Change in the sign of the Josephson interaction in a controlled *SINIS* Josephson junction

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This letter analyzes an *SINIS* Josephson junction in which an auxiliary current flows through the N electrode. In the absence of a voltage across the S electrodes, there is therefore a voltage V across the N and S electrodes. A calculation of the Josephson current as a function of V shows that the critical current I_c decreases with increasing V and changes sign at $V \cong 0.71\ \Delta/e$. Methods for experimentally observing a transition of an *SINIS* structure from an ordinary Josephson junction to a π junction are discussed. © *1995 American Institute of Physics.*

Charge-transport processes in *SIN* and *SINIS* systems (here S,I, and N are a superconductor, an insulator, and a normal metal, respectively) at low temperatures have recently attracted much interest. Studies have shown that S/Sm or $S/Sm/S$ junctions formed by a superconductor S with a heavily doped semiconductor Sm exhibit a similar behavior. In this case the role of potential barrier (or insulating layer) is played by the Schottky barrier which often arises at an S/Sm interface. An unusual dependence of the differential conductance $G(V)$ at S/Sm junctions—different from that predicted by the standard theory of the *SIN* junction—was apparently first observed in Ref. 1. Specifically, as the temperature is lowered ($T<1$ K), the plot of $G(V)$ acquires a peak at $V=0$, and the height of this peak increases with decreasing T (this is the zero-bias anomaly). A peak in the conductance at $V=0$ was subsequently observed in other systems also.[2–5] The height of this peak is suppressed by a weak magnetic field H ($H<100$ Oe). The magnetic-field dependence of the conductance of *SINIS* systems (Fig. 1a), in which the superconductors are connected by a superconducting loop, was studied experimentally in Refs. 4c–6. It was observed that the resistance R of the system (i.e., the resistance between the N electrode and the S loop) oscillates as H is varied at small values of V, with an oscillation period corresponding to the magnetic flux quantum in the superconducting loop.

A theory for the conductance in systems of this sort at low temperatures and low voltages ($V<\Delta/e$) was derived in Refs. 4b and 7–12. According to the theory in Refs. 7–9, the conductance of *SIN* junctions under the conditions $(eV,T)\ll\Delta$ is due to an anomalous proximity effect and is associated with a component of the current which gives rise to the so-called interference current in the case of *SIS* junctions. This current component is determined by the product of the condensate functions of the N and S electrons (F_N and F_S). The quantity F_N is proportional to the transmission of the barrier, which is small in the case of a low transmission. As the energy ϵ ($\epsilon\cong\{eV,T\}$) decreases,

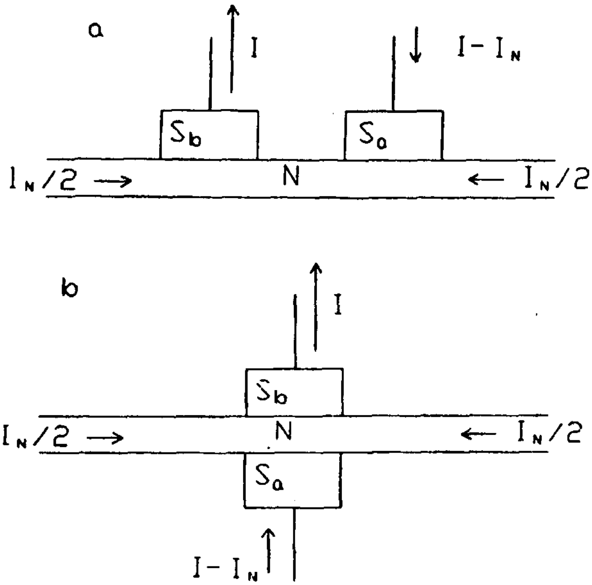

FIG. 1. Schematic diagram of *SINIS* junctions. The system in part a was used in an experimental study[5] carried out to measure the resistance between N and S films as a function of an applied magnetic field. In this case the superconductors were connected by a superconducting loop.

however, this quantity increases; it can reach values on the order of F_S ($|F_S|\cong 1$ under the condition $\epsilon \ll \Delta$). The oscillations in $R(H)$ in an *SINIS* system stem from a nonlocal relationship between F_N and $F_{Sa,b}$. As a result of this relationship, F_N depends on the cosine of the difference between the phases (ϕ) of the order parameter in superconductors S_a and S_b. The phase difference ϕ increases with increasing H, since a screened current arises in the superconducting circuit.

The inverse problem, i.e., the effect of the auxiliary current through the N electrode on the characteristics of a Josephson *SINIS* junction—was studied in Ref. 13 (Fig. 1, a and b). It was found, in particular, that for a low transmission of the barriers the critical current of the transition, I_c, can change sign if the voltage across the S and N electrodes, V, exceeds a certain value. In this letter we examine the transition from an ordinary junction ($I_c>0$) to a π junction ($I_c<0$), and we point out some possibilities for observing this transition.[2)]

Let us consider the system in Fig. 1b (our results also apply to the system in Fig. 1a if the distance between the superconductors is shorter than ξ_N). We assume that the impurity concentrations in the S and N films are high (the dirty limit), that the thickness of the N film is small in comparison with ξ_N, and that the width w is large in comparison with $\xi_S=\sqrt{D/\Delta}$. To calculate the current across the junction at low temperatures and low voltages ($T,eV \ll \Delta$), we need to find the condensate Green's functions in the N layer, $F^{R(A)}$ (for brevity, we omit the label N). Taking an average of the microscopic equations for the matrix Green's functions $\hat{G}$, and using the boundary conditions at the *SIN* junctions,[14] we find the equation

$$\epsilon_a[\hat{G},\hat{G}_a]_- + \epsilon_b[\hat{G},\hat{G}_b]_- + i\epsilon[\hat{\sigma}_z,\hat{G}]_- - (\gamma/2)[\hat{\sigma}_z\hat{G}\hat{\sigma}_z,\hat{G}]_- = 0. \tag{1}$$

Here the square brackets are commutators; $\epsilon_{a,b} = D/(2R_{a,b}\sigma d_N)$ are characteristic energies associated with the transmission of the *SIN* junctions; D, σ, and d_N are the diffusion coefficient, the conductivity, and the thickness, respectively, of the N film; $R_{a,b}$ are the resistances of the $S_{a,b}IN$ junctions per unit area; γ is the depairing parameter in the N film; and $\hat{G}^{R(A)}_{a,b}$ and $\hat{G}^{R(A)}$ are the matrix Green's functions in the superconductors $S_{a,b}$ and the N layer, respectively [we omit the labels $R(A)$ in (1)]; $\hat{G}^{R(A)} \equiv G^{R(A)}\hat{\sigma}_z + \hat{F}^{R(A)}$.

Equation (1) is solved in two limiting cases, in which the depairing parameter γ is respectively large and small in comparison with $\epsilon_{a,b}$ (Ref. 8; in an experiment[5] on an Al/Cu/Al junction, the values of γ and $\epsilon_{a,b}$ corresponded to frequencies of 10^{10} and $10^8\ \mathrm{s}^{-1}$, respectively). If the conditions $\epsilon_{a,b} \ll \gamma \ll \Delta$ hold (case 1), the condensate functions $F^{R(A)}$ are small in the N layer, and the spectrum there is gapless, although there are structural features at $\epsilon \cong \gamma$ and $\epsilon \cong \Delta$. Linearizing (1), we find

$$\hat{F}^{R(A)} = \pm i(\epsilon_a\hat{F}_a + \epsilon_b\hat{F}_b)^{R(A)}/(\epsilon \pm i\gamma). \tag{2}$$

Here $\hat{F}^{R(A)}_a = i\hat{\sigma}_y\Delta/\xi^{R(A)}_\epsilon$; $\hat{F}^{R(A)}_b = i(\hat{\sigma}_x\sin\phi + \hat{\sigma}_y\cos\phi)\Delta/\xi^{R(A)}_\epsilon$, $\xi^{R(A)}_\epsilon = ((\epsilon \pm i0)^2 - \Delta^2)^{1/2}$ are condensate Green's functions in superconductors S_a and S_b; and ϕ is the difference between the phases of the order parameter in S_a and S_b.

In the case of a low depairing rate ($\gamma \ll \epsilon_{a,b} \ll \Delta$; case 2), the condensate functions in the N film are not small. Ignoring γ in (1), we find

$$G^{R(A)} = \epsilon/(\epsilon - (m^{R(A)}_\epsilon)^2)^{1/2}, \quad F^{R(A)}_x = [m^{R(A)}_\epsilon/(\epsilon^2 - (m^{R(A)}_\epsilon)^2)^{1/2}\sin\vartheta,$$
$$F^{R(A)}_y = [m^{R(A)}_\epsilon/(\epsilon^2 - (m^{R(A)}_\epsilon)^2)^{1/2}]\cos\vartheta. \tag{3}$$

Here the $F^{R(A)}_{x,y}$ are the components of $\hat{F}^{R(A)}$: $\hat{F}^{R(A)} = i(\hat{\sigma}_x F_x + \hat{\sigma}_y F_y)^{R(A)}$; $m^{R(A)}_\epsilon = \epsilon_g(i\Delta/\xi^{R(A)}_\epsilon)$, $\epsilon_g = (\epsilon_a^2 + \epsilon_b^2 + 2\epsilon_a\epsilon_b\cos\phi)^{1/2}$ is the energy gap in the spectrum of excitation in the N layer, which depends on the phase difference ϕ; and the phase ϑ is related to ϕ by $\epsilon_g\sin\vartheta = \epsilon_b\sin\phi$.

The condensate current or the Josephson current across the S_aIN junction is found from the formula

$$I_{S,a} = -(w/16R_a)\mathrm{Tr}\hat{\sigma}_z\int d\epsilon\{(\hat{F}^R\hat{F}^R_a - \hat{F}^A\hat{F}^A_a)(f_0 + f_{0a})$$
$$+ (\hat{F}^R\hat{F}^A_a - \hat{F}^A\hat{F}^R_a)(f_0 - f_{0a})\}, \tag{4}$$

where w is the width of the S films, and f_0 and f_{0a} are the distribution functions in the N and S_a films. We assume that they are the equilibrium functions, assuming that the condition $\epsilon_{a,b} \ll D/w^2$ holds. Then we have $f_0 = [\tanh(\epsilon+V)\beta + \tanh(\epsilon-V)\beta]/2$ and $f_{0a} = \tanh(\epsilon\beta)$, where we have $\beta = (2T)^{-1}$, the charge e is incorporated in V, and V is the voltage across the N and S films. Substituting expressions (2) and (3) into (4), we find the following expression for the Josephson current I_S:

$$I_S(V,T) = I_c(V,T)\sin\phi. \tag{5}$$

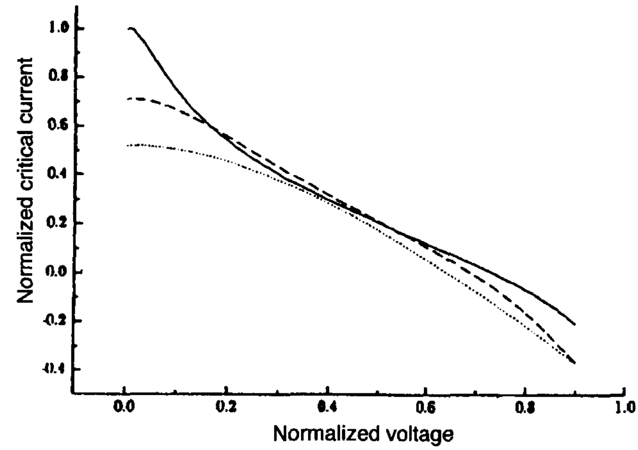

FIG. 2. Normalized critical current of the system, $I_c(V,T_i)/I_c(0,T_1)$, as a function of the normalized voltage eV/Δ at various temperatures ($i=1,2,3$). Solid curve—$\pi T_1/\Delta=0.01$; dashed curve—$\pi T_2/\Delta=0.3$; dotted curve—$\pi T_3/\Delta=0.6$. The depairing parameter is taken to be $\gamma=0.05\Delta$ and is assumed to be much larger than ϵ_0 (case 1).

We can write expressions for the critical current $I_c(V,T)$, which depends on the voltage V, and the temperature, assuming that the temperature is low ($T\ll\gamma$ in case 1 and $T\ll\epsilon_g$ in case 2):

$$I_c(V,0)R\cong 2\epsilon_0\begin{cases}(1/2)\ln[(\Delta^2-V^2)/(V^2+\gamma^2)]+(\pi/2)\gamma/\Delta, & \epsilon_0\ll\gamma\ll\Delta\\ \ln(2\Delta/\epsilon_g)-L(V), & \gamma\ll\epsilon_0\ll\Delta\end{cases}. \tag{6}$$

Here $R=R_a+R_b$ is the total resistance of the system; $\epsilon_0=(\epsilon_a+\epsilon_b)/2$; and the function $L(V)$ is defined by the expressions $L(V)=0$ at $V<\epsilon_g$, $L(V)=\ln[2\Delta V/\epsilon_g(\Delta^2-V^2)^{1/2}$ at $\epsilon_g\ll V<\Delta$, and $L(V)=\ln(V+(V^2-\epsilon_g^2)^{1/2}/\epsilon_g)$ at $V\ll\Delta$. It follows from (5) that the Josephson currents across the S_aIN and NIS_b junctions are equal. The order of magnitude of the critical current I_c is the same for the gapless case (1) and for the case with a nonzero energy gap in the N film, (2). The curves of $I_c(V,0)$ for these cases differ only at low voltages. At $V\gg\gamma$, ϵ_g, these curves are nearly identical. It can be seen from (6) that the critical current $I_c(V,0)$ changes sign, and the *SINIS* system converts into a Josephson π junction if the voltage V exceeds the value $V_0\cong\Delta/\sqrt{2}\cong0.71\Delta$.

As the temperature T is raised, the critical current $I_c(V,T)$ decreases. For example, under the conditions $\epsilon_g,\gamma\ll T\ll\Delta$, we find $I_c(0,T)R\cong 2\epsilon_0[\ln(2\Delta/\pi T)+C]$, where $C=0.577...$ is the Euler constant. As the voltage V increases ($\epsilon_g,\gamma\ll V$), however, and under the condition on the temperature specified above, we find $I_c(V,T)R\cong 2\epsilon_0\ln[(\Delta^2-V^2)^{1/2}/V]$. This result means that at $T\ll\Delta$ the voltage (V_0) at which I_c changes sign depends only weakly on the temperature. Figure 2 shows $I_c(V,T)$ versus V for case 1 for several temperatures.

If the width of the S films, w, is not large in comparison with $\xi_\Delta=\sqrt{D/\Delta}$, then the proximity effect weakens, and the expressions for $F^{R(A)}$ change. In the region of overlap of the films, we then find the following result for the gapless case, in place of (2):

$$\hat{F}^R=(\hat{F}^A)^*=(w/\sqrt{2D})[\epsilon_a\hat{F}_a^R+\epsilon_b\hat{F}_b^R]/(-i\epsilon+\gamma)^{1/2}. \tag{7}$$

It can be shown that again in this case the critical current vanishes if the voltage on the central electrode exceeds the value V_0, which is determined by the following condition at $T=0$:

$$\pi-\arctan g\sqrt{V_0/\Delta}+0.5\ln[(1-\sqrt{V_0/\Delta})/(1+\sqrt{V_0/\Delta})]=0. \tag{8}$$

We thus find $V_0/\Delta\cong0.96$. In other words, the voltage on the central electrode in this case must be very close to Δ.

The possibility of experimentally observing a negative Josephson current has been discussed in many papers (see, e.g., Ref. 15 and the papers cited there). In a superconducting loop containing a π junction, for example, a spontaneous circulating current can arise, along with the related magnetic flux, if the circuit inductance L is sufficiently large: $L>L_0\cong\hbar c^2/2eI_c$. This condition may be diff ·ult to arrange in an *SINIS* system, since the critical current I_c is small in this case. Apparently a more suitable system for observing a change in the sign of I_c is a two-junction SQUID with one ordinary junction (I_{c2}) and one π junction (I_{c1}). The maximum superconducting current of the SQUID is then $I_m=|I_{c1}|[1+r^2\pm2r\ \cos(2\pi\cdot\Phi_{ex}/\Phi_0)]^{1/2}$ with I_{c1} positive (negative), where $\Phi_{ex}=H_{ex}S$ is the flux of the external magnetic field, S is the area of the loop, and $r=I_{c2}/|I_{c1}|$ (we are ignoring the magnetic flux of the screening currents, assuming $L\ll L_0$). The quantity I_m then reaches a maximum (minimum) at $H=0$ in the case of the ordinary junction (the π junction).

There is another way to determine the type of Josephson junction (a π junction or an ordinary junction): by measuring the critical current I_c as a function of the voltage V across the film and the S electrodes. At $T<\Delta$, we should observe a nonmonotonic dependence of the critical current I_c on V: The current I_c should vanish at $V\cong V_0$ and rise again at $V>V_0$. The Joule heating of the junction should be at a minimum.

In addition, measurements of the resistance (or conductance) between the N and S electrodes as a function of the superconducting current $I=I_c\sin\phi$ would also make it possible to determine the sign of I_c. If the barriers have a low transmission, the conductance measured between the N and S films at $T=0$ is[16]

$$G=\mathrm{Re}\{[1-\sinh\alpha/\alpha+2\sinh^2(\alpha/2)/\alpha]+(\sinh^2(\alpha/2)/\alpha)\exp(-\alpha-kL)\cos\phi\}/k^2, \tag{9}$$

where $\alpha=kw$ and $k^2=2(-ieV+\gamma)/D$. The difference between the phases is associated with the condensate current $I=I_c\sin\phi$. In the case of an ordinary junction we would have $\phi=\arcsin(I/I_c)$ and thus $\cos\phi=(1-(I/I_c)^2)^{1/2}$; in the case of a π junction we would have $\phi=\pi+\arcsin(I/I_c)$ and thus $\cos\phi=-(1-(I/I_c)^2)^{1/2}$. Consequently, the sign of the second term changes at $V=V_0$.

The depairing parameter is determined by paramagnetic impurities and by the condensate velocity v_s: $\gamma=\tau^{-1}+Dp_s^2$, where τ^{-1} is the rate of scattering by magnetic impurities with spin flip, and $p_s=mv_s$ is the condensate momentum. For example, in a magnetic field we would have $p_s=\lambda H/\Phi_0$, where λ is the London depth of the superconductor. In addition, the depairing rate is determined by inelastic scattering processes. In this case, γ depends on the energy ϵ, and the γ dependence of F^R is more complex.

However, the conclusions remain qualitatively the same as in the case in which inelastic relaxation is predominant. We then have $\gamma=\langle\tau_\epsilon^{-1}\rangle$, where $\langle\tau_\epsilon^{-1}\rangle$ is an average rate of inelastic relaxation.

The method discussed here for producing a π junction differs from methods which have been proposed previously in that this new method makes it possible to switch from an ordinary junction to a π junction by changing the voltage on the N film with respect to the S films.

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[2)]Some results of this study were reported in Ref. 17.

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