\theta$, $l_x+il_y=\sin\theta\exp(i\varphi)$, the xy plane coincides with the easy plane, and a vortex corresponds to a solution of the type

$$\theta=\theta_0^{(\pm)}(r),\quad \varphi=q\chi+\varphi_0,\quad \theta_0^{(\pm)}(\infty)=\pi/2,\quad \theta^{(+)}(0)=0,\quad \theta^{(-)}(0)=\pi, \qquad (1)$$

Here $q=\pm 1$, ± 2 is the topological charge or vorticity; and $x+iy=r\exp(i\chi)$. The functions $\theta_0^{(+)}(r)$ and $\theta_0^{(-)}(r)$ describe two vortices with the same value of q which differ in the magnitude of a second topological charge: the polarization $p=\cos\theta(0)=\pm 1$ (Ref. 5, for example). The function $\theta_0^{(\pm)}(r)$ differs from $\pi/2$ in a vortex-core region with a diameter Δ_0. For slightly anisotropic antiferromagnets we have $\Delta_0 \gg a$, where a is the atomic spacing. In this region there is a large number [on the order of $(\Delta_0/a)^2$] of spins, whose directions change upon a quantum transition between states of the vortex with $p=+1$ and $p=-1$.

2. A transition of this sort is forbidden for easy-plane ferromagnets by conservation of the z projection of the resultant magnetization M_z. In contrast with the case of small particles, this prohibition is absolute, since incorporating sufficiently strong interactions,

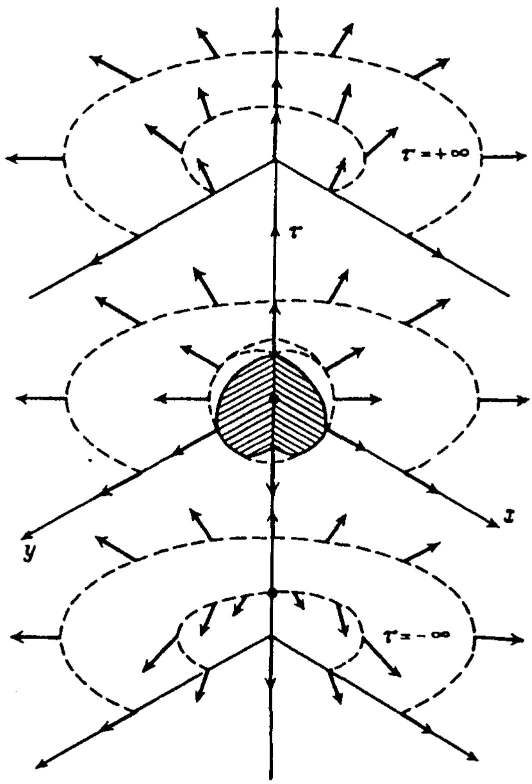

FIG. 1. Distribution of the vector **l** in an instanton solution in the limits $\tau \to \pm\infty$ (vortex with $p=+1$ and $p=-1$; upper and lower parts of the figure, respectively) and also for finite τ. The sphere near the origin of coordinates corresponds to a hedgehog solution.

which would disrupt the M_z conservation, would simultaneously disrupt the purely easy-plane symmetry. In such a case there would be no vortex solutions. For an antiferromagnet, in contrast, estimates are fairly promising. In contrast with the problems discussed previously,[1-3] the variable corresponding to the tunneling is not "soft," and polarization reversal of a vortex cannot be described in a quantum-mechanical problem for one variable. In this case, there is no alternative to an instanton approach. The effect is described by an instanton solution $\mathbf{l}=\mathbf{l}(\tau,x,y)$ found after transformation to an imaginary time $\tau=it$ in the equations of motion for the vector **l**. Since the dynamics of **l** is governed by the Lorentz-invariant Lagrangian of the σ model,[4] there is no problem in switching from the Lagrangian $L\{\mathbf{l}\}$ to the Euclidean action $S_E\{\mathbf{l}\}$. An instanton solution $\mathbf{l}=\mathbf{l}_0(\tau,x,y)$ describes the macroscopic quantum-tunneling effect of interest in a vortex if the functions $\mathbf{l}(\pm\infty,x,y)$ describe equilibrium states of a vortex with different values of p in the limits $\tau \to +\infty$ and $\tau \to -\infty$. The tunneling probability or the tunneling splitting of the levels, Γ, is given by the usual formula $\Gamma=\omega_0 \exp\{-S_E/\hbar\}$, $S_E=S_E\{\mathbf{l}_0\}$; the quantity ω_0 is on the order of the frequency of linear out-of-plane magnons (more on this below). Figure 1 shows a distribution of the vector **l** in the instanton solution. In the 3D Euclidean space τ,x,y, it describes a topological configuration of the hedgehog type (homotopy group π_2) and has a singularity at $\tau=0$, $x=y=0$. The value $\tau=0$ corresponds to a so-called in-plane vortex, with $l_z \equiv 0$. Such a vortex is realized in highly anisotropic magnetic materials, but it is unstable in our case of a slightly anisotropic antiferromagnet.[6]

3. After the transformation $t \to i\tau$ in the Lagrangian of the σ model, the Euclidean action S_E takes the form

$$S_E\{\mathbf{l}\}=A\int_{-\infty}^{+\infty}d\tau\int dxdy\{(1/c^2)(\partial\mathbf{l}/\partial\tau)^2+[(\nabla_2\mathbf{l})^2-(\nabla_2\mathbf{l}^{(0)})^2]+(1/\Delta_0^2)[l_z^2-(l_z^{(0)})^2]\}, \tag{2}$$

where A and K are exchange and anisotropy constants; c is the phase velocity of the magnons; $\Delta_0=\sqrt{A/K}$ is a magnetic length, which determines either the radius of the vortex core or the thickness of the domain wall; ∇_2 is a gradient in terms of the variables x,y; and $\mathbf{l}^{(0)}$ describes an equilibrium vortex. Away from the singularity (for $[(\tau c)^2+x^2+y^2]\gg a^2$), the condition $\mathbf{l}^2=1$ holds, and the equations for θ,φ become

$$\nabla_E^2\theta+\sin\theta\cos\theta[\Delta_0^2-(\nabla_E\varphi)^2]=0,\quad \nabla_E(\sin^2\theta\nabla_E\varphi)=0, \tag{3}$$

where ∇_E is the Euclidean gradient in $(c\tau,x,y)$ space. In the region $a\ll\rho\ll\Delta_0$, $\rho=(c^2\tau^2+x^2+y^2)^{1/2}$, system (3) has an exact, centrally symmetric solution of the hedgehog type: $\cos\theta=c\tau/\rho$, $\tan\varphi=y/x$. As $\rho\to 0$, for a vortex (as for a Bloch point in a ferromagnet) we need to consider the change in $\mathbf{l}$ over distance. In this case we have $\mathbf{l}=l(\rho)\mathbf{n}$ and $l(0)=0$ and $\mathbf{n}$ corresponds to a hedgehog.[7] Taking this circumstance into account, we see that the contribution of the singularity region itself to S_E is small (as for a Bloch point). The contribution of the region of small distances $\rho<R\ll\Delta_0$ to the Euclidean action is given by

$$S_E[\rho<R]=4\pi(A/c)R. \tag{4}$$

For large distances, $\rho\gtrsim\Delta_0$, it is not possible to construct an exact solution, which would have to be cylindrically symmetric: $\theta=\theta(u,r)$, $\tan\varphi=y/x$, $u=c\tau$. We accordingly use a variational procedure, adopting a trial function

$$\theta(\tau,x,y)=\pi/2+F(\tau)[\pi/2-\theta_0^{(+)}(r)],\quad r=(x^2+y^2)^{1/2},$$

where $\theta_0^{(+)}(r)$ describes an equilibrium vortex with $p=1$, and we have $F(\tau)\to+1$ and $F(\tau)\to-1$ as $\tau\to+\infty$ and $\tau\to-\infty$, respectively. A simple estimate shows that we can describe the contribution of the region $\rho>R$ to S_E by

$$S_E[\rho>R]=(2\pi A/c)[\xi_1\Delta_1\ln(\Delta_0/R)+\xi_2\Delta_1+\xi_3\Delta_0^2/\Delta_1], \tag{5}$$

where Δ_1 is the region in which the function $F(u)$ is localized; $u=c\tau$; and ξ_1, ξ_2, and ξ_3 are numerical coefficients on the order of one. Minimizing $S_E=S_E[\rho<R]+S_E[\rho>R]$ with respect to Δ_1 and R, we find $\Delta_1\sim R\sim\Delta_0$. We estimate the value of S_E for the instanton solution to be $2\pi\xi A\Delta_0/c$. By virtue of this estimate, the tunneling-induced splitting of levels is given by

$$\Gamma=\omega_0\exp(-4\pi\xi A^{3/2}/k^{1/2}c), \tag{6}$$

where $\omega_0=c/\Delta_0$ is a quantity on the order of the activation of out-of-plane magnons in an antiferromagnet, which typically has values $\omega_0\sim(10^{11}-10^{12})\ \mathrm{s}^{-1}$. Since the interaction of the field of the vector $\mathbf{l}$ with the magnetic field $\mathbf{H}$ is described by $w_{\mathrm{int}}=\chi_{\|}(\mathbf{lH})^2/2$ (Ref. 4), a magnetic field inclined with respect to the easy plane must be used to excite resonance transitions.

4. Adopting $A=Js^2$, $c=Js/\hbar a$ (where J is the exchange integral of the antiferromagnet, and s is the spin of the atom), and $\xi\simeq 2$, we find $S_E/\hbar\simeq 4\pi s\Delta_0/a$. For a spin $s=1$ the standard condition $S_E/\hbar<30$ leads to the rather tight restriction $\Delta_0<3a$. This condition does not contradict the condition for stability of out-of-plane vortices,[6] $\Delta_0>1.5a$. The macroscopic approach which we have taken here presupposes $\Delta_0\gg a$, but it is also applicable for $\Delta\sim(2-3)a$ (Ref. 7). With $S\simeq 1$ and $\Delta_0=3a$, the total spin of the sublattice changes by an amount $s_{\rm tot}=2\pi s(\Delta_0/a)^2\sim 60$ in the course of the tunneling; i.e., this effect may be thought of as a macroscopic quantum-tunneling effect. If the anisotropy is slight, and Δ_0 large (as, for example, in the case of a genuinely 2D antiferromagnet: a Langmuir film of manganese stearate[8]), the value of Δ_0 can be reduced by applying a static magnetic field H perpendicular to the easy plane: $\Delta(H)=\Delta_0(1+H^2/H_eH_a)^{1/2}$, where H_e and H_a are the exchange and anisotropy fields. At $H\gg(H_eH_a)^{1/2}$ we have a value $\Delta(H)\sim aH/H_e$.

We need to stress that the structure of the vortex and thus the quantities Γ and S_E are determined exclusively by parameters of the antiferromagnet. Consequently, the spread in the values of the parameters characterizing the macroscopic quantum tunneling, e.g., the values of the quantum splitting of the levels for the ensemble of vortices must be negligible.

Effects of a macroscopic quantum tunneling can be observed only at a sufficiently low temperature $T<T_c$, $\Delta U/T_c\simeq S_E/\hbar$, where ΔU is the energy barrier.[1] Since we have[6] $\Delta U\sim Js^2$, the value of T_c is not small: $T_c\sim Js^2(a/\Delta_0)$.

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