Anomalous vortex electron fluxes produced as a result of inverse bremsstrahlung of an rf field

K. N. Ovchinnikov, V. P. Silin, and S. A. Uryupin
P. N. Lebedev Physics Institute, Russian Academy of Sciences, 117924 Moscow, Russia

(Submitted 15 February 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **61**, No. 7, 541–546 (10 April 1995)

The nonlocal vortex current and the nonlocal vortex heat flux of electrons in a plasma heated by inverse bremsstrahlung of an rf field are derived. The magnitudes of the fluxes are established for all possible values of the ratio of the length scale of the variations in the amplitude of the electromagnetic field, $\sim 1/k$, to the mean free path of a thermal electron, l. As the parameter kl increases, the vortex current initially reaches a maximum j_1 at $kl \approx 0.02$ and then falls off, changing the sign of the helicity at $kl \approx 0.05$. At $kl \approx 0.9$, the absolute value of the current reaches an anomalously high value $j_2 \approx 6 j_1$ and then falls off monotonically. © *1995 American Institute of Physics.*

The problem of generating megagauss magnetic fields by means of rf radiation in a hot laser-produced plasma requires the derivation of a theory for the excitation of quasisteady vortex currents (see, for example, Refs. 1–5). The reason for the onset of quasisteady currents may be either an inhomogeneity of the plasma or a nonuniformity of the electromagnetic field heating the plasma.[5] We should stress that the vortex current due to the nonpotential part of the ponderomotive force[6] is the main source of magnetic-field generation in a plasma with smooth profiles of the electron temperature and density.[5] A systematic kinetic theory for the excitation of quasisteady currents by electromagnetic radiation whose spatial variations have a length scale $\sim 1/k$ much larger than the mean free path of a thermal electron, l, is based on the Hilbert–Enskog–Chapman method.[3,5] In real laser plasmas, conditions may be such that the parameter kl is by no means small in comparison with unity. In this case it becomes necessary to derive a kinetic theory for the excitation of currents under conditions such that the spatial nonlocality of the electron transport is prominent. The results of a corresponding analysis are presented below for conditions such that the plasma is heated by inverse bremsstrahlung, as is typical of laser fusion. We derive analytic solutions for the kinetic equations in the limits of small and large values of the nonlocality parameter kl. We also derive the necessary expressions for the electron current density, which are quadratic in the electromagnetic field heating the plasma. The corresponding asymptotic expressions, however, are not suitable over the extremely broad range

$$0.01 \leq kl \leq 10. \tag{1}$$

In this range, the necessary behavior must be found numerically. We show that the increase in the vortex current with increasing value of kl at small values of the latter product leads to a maximum value at $kl \approx 0.02$. Here we already see a significant difference from the result of Refs. 3 and 5, based on the Hilbert–Enskog–Chapman approximation. Later, at $kl \approx 0.05$, the current vanishes. This event corresponds to a change in the sign of the helicity of the vortex current. With a further increase in the nonlocality parameter, the absolute value of the density of the excited current reaches a maximum, at $kl \approx 0.9$. The absolute value of this maximum is anomalously high—greater than the maximum for the opposite helicity by a factor of 6. Finally, as kl is raised further, the current falls off monotonically. We offer a corresponding description of the vortex electron heat flux, for which we again see a change in the sign of the helicity at $kl \approx 0.1$. We wish to stress that in intermediate region (1) the absolute value of the vortex electron heat flux is anomalously greater than the asymptotic values found in Ref. 3 in the limit of small values of the nonlocality parameter and in Ref. 7 in the limit $kl \geqslant 10$.

As the starting point for determining the quasisteady vortex electron fluxes we use the equation for a small nonequilibrium increment δf in the spatially uniform Maxwellian distribution function f_m. This increment arises because of inverse bremsstrahlung of the rf electromagnetic field by electrons:

$$\tfrac{1}{2}\mathbf{E}e^{-i\omega t}+\text{c.c.},$$

where the frequency ω is assumed to be much larger than the rate of electron–ion collisions and also much larger than the reciprocal time taken by a thermal electron to traverse a distance on the order of the size of the nonuniformity of the electromagnetic field. The reason why δf is small in comparison with f_m is the small ratio of the amplitude of the electron oscillation velocity in the rf field, $v_E = eE/m\omega$, to the electron thermal velocity $v_T = \sqrt{\kappa T/m}$, where e and m are the charge and mass of an electron, κ is the Boltzmann constant, and T is the electron temperature. Under these conditions the electron distribution function can be written[7]

$$f = f_m\left(1 - \frac{v_E^2}{2v_T^2} - \frac{e\,\delta\varphi}{mv_T^2} + \frac{1}{8v_T^4}\,v_i v_j V_{ij}\right) + \delta f.$$

Here $\delta\varphi$ is the perturbation of the electric potential, $V_{ij} = (e^2/m^2\omega^2)(E_i E_j^* + E_i^* E_j)$ is the oscillation-velocity tensor, and the nonequilibrium increment δf, which is determined by electron collisions, obeys the equation

$$\begin{aligned} &\mathbf{ikv}\,\delta f - \frac{1}{2}\,\nu(v)\frac{\partial}{\partial v_i}(v^2\delta_{ij} - v_i v_j)\frac{\partial}{\partial v_j}\,\delta f - \mathbf{St}(\delta f) \\ &= -\frac{v_E^2}{6v_T^2}\frac{\partial}{\partial v_i}(v_i \nu(v) f_m) + \left(v_i v_j - \frac{1}{3}\,\delta_{ij} v^2\right)V_{ij}\left(3 - \frac{v^2}{2v_T^2}\right)\frac{\nu(v)}{4v^2 v_T^2}\,f_m. \end{aligned} \tag{2}$$

In writing Eq. (2) we assumed that the quantities v_E^2, V_{ij}, and δf have a spatial variation $\exp(\mathbf{ikr})$, and we have used the following notation: $\mathbf{St}(\delta f)$ for the electron–electron collision integral and $\nu(v)$ for the rate of electron–ion collisions. Here we have $\nu(v) = 4\pi e^4 Z n\Lambda/m^2 v^3$, where Z is the degree of ionization of the ions, n is the density of electrons, and Λ is the Coulomb logarithm. Below we ignore the logarithmic dependence of Λ on the velocity.

We wish to derive the vortex fluxes which are oriented perpendicular to the vector **k**, which we assume to be directed along the z axis. We restrict the discussion to a plasma with multiply charged ions, with $Z \gg 1$. In this case we can ignore the electron–electron collision integral. The components of the vortex current $j_\alpha = e\int d\mathbf{v} v\sqrt{1-\xi^2}\delta f_\alpha$ and of the vortex heat flux density $q_\alpha = (m/2)\int d\mathbf{v} v^3\sqrt{1-\xi^2}\delta f_\alpha [\alpha=(x,y), \xi=\cos\theta$, and θ is the angle between the vectors **k** and **v**] are determined only by δf_α, i.e., by that part of the nonequilibrium increment δf which is proportional to the tensor $V_{\alpha z}$. Taking an average of Eq. (2) over φ, which is the azimuthal angle of the velocity vector, with weight factors $\cos\varphi$ or $\sin\varphi$, we find the following result for the functions δf_α:

$$ikv\xi\delta f_\alpha - \frac{1}{2}\nu(v)\left(\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial}{\partial\xi} - \frac{1}{1-\xi^2}\right)\delta f_\alpha$$
$$= \xi\sqrt{1-\xi^2}V_{\alpha z}\left(3-\frac{v^2}{2v_T^2}\right)\frac{\nu(v)}{4v_T^2}f_m. \tag{3}$$

Solving Eq. (3), we find

$$\delta f_\alpha = \frac{1}{2v_T^2}\left(3-\frac{v^2}{2v_T^2}\right)V_{\alpha z}\frac{g(v,\xi)}{\sqrt{1-\xi^2}}. \tag{4}$$

For the function $g=g(v,\xi)$ we have the second-order ordinary differential equation

$$(1-\xi^2)\frac{d^2}{d\xi^2}g - i\gamma\xi g = -\xi(1-\xi^2), \tag{5}$$

with the boundary conditions $g(v,\xi=1)=g(v,\xi=-1)=0$. The solution of Eq. (5) depends on the value of the parameter $\gamma = 2klv^4/v_T^4$, where $l=v_T/\nu$ and $\nu=\nu(v_T)$. For values of γ much less than one, we find, by perturbation theory,

$$g \approx \frac{1}{6}(1-\xi^2)\left\{\xi - i\frac{\gamma}{12}(1+\xi^2) - \xi\frac{\gamma^2}{24}\left(\frac{1}{3}+\frac{1}{10}(1+\xi^2)\right)\right.$$
$$\left. + i\frac{\gamma^3}{1440}\left(\frac{13}{6}(1+\xi^2)+\frac{1}{5}(1+\xi^2+\xi^4)\right)\right\}. \tag{6}$$

In the limit of large values of γ, we use the Pade approximation and find

$$g \approx \frac{1}{i\gamma}(1-\xi^2)\left(1-\frac{2}{i\gamma\xi}\right) = \frac{\xi(1-\xi^2)}{2+i\gamma\xi}. \tag{7}$$

Expressions (6) and (7) can be used to determine the vortex current in the limiting cases of small and large values of the parameter kl. At anomalously small values of kl, we find the following expression, using solution (4), (6) and the expression for the current j_α:

$$j_\alpha = \frac{16i}{15\sqrt{2\pi}}\frac{en}{v_T}V_{\alpha z}kl\left(1-\frac{5}{6}(80kl)^2\right). \tag{8}$$

It can be seen from (8) that a solution can be constructed by perturbation theory under the condition $kl<0.01$. In this case the leading terms in the expansion in kl are small. For

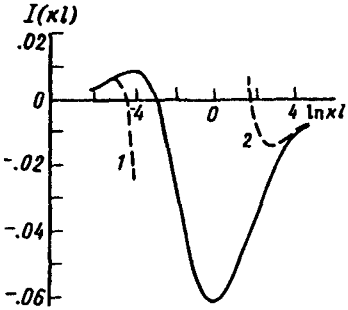

FIG. 1. The dimensionless vortex current $I(kl)$ as a function of the nonlocality parameter kl. Curves 1 and 2 correspond to the asymptotic analytic results $I_1(kl)$ in (8) and $I_2(kl)$ in (10).

$kl \ll 0.01$, expression (8) becomes an expression derived previously in a kinetic theory based on the Hilbert–Enskog–Chapman method.[3,5] Expression (8) changes sign at $kl \approx 0.014$, at which the approximation is no longer valid.

In the opposite limit, of large values of kl, we use solution (4), (7) and rewrite the current j_α as follows:

$$j_\alpha = -\frac{i}{\sqrt{8\pi}}\frac{en\nu}{kv_T^2}V_{\alpha z}\int_0^\infty \frac{dv}{v}\left(3-\frac{v^2}{2v_T^2}\right)$$
$$\times \exp\left(-\frac{v^2}{2v_T^2}\right)\left\{\frac{2}{3}+\left(\frac{v_c}{v}\right)^8-\left(\frac{v_c}{v}\right)^{12}\left(1+\left(\frac{v}{v_c}\right)^8\right)\arctan\left(\left(\frac{v}{v_c}\right)^4\right)\right\}, \quad (9)$$

where $v_c = v_T(kl)^{-1/4} \ll v_T$. The integration over velocity in (9) can be carried out approximately by breaking up the integration range into $v < v_*$ and $v > v_*$, where the velocity v_* satisfies the inequality $v_c \ll v_* \ll v_T$. For the current we then find

$$j_\alpha = -\frac{i}{\sqrt{8\pi}}\frac{en\nu}{kv_T^2}V_{\alpha z}\left(\frac{1}{2}\ln(4kl)-1-\mathrm{C}\right), \quad (10)$$

where $\mathrm{C}=0.577...$ is the Euler constant. The result in (10) is asymptotically exact at $kl \gg 10$. It corresponds to a monotonic decrease in the current with increasing kl.

In the transition region of values of the parameter kl, the solution of Eq. (5) and the subsequent calculation of the vortex current j_α were carried out numerically. These results of these calculations are shown in Fig. 1, which is a plot of the function

$$I(kl) = -i\int_0^\infty du\,u(3-u)e^{-u}\int_{-1}^{1} d\xi\, g(v_T\sqrt{2u},\xi),$$

which determines the current in accordance with

$$j_\alpha = \frac{i}{\sqrt{2\pi}}\frac{en}{v_T}V_{\alpha z}I(kl).$$

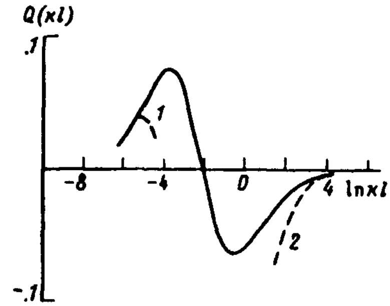

FIG. 2. Dimensionless vortex electron heat flux $Q(kl)$ as a function of the parameter kl. Curves 1 and 2 correspond to the functions $Q_1(kl)$ in (11) and $Q_2(kl)$ in (12).

Shown for comparison in Fig. 1 are plots of the functions $I_1(kl)=(16kl/15)[1-(5/6)\times(80kl)^2]$ and $I_2(kl)=-(1/4kl)[\ln(4kl)-2-2\,C]$, which correspond to analytic expressions (8) and (10). The asymptotic analytic results agree well with the numerical calculations at extremely small values of the nonlocality parameter ($kl<0.01$) and again at large values ($kl>10$). Over the wide transition region of values of the parameter kl in (1), the vortex current vanishes, and it changes the helicity direction at $kl\sim0.05$. This figure differs by a factor of about 4 from the value corresponding to Eq. (8). It is extremely interesting to note the anomalously large value of the maximum (in absolute value) value of the vortex current at the point $kl\approx0.9$, where it is six times the maximum current at small values $kl\approx0.02$.

We can also use the increment in the distribution function, δf_α in (4), to find the vortex electron heat flux q_α. Using relation (4) and the limiting analytic expressions for the functions $g(v,\xi)$ in (6) and (7), we find

$$q_\alpha=\frac{128}{15\sqrt{2\pi}}inmv_TV_{\alpha z}kl[1-(80kl)^2], \tag{11}$$

for $kl<0.01$ and

$$q_\alpha=-\frac{i}{3\sqrt{2\pi}}nmv_TV_{\alpha z}\frac{1}{kl}, \tag{12}$$

for $kl>10$. Expression (12) for the vortex flux, which was recently derived in Ref. 7, was found to be a very nonlocal expression. The heat flux along the direction of variation was also found in Ref. 7. According to Ref. 7, the nonpotential part of the heat flux is the reason why the gradient of the effective electron temperature is not collinear with the flux. It can be seen from a comparison of (11) and (12), which have different signs and magnitudes, that the degree of deviation from collinearity depends strongly on the nonlocality parameter kl. The latter point is particularly clear in Fig. 2, which shows the dimensional electron vortex heat flux $Q(kl)$ found numerically, from the expressions

$$q_\alpha=\frac{i}{\sqrt{2\pi}}nmv_TV_{\alpha z}Q(kl),$$

$$Q(kl) = -i\int_0^\infty duu^2(3-u)e^{-u}\int_{-1}^{1} d\xi g(v_T\sqrt{2u},\xi).$$

Figure 2 also shows values of $Q_1(kl) = (128kl/15)[1-(80kl)^2]$ and $Q_2(kl) = -1/3kl$, corresponding to the analytic results in (11) and (12). It can be seen from Fig. 2 that the analytic expressions are quite accurate at $kl < 0.01$ and $kl > 10$. In the transition region of values of the nonlocality parameter, the vortex heat flux vanishes at $kl \sim 0.1$ and has two extrema, differing in sign. These extreme values are far greater than the estimates corresponding to the asymptotic formulas for both small and large values of kl.

The analytic and numerical results presented here, from a study of the electron vortex fluxes which arise during inverse bremsstrahlung of an rf field, span the entire range of possible values of the nonlocality parameter kl. The behavior found here can be used to analyze the possibility of generating a magnetic field through excitation of a vortex current and also to analyze possible deviations of the heat flux from the direction in which the effective electron temperature falls off.

This study was carried out as part of Project 94-02-03631 of the Russian Fund for Fundamental Research.

[1] M. G. Hains, in *Inertial Confinement Fusion*, ed. by K. A. Brueckner (AIP, New York, 1992).
[2] I. B. Bernstein, C. E. Max, and J. J. Thomson, Phys. Fluids **21**, 905 (1978).
[3] I. P. Shkarofsky, Phys. Fluids **23**, 52 (1980).
[4] A. Sh. Abdullaev, Yu. M. Aliev, V. Yu. Bychenkov, and A. A. Frolov, Zh. Éksp. Teor. Fiz. **94**(3), 133 (1988) [Sov. Phys. JETP **67**, 507 (1988)].
[5] K. N. Ovchinnikov, V. P. Silin, and S. A. Uryupin, Fiz. Plazmy **17**, 1116 (1991) [Sov. J. Plasma Phys. **17**, 648 (1991)].
[6] V. I. Perel' and Ya. M. Pinskiĭ, Zh. Éksp. Teor. Fiz. **54**, 1889 (1968) [Sov. Phys. JETP **27**, 1014 (1968)].
[7] V. P. Silin, JETP Lett. **60**, 775 (1994).

Translated by D. Parsons