Minimal nature of a nonminimal electromagnetic interaction

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Within the framework of supersymmetric electrodynamics, the interaction of spin-1/2 particles, which have an anomalous magnetic moment, with the fields of the Maxwell supermultiplet can be described on the basis of a generalized minimality principle. A new mechanism thus becomes available for dealing with the electromagnetic interactions of photinos with charged and neutral fermions, by means of their anomalous magnetic moment. © *1995 American Institute of Physics.*

The principle of minimality in the incorporation of the electromagnetic interaction and the generalization of this principle to Yang–Mills theories are leading geometric principles in the derivation of a theory of interacting fields.[1] However, the use of this one principle alone in ordinary Minkowski space complicates efforts to describe the electromagnetic interactions of particles which have an anomalous magnetic moment. In the present letter we show that a switch to the superspace $z^M=(x^\mu,\theta^\alpha,\bar{\theta}_{\dot{\alpha}})$, which contains both the ordinary spatial coordinates x^μ and the Grassmann spinors θ^α, $\bar{\theta}_{\dot{\alpha}}$, makes it possible to formulate a generalized minimality principle which is sufficient for describing electromagnetic interactions of particles with an anomalous magnetic moment.

The introduction of the concept of a superspace z^M makes possible a natural geometric realization of the expected supersymmetry in the world of elementary particles.[2,3] The principle of supersymmetry presupposes that the ordinary spatial coordinates have the same rank as the auxiliary Grassmann coordinates, with the result that the standard transformation from the electromagnetic field $a_\mu(x)$ to the superfield $A_\mu(x,\theta,\bar{\theta})$ is accompanied by the introduction of some auxiliary spinor connections $A_\alpha(x,\theta,\bar{\theta})$ and $\bar{A}^{\dot{\alpha}}(x,\theta,\bar{\theta})$. It thus becomes possible to construct an invariant supersymmetric 1-form

$$\omega^M A_M=[dx^\mu-i(d\theta\sigma^\mu\bar{\theta}-\theta\sigma^\mu d\bar{\theta})]A_\mu+d\theta^\alpha A_\alpha+d\bar{\theta}_{\dot{\alpha}}\bar{A}^{\dot{\alpha}},$$

where $\omega^M=(\omega^\mu,d\theta^\alpha,d\bar{\theta}_{\dot{\alpha}})$. It also becomes possible to write an interaction functional $S_{\text{int}}^{(e)}$ for the interaction of a charged superparticle in an external superfield $A_M=(A_\mu,A_\alpha,\bar{A}^{\dot{\alpha}})$ as an integral of this 1-form along the world line of the particle in superspace:[4,5]

$$S_{\text{int}}^{(e)}=ie\int d\tau\omega_\tau^M A_M(x,\theta,\bar{\theta}). \tag{1}$$

The use of this geometric formulation is equivalent to the ordinary principle of minimality.

Here we would like to call attention to the possibility of expanding superfield functional (1) by lengthening the connection A_M, i.e., by making the transformation

$$eA_M \mapsto eA_M + i\mu W'_M, \tag{2}$$

where μ is the anomalous magnetic moment of the particle, W'_M is the $U(1)$-invariant superfield, and $W'_M \equiv (W'_\mu, W'_\alpha, \bar{W}'^{\dot\alpha})$. Natural entities for the construction of the invariants W'_M are the components of the superfield strength $F_{MN}(x,\theta,\bar\theta)$. Since the constant μ has the dimensionality of a length, $[\mu]=L$ (in a system of units with $\hbar=c=1$), the components of the superfield W'_M must have strictly fixed dimensionalities: $[W'_\mu]=L^{-2}$, $[W'_\alpha]=L^{-3/2}$. It is not difficult to verify that, within the framework of the conventions adopted here, it is impossible to work from the components F_{MN} to construct a Lorentz vector W'_μ of dimensionality L^{-2} which is linear in these components. On the other hand, a contraction of the type $F_{\mu\dot\alpha}\tilde\sigma^{\mu\dot\alpha\alpha}$ gives us an invariant spinor superfield of the necessary dimensionality, $L^{-3/2}$. As the invariant W'_M we can therefore choose a superfield W_M of the form

$$W'_M = W_M \equiv \frac{i}{4}(0, -\sigma_{\mu\alpha\dot\alpha}F^{\mu\dot\alpha}, \tilde\sigma^{\mu\dot\alpha\alpha}F_{\mu\alpha}), \tag{3}$$

where $F_{\mu\alpha}$ is defined, along with the other quantities used here, in Ref. 3. According to (3), the action in (1) becomes

$$S_{\rm int}^{(e,\mu)} = i\int d\tau[\omega^\mu_\tau eA_\mu + \dot\theta^\alpha(eA_\alpha + i\mu W_\alpha) + \dot{\bar\theta}_{\dot\alpha}(e\bar A^{\dot\alpha} + i\mu\bar W^{\dot\alpha})]. \tag{4}$$

Pointing out that the superfields eA_M and $i\mu W_M$ are of equal rank from the standpoint of the minimality principle, we introduce a two-component "charge" $q^\Lambda = (e, i\mu)$ and a two-component "connection" $G^\Lambda_M = \binom{A_M}{W_M}$. Action (4) can then be written in the more compact form

$$S_{\rm int}^{(e,\mu)} = i\int d\tau\omega^M_\tau q^\Lambda G^\Lambda_M. \tag{5}$$

It is clear from this result that we have "$e-\mu$" universality, i.e., a symmetry between (e,A_M) and $(i\mu,W_M)$ pairs. The lengthening of constraint (2) leads to an auxiliary shift of the standard "long" derivatives ∇_M (Ref. 3):

$$\nabla_M \equiv D_M + eA_M \mapsto \tilde\nabla_M = D_M + q^\Lambda G^\Lambda_M = D_M + eA_M + i\mu W_M, \tag{6}$$

where $D_M = (\partial_\mu, D_\alpha, \bar D^{\dot\alpha})$. The new expanded $U(1)$-invariant "field strengths" G^Λ_{MN} are constructed from $\tilde\nabla_M$ and G^Λ_M by the ordinary rules. Using (3), we express them in terms of F_{MN} (Ref. 3) as follows:

$$q^\Lambda G^\Lambda_{\mu\nu} = eF_{\mu\nu}, \quad q^\Lambda G^\Lambda_{\mu\alpha} = eF_{\mu\alpha} + i\mu\partial_\mu W_\alpha, \quad q^\Lambda G^\Lambda_{\mu\dot\alpha} = eF_{\mu\dot\alpha} + i\mu\partial_\mu \bar W_{\dot\alpha},$$
$$q^\Lambda G^\Lambda_{\alpha\beta} = eF_{\alpha\beta} + i\mu(D_\alpha W_\beta + D_\beta W_\alpha), \quad q^\Lambda G^\Lambda_{\dot\alpha\dot\beta} = eF_{\dot\alpha\dot\beta} + i\mu(\bar D_{\dot\alpha}\bar W_{\dot\beta} + \bar D_{\dot\beta}\bar W_{\dot\alpha}),$$
$$q^\Lambda G^\Lambda_{\alpha\dot\beta} = eF_{\alpha\dot\beta}. \tag{7}$$

The generalized equations of motion found from the total action

$$S=-\frac{1}{2}\int d\tau\left[\frac{\omega_\tau^\mu\omega_{\tau\mu}}{g}+gm^2\right]+i\int d\tau\omega^M q^\Lambda G_M^\Lambda, \tag{8}$$

where g is a Lagrange multiplier, take the following form for a particle with a charge e and an anomalous magnetic moment μ:

$$(g^{-1}\omega_{\tau\mu})^{\cdot}=i\omega_\tau^M q^\Lambda G_{M\mu}^\Lambda, \quad g^{-1}\omega_{\tau\mu}(\sigma^\nu\dot{\bar\theta})_\alpha=-\tfrac{1}{2}\,\omega_\tau^M q^\Lambda G_{M\alpha}^\Lambda,$$

$$g^{-1}\omega_{\tau\mu}(\dot\theta\sigma^\mu)_{\dot\alpha}=\tfrac{1}{2}\,\omega_\tau^M q^\Lambda G_{M\dot\alpha}^\Lambda. \tag{9}$$

For the case of a massive particle these equations can be simplified by choosing the gauge $mg=1$, which is equivalent to the condition $\omega_\tau^\mu\omega_{\tau\mu}=1$. In the case of a massless particle, we would do this by choosing the gauge $\dot g=0$ and the condition $\omega_\tau^\mu\omega_{\tau\mu}=0$.

To analyze the equations of motion of a superparticle in the fields of a physical Maxwell multiplet, we need to incorporate on the right sides of Eqs. (9) for the constraints:[3] $F_{\alpha\beta}=F_{\dot\alpha\dot\beta}=F_{\alpha\dot\beta}=0$. Taking these constraints into account, we can express all the fields F_{MN} in terms of the spinor superfields W_α, $\bar W^{\dot\alpha}=(W_\alpha)^*$[see (3)], which have the component expansions

$$W_\alpha=-i\lambda_\alpha(y)+\left[\delta_\alpha^\beta D(y)-\frac{i}{2}(\sigma^\mu\tilde\sigma^\nu)_\alpha^\beta(\partial_\mu v_\nu(y)-\partial_\nu v_\mu(y))\right]\theta_\beta$$

$$+\theta\theta\sigma^\mu_{\alpha\dot\alpha}\partial_\mu\bar\lambda^{\dot\alpha}(y),$$

$$\bar W^{\dot\alpha}=i\bar\lambda^{\dot\alpha}(y^+)+\left[\delta_{\dot\beta}^{\dot\alpha}D(y^+)+\frac{i}{2}(\tilde\sigma^\mu\sigma^\nu)_{\dot\beta}^{\dot\alpha}(\partial_\mu v_\nu(y^+)-\partial_\nu v_\mu(y^+))\right]\bar\theta^{\dot\beta}$$

$$-\bar\theta\bar\theta\tilde\sigma^{\dot\alpha\alpha\mu}\partial_\mu\lambda_\alpha(y^+). \tag{10}$$

Here $y=x+i\theta\sigma\bar\theta$, $y^+=x-i\theta\sigma\bar\theta$, v_μ is the electromagnetic field, λ_α is the photino field; and D is an auxiliary field. In terms of the superfields W_α and $\bar W^{\dot\alpha}$, equations of motion (9) take the form of generalized Lorentz equations,

$$(g^{-1}\omega_{\tau\mu})^{\cdot}=-\frac{i}{2}\,e\omega_\tau^\nu(\bar D\tilde\sigma_{\nu\mu}\bar W-D\sigma_{\nu\mu}W)+\dot\theta^\alpha\Xi_{\mu\alpha}+\bar\Xi_{\mu\dot\alpha}\dot{\bar\theta}^{\dot\alpha},$$

$$g^{-1}\omega_{\tau\mu}(\sigma^\mu\dot{\bar\theta})_\alpha=-\frac{i}{2}(\omega_\tau^\mu\Xi_{\mu\alpha}+\mu\dot\theta^\beta G_{\beta\alpha}),$$

$$g^{-1}\omega_{\tau\mu}(\dot\theta\sigma^\mu)_{\dot\alpha}=\frac{i}{2}(\omega_\tau^\mu\bar\Xi_{\mu\dot\alpha}+\mu\dot{\bar\theta}_{\dot\beta}G_{\dot\alpha}^{\dot\beta}), \tag{11}$$

where Ξ and $\bar\Xi$ are given by

$$\Xi_{\mu\alpha}\equiv e(\sigma_\mu\bar W)_\alpha+\mu\partial_\mu W_\alpha, \quad \bar\Xi_{\mu\dot\alpha}\equiv e(W\sigma_\mu)_{\dot\alpha}+\mu\partial_\mu\bar W_{\dot\alpha}.$$

Nonlinear equations (11) are nontrivial in that a relativistic particle is subjected to some additional forces which are caused by the interaction with photinos and which are proportional to the electric charge and also the anomalous magnetic moment μ of the particle.

The physical meaning of the constant μ—the anomalous magnetic moment of a particle—follows from an analysis of action (4). After the first pair of Maxwell's equations, $\epsilon^{\mu\nu\rho\lambda}\partial_\nu v_{\nu\rho}=0$, is taken into account, this action becomes

$$S_{\rm int}^{(e,\mu)}\Big|_{\substack{e=0\\ \text{photon}}}=i\mu\int d\tau[(\dot\theta\sigma^{\mu\nu}\theta)-(\dot{\bar\theta}\tilde\sigma^{\mu\nu}\bar\theta)]v_{\mu\nu}$$

$$+\frac{1}{2}\mu\int d\tau[\theta\theta(\dot\theta\sigma^\mu\bar\theta)+\bar\theta\bar\theta(\theta\sigma^\mu\dot{\bar\theta})]\partial^\rho v_{\rho\mu}. \tag{12}$$

Of the two terms remaining in action (12), the second (which contains the current $\partial^\rho v_{\rho\mu}$) describes the spin–orbit and other relativistic interactions, which correspond to succeeding terms in the expansion in powers of $1/c$. Accordingly, we can see the physical meaning of the constant μ by restricting the analysis to the first term in (12). Here it is convenient to switch from the pair of Weyl spinors θ_α, $\bar\theta^{\dot\alpha}$ and the matrix $(\sigma^{\mu\nu})_\alpha^\beta$ to the Dirac bispinor Ψ and the operator for the spin of the relativistic particle, $\Sigma_{\mu\nu}$ (Ref. 1):

$$\Psi=\begin{pmatrix}\theta_\alpha\\ \dot{\bar\theta}^{\dot\alpha}\end{pmatrix},\quad \Sigma_{\mu\nu}=\frac{i}{4}[\gamma_\mu,\gamma_\nu]. \tag{13}$$

Here the γ_μ are the Dirac matrices in a Weyl basis. The contribution of the first term to the action in (12) can then be written as a standard Pauli term

$$S_{\rm int}^{(e,\mu)}=\mu\int d\tau(\bar\psi\Sigma^{\mu\nu}\psi)v_{\mu\nu}+\text{(higher-order corrections and other interactions)}. \tag{14}$$

The physical meaning of the constant μ is obvious from this expression: It is the anomalous magnetic moment of the particle, expressed in Bohr magnetons.

We thus see that the generalized minimality principle discussed above, which assumes an additional lengthening of the standard spinor covariant derivatives ∇_α and $\bar\nabla_{\dot\alpha}$, does indeed describe the contribution of the anomalous magnetic moment to the electromagnetic interactions of spin-1/2 particles. This description has not been conducted in the ordinary space–time x^μ, since the necessary lengthening occurs along the spinor directions of the superspace. From the geometric point of view, the lengthening in (6) is analogous to a lengthening of Riemann connections on the twisting tensor. It can thus be said that a particle "senses" the twisting of the superspace by means of its anomalous magnetic moment.

We would like to emphasize in particular that action (9) contains several new terms, which correspond to electromagnetic interactions of spin-1/2 relativistic particles via their anomalous magnetic moment with the photino field $\lambda^\alpha(x)$[and also with the auxiliary field $D(x)$]. These terms have some interesting consequences, particularly in the case of neutrinos. Since a neutrino can be described by a Majorana spinor $\nu(x)$, it is natural to assume $\nu(x)=[\theta_\alpha(x)/\bar\theta^{\dot\alpha}(x)]$ by virtue of (13) and in accordance with the Goldstone interpretation.[2] The neutrino-interaction Lagrangian which follows from (4) can then be written

$$\mathscr{L}_{\rm int}^{\rm neutrino}=\mu[(i\bar\nu\gamma_5\lambda)-(\bar\nu\nu>D-\tfrac{1}{2})(\bar\nu\gamma_5\gamma_\mu\nu)(\bar\nu\partial^\mu\lambda)]. \tag{15}$$

The first term in (15) indicates the possibility of neutrino–photino oscillations due to the anomalous magnetic moment of neutrinos. Incorporating such oscillations may be sufficient for generating masses for neutrinos and photinos. The oscillations might thus give us a natural mechanism for the spontaneous breaking of supersymmetry. The second term suggests an alternative mechanism (analogous to the Higgs mechanism) for the generation of a neutrino mass under the condition that the auxiliary field $D(x)$ has a nonzero vacuum expectation value. Finally, the third term indicates the possibility of a transformation of photinos into three neutrinos and of the inverse transformation.

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