Microscopic oscillations of persistent currents in a metal ring with a magnetic impurity (the Kondo case)

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When there is a low concentration of magnetic impurities (the Kondo case), there are oscillations in the persistent current in a metal ring with a fractional period Φ_0/N (N is the number of electrons in the ring). These oscillations are characteristic of highly correlated electron systems in which charge and spin degrees of freedom are separated. The interaction with impurities gives rise to an effective interaction between electrons, which is manifested in the appearance of fractional microscopic oscillations.

Recent years have seen increasing research interest in persistent currents in mesoscopic metallic and semiconducting rings. The primary reason for this interest has been the report of experimental studies of these persistent currents.[1–3] These experiments stimulated a series of theoretical papers, which have examined, in particular, the persistent currents in interacting electron systems.[4–10] Persistent currents in multiparticle systems are manifestations of the Aharonov–Bohm and Aharonov–Casher effects, which are quite familiar for particles.[11,12] In other words, there is a change in the phase of the wave function of charged particles (particles with a magnetic moment) as these particles loop a magnetic (or electrical) topological defect, e.g., an infinite solenoid or charge filament. The interaction of electrons in (for example) the Hubbard model[4–7] is manifested by changes in the amplitude, the phase, and even the period of oscillations of the persistent currents in comparison with those in noninteracting systems. One of the basic manifestations of the interaction, for example, is the absence of mesoscopic oscillations, on the order of L^{-1} (L is the number of sites in the ring) at half-filling of the band, corresponding to the insulating phase. Persistent currents are thus a good signal of a metal–insulator phase transition[13] in highly correlated electron systems. The amplitude of the persistent currents in the metallic phase is manifested as finite-size corrections (L^{-1}) and is related to the exponents of the correlation functions of the gapless excitations at large distances.[14,15] They are found by conformal field theory.[16–18]

Mesoscopic oscillations in persistent currents in correlated electron systems have been studied by the Bethe-ansatz method[4–7] and also by a bosonization method.[8] In a Hubbard ring with a repulsion of electrons at sites, these mesoscopic oscillations have a fundamental period $\Phi_0 = hc/e$ (c is the velocity of light, and e the charge of an electron). Alternatively, depending on the number of electrons and their magnetic moment, they have a period $\Phi_0/2$, as the result of a level crossing.[7] When the filling of the band is only slight, $N \ll L$ (N is the number of electrons), or in the case of a strong Hubbard repulsion, $U \gg t$, where t is a hopping integral, there can be so-called microscopic oscillations with a fractional period[7,19,20] Φ_0/N. While the mesoscopic oscillations are associated with the motion of one electron in the ground state (the virtual transfer of excitations from one Fermi point to another), and the result is a fundamental periodicity, the microscopic oscillations are associated with the motion of all the electrons, as a whole, in the ring. These oscillations are possible only in systems in which spin and charge are separated. If the magnetic flux increases the quasimomentum of charge excitations (holons), it can be reduced as a result of the creation of spin excitations (spinons). At large values of U, the disadvantage in terms of energy is slight (t^2/U), so there is a quasiperiodicity in terms of the flux Φ, with a period Φ_0/N (Refs. 7, 19, and 20).

It was recently shown[21,22] that in a metal ring with a magnetic impurity of the $s-d$ type (the Kondo situation) or of the Anderson type, in which the presence of magnetic impurities (even a small number thereof) leads to an effective interaction between "bulk" electrons, this interaction is not manifested in the thermodynamic characteristics of the bulk electrons.[23–25] It is, however, manifested in mesoscopic, finite-size corrections (or, correspondingly, in elementary excitations), i.e., in oscillations of persistent currents.[21,22]

In this letter we calculate microscopic oscillations with a period Φ_0/N in a metal ring with $s-d$ impurities (N^i is the number of impurities). We show that in this system there are oscillations of the persistent currents with a fractional period Φ_0/N in the ground state. In contrast with the Hubbard model, these oscillations are manifested at arbitrary values of the electron–impurity interaction constant, but at a low impurity concentration, $N^i \ll N$.

Let us consider the equations of the Bethe ansatz for N electrons (M of which have spin down) in the presence of N^i impurities, which do not interact directly with each other (we are adopting the notation of a review[23]). The magnetic flux Φ changes the periodic boundary conditions into "twisted" conditions[21] in the equations of the ansatz for quantum numbers which parametrize the eigenfunctions and eigenvalues of the Hamiltonian of the system:[22–24]

$$Lk_j^e = 2\pi(n_j + \Phi/\Phi_0) - \sum_{\gamma=1}^{M} [\pi - \theta(2\Lambda_\gamma - 2)] - N^i\phi, \tag{1}$$

$$N\theta(2\Lambda_\gamma - 2) + N^i\theta(2\Lambda_\gamma) = \sum_{\gamma=1}^{M} \theta(\Lambda_\gamma - \Lambda_\delta) - 2\pi I_\gamma, \quad \gamma = 1,\dots,M. \tag{2}$$

Here n_j and I_γ are either integers or half-integers, depending on whether N and $N-M$ are even or odd; $\theta(x) = -2\arctan(x/c)$; $c = 2J/(1-J^2)$ is an effective coupling constant; and $\phi = 2\pi + \theta(1+J^2)$. It can be seen from Eqs. (1) and (2) that an increase in the flux Φ can be offset by a change in the numbers I_γ or, equivalently, by the creation of spin excitations in the ring under consideration or by a change in their speeds. The disadvantage in terms of energy is seen from (2) to be proportional to the impurity concentration N^i/N. The energy of this system, $E = \Sigma_{j=1}^{N} k_j^e$, is given by the following expression, since the spin speeds satisfy Eq. (2):

$$E = (2\pi/L)\left\{ \sum_{j=1}^{N} n_j + N\Phi/\Phi_0 - \sum_{\gamma=1}^{M} I_\gamma - MN/2 - (N^i/2\pi)\left[\sum_{\gamma=1}^{M} \theta(2\Lambda_\gamma) + N\phi \right]\right\}.$$

At a low impurity concentration, $N^i \ll N$, we can ignore the M terms, which are proportional to N^i. A continuous increase in the magnetic flux can be "cancelled" by the discrete formation of spin excitations (by a change in the speeds of the spinons), with the result that there is an oscillatory dependence of the energy of the "sonov" state with a fractional period $N^{-1}\Phi_0$. In other words, we have, for example, $\Sigma_{\gamma=1}^{M} I_\gamma = p$ for $(2p-I)\Phi_0/2N < \Phi < (2p+I)\Phi_0/2N$, etc. Mesoscopic oscillations with a fundamental period[21] Φ_0 of course always occur and are manifested most prominently at high concentrations of magnetic impurities, as can be seen from this analysis.

The reason why the presence of microscopic oscillations does not depend on the exchange constant in this model is that the charge degrees of freedom of the electrons are noninteracting. This situation is of course determined by the nature of the $s-d$ interaction of electrons with impurities, which leads to a complete separation of the spin and charge excitations in the system. This property of course does not prevail in a study of microscopic oscillations in persistent currents in the impurity Anderson model. The results for that model, like the results of a study of microscopic oscillations of persistent spin currents with an electric-field flux in impure metal rings, will be published later.

In summary, we have shown in this letter that microscopic oscillations with a fractional period Φ_0/N should occur in the persistent currents in a metal ring with a small number of magnetic impurities (the Kondo case). These oscillations are generated by a virtual creation of spinons in the system (or by a change in the speeds of spinons) and are associated with the motion of N bulk electrons as a whole, in contrast with the mesoscopic oscillations (with a period Φ_0), which are associated with the motion of an individual electron (or excitation). These oscillations are a manifestation of an effective interaction between electrons in the system, caused by the interaction of electrons with impurities, even if the latter are present in only a very low concentration.

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