Quantum capacitance of a tunnel structure due to finite tunneling times

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The spectrum of low-frequency plasma waves in a semiconductor–(tunnel junction)–semiconductor structure is calculated. The impedance of the structure is also calculated. The finite transmission of the barriers and the nonzero time it takes electrons to tunnel through the barrier are taken into account. The capacitance of the junction has a significant quantum contribution, which is proportional to the phase tunneling time. This contribution does not vanish even in the limit of an opaque barrier. © *1995 American Institute of Physics.*

The time scale of the tunneling of an electron through a potential barrier, τ_r, continues to attract research interest despite the more than 40-year history of this problem.[1] In addition to being a problem of fundamental interest, it is of interest from the applications standpoint, since tunneling times set limits on the response frequencies of tunnel devices. In the present letter we take a kinetic approach to calculate the spectrum of low-frequency plasma waves propagating along a tunnel barrier and also the dynamic impedance of the tunnel structure. The latter is determined by these "tunnel plasmons." We show that the frequency and damping of the plasmons and also the parameters of the small-signal equivalent circuit are related to the tunneling transmission of the barrier and the phase delay time, averaged in a certain way over a Fermi distribution.

We consider a structure consisting of a semiconductor ($z<-d/2$), a tunnel junction ($|z|<d/2$), and another semiconductor ($z>d/2$). We assume that the electron gas in the semiconductor is degenerate and that the electron density is such that the condition $\omega_p \gg \gamma$ holds, where ω_p and γ are the plasma frequency and relaxation rate. The junction can be either a single-barrier junction or a double-barrier (resonant) junction. Plasma waves localized at the junction can propagate along a plasma–insulator–plasma structure.[2] Their structure consists of two branches. If tunneling, retardation, and nonlocal effects are all ignored, this spectrum is described by

$$\omega(\omega+i\gamma)=\frac{\omega_p^2}{1+[\tanh(qd/2)]^{\pm 1}}, \tag{1}$$

where q is the wave vector of a plasmon parallel to the interface. The minus sign in (1) corresponds to a gapless mode with antisymmetric distributions of the charge $-en_1$ and the potential Φ (see the inset in Fig. 1). Below we examine the effect of tunneling on the spectrum of this mode in the long-wave limit $qd \ll 1$.

To solve the problem we consider the self-consistent system of equations consisting of the Poisson equation

$$\Delta\Phi(\mathbf{r},t)=\begin{cases}4\pi e n_1(\mathbf{r},t)/k & |z|>d/2,\\ 0, & |z|<d/2,\end{cases} \tag{2}$$

(k is the dielectric constant of the lattice) and the Boltzmann equation

$$\frac{\partial f}{\partial t}+\mathbf{v}_p\frac{\partial f}{\partial \mathbf{r}}+e\,\frac{\partial\Phi}{\partial \mathbf{r}}\,\frac{\partial f}{\partial \mathbf{p}}=-\gamma(f_p-g_p). \tag{3}$$

In Eq. (2) we ignored the effect of charge near the barrier. The electron distribution function in the semiconductor, $fp(\mathbf{r},t)=f_p^0(\epsilon_p)+f_p^1(\mathbf{r},t)$, is related to a local equilibrium distribution function

$$g_p(\mathbf{r},t)=\{1+\exp[(\epsilon_p-e\Phi(\mathbf{r},t)-\mu(\mathbf{r},t))/T_L]\}^{-1}$$

by the charge-conservation condition

$$n(\mathbf{r},t)=n_0+n_1(\mathbf{r},t)=\sum_p f_p(\mathbf{r},t)=\sum_p g_p(\mathbf{r},t), \tag{4}$$

where $\mu(\mathbf{r},t)$ is the chemical potential, and T_L the temperature. At infinity, the boundary conditions on Eqs. (2) and (3) are

$$\Phi|_{z=\pm\infty}=0,\ \ f_p^1(v_z<0)|_{z=+\infty}=0,\ \ f_p^1(v_z>0)|_{z=-\infty}=0. \tag{5}$$

At the junction, at the points $z=\pm d/2$, we impose the standard condition that the potential $\Phi(z)$ and the normal electric field $\Phi'(z)$ are continuous.

The central point of this paper is the formulation of boundary conditions on $f_p(z)$ at the tunnel junction. We assume that the dispersion relation for the electrons in the semiconductor is isotropic and quadratic, and we assume that the tunneling occurs elastically and coherently, conserving the total energy and the tangential component of the momentum. At first, we ignore the time delays in the reflection and transmission of electrons at the barrier, and we postulate the boundary conditions

$$\begin{aligned}f(d/2,p_z=+\sqrt{2m(\epsilon_z+e\Phi(d/2))})&=R(\epsilon_z)f(d/2,p_z=-\sqrt{2m(\epsilon_z+e\Phi(d/2))})\\&\quad+T(\epsilon_z)f(-d/2,p_z=+\sqrt{2m(\epsilon_z+e\Phi(-d/2))});\end{aligned} \tag{6}$$

$$\begin{aligned}f(-d/2,p_z=-\sqrt{2m(\epsilon_z+e\Phi(-d/2))})&=R(\epsilon_z)f(-d/2,p_z\\&=+\sqrt{2m(\epsilon_z+e\Phi(-d/2))})+T(\epsilon_z)f(d/2,p_z=-\sqrt{2m(\epsilon_z+e\Phi(d/2))}).\end{aligned} \tag{7}$$

Physically, condition (6) reflects the fact that the number of electrons moving away from the barrier (to the right) in the semiconductor on the right is equal to the number of electrons which are moving toward the barrier (to the left) in the semiconductor on the right and which are reflected with a probability R, plus the number of electrons which are moving toward the barrier (to the right) in the semiconductor on the left and which are transmitted through the barrier with a probability T. Equation (7) has a corresponding meaning. Boundary conditions (6) and (7) were recently used along with $\Phi(\pm d/2)=0$ in Ref. 3 to calculate the static ohmic conductivity of a tunnel structure.

Self-consistent problem (2)–(4) with boundary conditions (5)–(7) reduces to a system of two coupled integral equations for the function $X(z) =\gamma n_1(z)/N_F+i(\omega+i\gamma)e\Phi(z)$, where $N_F=3n_0/mv_F^2$ is the density of states at the Fermi level. At $T(\epsilon_z)=0$, the resulting system of equations can be solved exactly; as a result, we find nonlocal corrections to (1). For an arbitrary transmission $T(\epsilon_z)$, a solution for the low-frequency (odd) mode can be found under the conditions $qv_F\ll|\omega+i\gamma|\ll\omega_p$ and by ignoring Landau damping, which is slight under these approximations. We will not write out that solution here, because it is quite lengthy. We can find the same result by combining a hydrodynamic approach with a boundary condition on the current,

$$j_z(d/2)\equiv(-e)\sum_p v_zf_p(d/2)=e\sum_{p+} v_zT\{(\partial f_p^0/\partial\epsilon_p)e[\Phi(d/2)-\Phi(-d/2)]$$
$$+[f_p^1(d/2,-v_z)-f_p^1(-d/2,v_z)]\}, \tag{8}$$

which follows from (6) and (7). Here $\{\mathbf{p}+\}$ means $\{\mathbf{p}_\parallel,p_z>0\}$. The difference between expression (8) and the standard expression[4] stems from a deviation of the distribution function from the locally equilibrium distribution function in the semiconductor plasma. The boundary values of the distribution function, $f_p^1(d/2,-v_z)$ and $f_p^1(-d/2,v_z)$, can be expressed in terms of integrals of the function $X(z)$ over the regions $z>d/2$ and $z<-d/2$ with the help of the Boltzmann equation. As a result, the expression for the current $j_z(d/2)$ becomes

$$j_z(d/2)=(-2e)\sum_{p+} T\partial f_p^0/\partial\epsilon_p\int_{d/2}^{\infty}dzX(z)\exp\left[i\,\frac{\omega-qv_x+i\gamma}{v_z}(z-d/2)\right]. \tag{9}$$

Writing a solution of the hydrodynamic equations at $z\geqslant d/2$ in the form

$$n_1(z)=n_1\exp[-q_{TF}(z-d/2)], \tag{10}$$

$$\Phi(z)=\Phi_1\exp[-|q|(z-d/2)]+[4\pi e/kq_{TF}^2]n_1\exp[-q_{TF}(z-d/2)] \tag{11}$$

($q_{TF}=1/\lambda_{TF}$ is the reciprocal of the screening length), using the continuity of the potential and the field at the junction, and joining the hydrodynamic current $j_z(d/2)=\Phi_1n_0e^2/im(\omega+i\gamma)+i\omega e\lambda_{TF}n_1$ with kinetic expression (9), we find a system of two equations for the amplitudes n_1 and Φ_1. Equating the determinant of the resulting system to zero, we find a dispersion relation for the tunnel plasmons:

$$(\omega+i\gamma)(\omega+iG_T/C_0)-\Omega^2(q)=0, \tag{12}$$

where $qd\ll1$; $qv_F\ll|\omega+i\gamma|\ll\omega_p$;

$$\Omega^2(q)=\omega_p^2|q|(d/2+\lambda_{TF})\langle R\rangle_2/\langle R\rangle_0, \tag{13}$$

and

$$C_0=\frac{k}{4\pi(d+2\lambda_{TF})}\frac{\langle R\rangle_0}{\langle R\rangle_2}\approx\frac{k}{4\pi(d+2\lambda_{TF})}\equiv C_{cl} \tag{14}$$

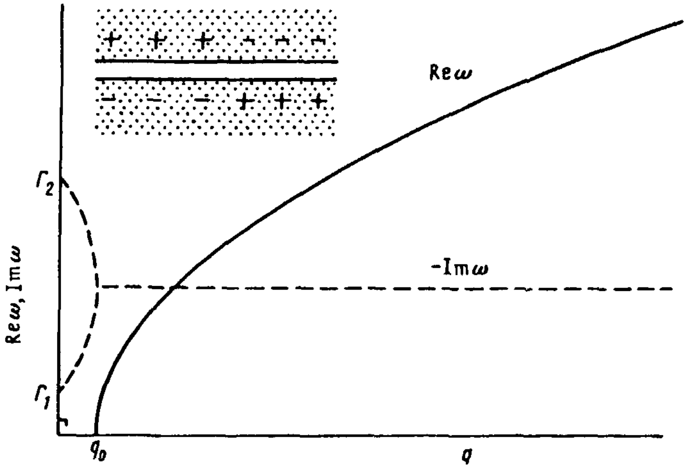

FIG. 1. Frequency (solid curve) and damping (dashed curve) of tunnel plasmons as a function of the wave vector. Here $\Gamma_1 = \min\{\gamma,\Gamma\}$ and $\Gamma_2 = \max\{\gamma,\Gamma\}$. The wave vector q_0 is defined by the condition $\Omega(q_0) = (\Gamma_2 - \Gamma_1)/2$. The inset shows the distribution of the charge of the tunnel plasmons (antisymmetric mode).

is the capacitance of the junction incorporating nonlocal corrections (because of λ_{TF}) and tunneling corrections (because of the finite transmission). The quantity

$$G_T = (3n_0e^2/4p_F)\langle T\rangle_1/\langle R\rangle_2 \tag{15}$$

represents the tunneling transmission of the barrier [see Eq. (20) below]. The averages $\langle ...\rangle_n$ are defined by

$$\langle ...\rangle_n = \frac{\Sigma_p(...)|v_z|^n\partial f_p^0/\partial\epsilon_p}{\Sigma_p|v_z|^n\partial f_p^0/\partial\epsilon_p}\,. \tag{16}$$

Figure 1 shows the frequency and damping of tunnel plasmons as a function of the wave vector. The finite tunneling transmission of the barrier thus leads to an additional damping of the tunnel plasmons, $\Gamma = G_T/C_0$. Under the conditions $\langle R\rangle_0 = \langle R\rangle_2 = 1$ and $2\lambda_{TF}/d \to 0$, the results in (12)–(15) reduce to those derived in Ref. 5.

To take the tunneling times into account, we modify boundary conditions (6) and (7):

$$\begin{aligned} f(\mathbf{r}_\parallel, \pm d/2; p_z = \pm\sqrt{2m(\epsilon_z + e\Phi(\mathbf{r}_\parallel, \pm d/2, t))}; t) &= Rf(\mathbf{r}_\parallel - \mathbf{s}_T, \pm d/2; p_z \\ &= \mp\sqrt{2m(\epsilon_z + e\Phi(\mathbf{r}_\parallel - \mathbf{s}_T, \pm d/2, t-\tau_T))}; t-\tau_T) + Tf(\mathbf{r}_\parallel - \mathbf{s}_T, \mp d/2; p_z \\ &= \pm\sqrt{2m(\epsilon_z + e\Phi(\mathbf{r}_\parallel - \mathbf{s}_T, \mp d/2, t-\tau_T))}; t-\tau_T), \end{aligned} \tag{17}$$

where $\mathbf{r}_\parallel = (x,y)$. Expression (17) incorporates the circumstance that there is a time delay τ_T in the scattering of the electron by the barrier, and there is a also a spatial shift $\mathbf{s}_T = \mathbf{v}_\parallel\tau_T$ in the lateral direction. Here $\tau_T(\epsilon_z)$ is the tunneling phase time, determined by the derivative of the phase of the transmission (or reflection) coefficient with respect to the energy. (In the case of the symmetric barrier, under consideration here with a zero bias voltage, the reflection time and the transmission time are equal, and they are furthermore identical for electrons which are moving toward the barrier from the right and from the left.) The introduction of a phase time τ_T in classical boundary conditions (17) is justified under the condition $k_F l \gg 1$, where k_F is the Fermi momentum, and l is the mean free path. Expression (17) assumes that the tunneling parameters T, R, and τ_T are independent of the coordinates and the time (e.g., they have no such dependence by virtue of a dependence on the wave potential). This assumption corresponds to the adoption of the inequalities $\omega\tau_T \ll 1$ and $\mathbf{q}\mathbf{s}_T \ll 1$.

Taking the finite tunneling times into account, and ignoring the small corrections $\langle T\rangle_0$ and $\langle T\rangle_2$, we put the dispersion relation for tunnel plasmons in the form

$$(\omega+i\gamma)\left(\omega+\frac{iG_T}{C_{cl}+C_T}\right)-\frac{\Omega^2 q}{1+C_T/C_{cl}}=0, \tag{18}$$

where the quantity

$$C_T=\frac{3n_0e^2\langle\tau_T(R-T)\rangle_1}{8p_F}=C_{cl}\frac{\sqrt{3}(1+q_{TF}d/2)\omega_p\langle\tau_T(R-T)\rangle_1}{4} \tag{19}$$

is an additional quantum capacitance of the tunnel junction. The lateral shift of the electron in the course of tunneling does not contribute to the plasmon spectrum in the linear order in q.

The same method can be used, under the same approximations, to solve the problem of the response to a small-amplitude alternating signal of a structure consisting of a metal $(z<-A/2)$, a semiconductor, a tunnel junction, another semiconductor, and another metal $(z>A/2)$. The impedance of the structure found under the condition $l \ll A$ is

$$Z(\omega)=R-i\omega L+[G_T-i\omega(C_{cl}+C_T)]^{-1}, \tag{20}$$

where R and L are the classical resistance and classical inductance of the semiconductor regions. The small-signal equivalent circuit of the structure contains an additional capacitance C_T in parallel with the classical capacitance C_{cl}.

Incorporating the tunneling time thus gives rise to an additional tunneling capacitance of the junction, C_T, and substantial changes in the frequency and damping of the tunnel plasmons (Fig. 2 gives an idea of the behavior of C_T as a function of the parameters of the barrier and the Fermi energy). The tunneling component of the admittance of the junction becomes complex: $G_T(\omega)=G_T-i\omega C_T$. As the barrier thickness increases, the tunneling component of the capacitance becomes relatively larger, since C_{cl} decreases, while C_T remains finite because of a contribution of reflected electrons. Consequently, the tunneling capacitance C_T and thus the tunneling time τ_T can be measured at classical (nontransmitting) junctions. (Note, however, that C_T falls off with increasing height of the potential barrier; Fig. 2b.) It follows from (19) that the contribution to C_T from reflected electrons $(\sim\tau_T R)$ is positive, while that from electrons which have undergone tunneling $(\sim\tau_T T)$ is negative. Accordingly, a sharp decrease in C_T near a tunneling resonance should be observed in double-barrier structures.

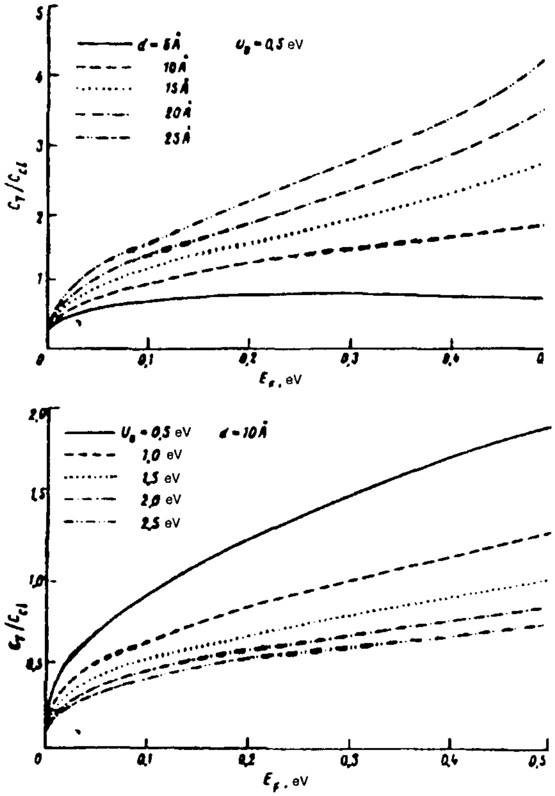

FIG. 2. Reduced tunneling capacitance versus the Fermi energy E_F in a single-barrier tunnel structure. a—For a fixed height $U_0 = 0.5$ eV and various thicknesses d of the barrier; b—for a fixed thickness $d = 10$ Å and various heights of the barrier.

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