Macroscopic model of magnetic dipole resonance in spherical nuclei

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A macroscopic mechanism for nuclear magnetization associated with a magnetic dipole resonance is analyzed. The giant $M1$ resonance is shown to be a consequence of oscillations excited in the magnetization current in a peripheral layer of finite depth. The collective model discussed here gives a satisfactory description of the empirical behavior of the energy position of the peak as a function of the mass number: $E(M1)=40.8A^{-1/3}$ Mev. The model predicts that the resultant branching ratio will depend on the atomic number and mass number in accordance with $B(M1)=8.5\times 10^{-2}Z^2A^{-2/3}\mu_N^2$. © *1995 American Institute of Physics.*

The most reliable sources of information on the giant magnetic dipole resonance are experiments on large-angle inelastic scattering and measurements of the cross section for the elastic scattering of photons by the method of resonant fluorescence.[1–4] An empirical expression[3,4] is available for the peak energy of the $M1$ resonance as a function of the number of nucleons in the nucleus: $E(M1)\approx 41A^{-1/3}$ MeV. The data which have been accumulated (Table I) indicate a systematic increase in the total branching ratio for the excitation of this resonance, $B(M1)$, with increasing mass number. In an effort to find an analytic expression for $B(M1)$ as a function of Z and A, we consider a collective model of a magnetic dipole resonance, working from the results of Refs. 5–7. There, giant magnetic resonances of multipolarity $\lambda\geqslant 2$ were treated as a manifestation of shear (torsional) vibrations within the framework of the theory of continuous media. That macroscopic interpretation is based on a picture of the nucleus as a spherical particle of an elastic Fermi continuum, as introduced in Ref. 8. We should stress that the standard liquid-drop model completely forbids collective excitations of a magnetic type: excited states with a nonzero magnetic multipole moment. The collective model of a torsional magnetic response of a nucleus which was developed in Refs. 5–7 gives a satisfactory description of the data which have been accumulated on the giant $M2$ resonance. That model generates concrete predictions for the energy position of the peak and the total branching ratio of giant magnetic resonances of higher multipolarities. The literature reveals no study of a macroscopic mechanism for dipole magnetization of a nucleus which leads to an $M1$ resonance.

Here are the physical assumptions underlying the macroscopic approach to this problem which we are discussing here. A heavy nucleus of radius $R=r_0A^{1/3}$ is modeled as a spherical macroscopic particle of incompressible nuclear matter with homogeneous

TABLE I. Energy position of the peak, $E(M1)$, and total branching ratio $B(M1)$ for the excitation of a magnetic dipole resonance. Experimental data from Ref. 4.

Nucleus	Experimental $E(M1)$, MeV	Experimental $B(M1)\uparrow$, μ_N^2	Model $E(M1)$, MeV	Model $B(M1)\uparrow$, μ_N^2
^{90}Zr	9.1	6.7	9.1	6.8
^{120}Sn	8.3	8.8	8.3	8.8
^{140}Ce	7.9	7.5	7.9	10.6
^{206}Pb	7.5	19.0	6.9	16.4
^{208}Pb	7.3	17.5	6.9	16.3

distributions of the charge density $n_e = e(Z/A)n_0$ and the mass density $\rho_0 = mn_0$, where m is the mass of a nucleon, and $n_0 = (2/3\pi^2)k_F^3$ is the number density of particles. The dynamics of collective motions of nucleons is formulated in terms of the theory of continuous media and is described by the equations of an elastic continuum.[7] Giant resonances are associated with the excitation of fluctuations of the electric current in the interior of the nucleus:

$$\mathbf{j} = n_e \delta \mathbf{V}. \tag{1}$$

The velocity field of the latter, $\delta\mathbf{V}(\mathbf{r},t) = \mathbf{a}(\mathbf{r})\dot{\phi}(t)$[$\mathbf{a}(\mathbf{r})$ is the field of instantaneous displacements], is found as the solution of the vector Laplace equation[9]

$$\Delta \delta \mathbf{V} = 0, \quad \mathrm{div}\, \delta \mathbf{V} = 0. \tag{2}$$

A solution corresponding to giant magnetic resonances is specified by a toroidal rotational field, the explicit expression for which is the same as that for the velocity field of shear torsional vibrations of a spherical particle of elastic matter.[10] The frequencies of these oscillations are found as the natural modes of the oscillator Hamiltonian

$$H = \frac{J\dot{\phi}^2}{2} + \frac{K\phi^2}{2}. \tag{3}$$

The torsional moment of inertia J and the stiffness of the torsional vibrations, K, are given by

$$J = \int \rho_0 a_i a_i d\tau, \quad K = \frac{1}{2}\int \mu \left(\frac{\partial a_i}{\partial x_j} + \frac{\partial a_j}{\partial x_i}\right)^2 d\tau, \tag{4}$$

where $\mu = 1/5\rho_0 v_F^2$ is the shear modulus, and $v_F = (\hbar/2mr_0)(9\pi)^{1/3}$ is the limiting velocity of the Fermi motion of the nucleons. The details of the analytic calculations of J and K are given in Ref. 7. From the microscopic standpoint, the restoring force of the elastic response is interpreted as the response of a finite Fermi system to anisotropic distortions of the Fermi sphere. Following Refs. 5–7, we consider the excitation of the $M1$ resonance as a transition from a ground state to a "magnetized" excited state characterized by a nonzero magnetic dipole moment,

$$\mathcal{M}(M1) = -\frac{1}{2c}\int \mathbf{j}\cdot[\mathbf{r}\times\nabla] r P_1(\cos\theta) d\tau, \tag{5}$$

which is caused by the excitation of oscillations of a charged collective flux which are induced by inelastically scattered electrons or by elastically scattered photons. Here and below, c is the velocity of light. The energy position of the peak, and the total branching ratio for the excitation of the 1^+ resonance are calculated from

$$E(M1) = \hbar\sqrt{\frac{K}{J}},\quad B(M1) = 3\langle|\mathcal{M}(M1)|^2\rangle_t, \tag{6}$$

where $\langle ...\rangle_t$ means a time average. In particular, we have $\langle\dot{\phi}^2\rangle_t = 1/2\phi_0^2\omega^2$, where $\phi_0 = (\hbar\omega/2K)^{1/4}$ is the frequency of zero-point vibrations.[11] The problem of finding integral characteristic parameters of the $M\lambda$ resonance thus reduces to one of solving the Laplace equation for the velocity field and using this field to evaluate integrals which determine the torsional moment of inertia, the stiffness, and the collective magnetic moment.

In the discussion below, we work from the following results of the nuclear shell model. In this model, a magnetic dipole resonance is interpreted as the result of transitions, induced by some external perturbation, across the Fermi surface between states with large orbital angular momenta l which are spin–orbit partners $(j_1 = l\pm s \to j_2 = l\mp s)$. From the shell model we also know that probable position of a nucleon shifts away from the center toward the surface with increasing angular momentum: With increasing value of l, the nucleon moves closer to the nuclear surface. The latter circumstance indicates that a giant magnetic resonance is formed by coherent motions of nucleons which are localized primarily in a peripheral layer of finite depth, not throughout the nuclear volume. This observation may be reflected in the following macroscopic picture. The excitation of a giant $M1$ resonance by inelastically scattered electrons or by elastically scattered photons causes a dynamic dissociation of the nucleus. This dissociation is accompanied by oscillations of a solenoidal current excited in the outer layer, while the inner region, of radius R_c, is not affected by the perturbation. The distributions of charge and mass remain the same over the entire volume of the nucleus. We wish to stress that this stratification of the nucleus into a mobile layer and an inert core is a dynamic effect; i.e., it arises and exists only during excitation of the nucleus. This model should therefore be thought of as merely a semiclassical method for describing a dynamic dipole magnetization of a nucleus associated with a giant $M1$ resonance.

The velocity field corresponding to this picture can be determined unambiguously by specifying the following boundary conditions: 1) The surface of the nucleus undergoes solid-like rotational oscillations,

$$\delta\mathbf{V} = [\mathbf{r}\times\mathbf{\Omega}_0(t)]_{r=R}, \tag{7}$$

where $\mathbf{\Omega}_0(t) = \mathbf{e}_z\dot{\phi}(t)$ is a sinusoidal function of the time. 2) At the inner effective boundary of the unperturbed region we have

$$\delta\mathbf{V} = 0|_{r=R_c}. \tag{8}$$

As a result, we find a rotational velocity field

$$\delta \mathbf{V}=\text{rot } \mathbf{r}\left[\Gamma\left(\frac{r}{R_c^3}-\frac{1}{r^2}\right)\right]\sin\theta\, \dot{\phi}(t), \quad \Gamma=\frac{R^3R_c^3}{R^3-R_c^3}, \tag{9}$$

which is an exact solution of Eq. (2) under boundary conditions (7) and (8).

In a spherical coordinate system with a fixed polar axis, the components of the instantaneous-displacement field are

$$a_r=0, \quad a_\theta=0, \quad a_\phi=\Gamma\left(\frac{r}{R_c^3}-\frac{1}{r^2}\right)\sin\theta. \tag{10}$$

Since the charge density does not change, we find, by substituting (9) into expression (1) for the current density, that the latter reduces to the standard expression for the magnetization current:

$$\mathbf{j}=c \text{ rot } \mathbf{M}, \tag{11}$$

with the magnetization field

$$\mathbf{M}(\mathbf{r},t)=\mathbf{r}\left[\frac{2m}{\hbar} n_e \Gamma\left(\frac{r}{R_c^3}-\frac{1}{r^2}\right)\cos\theta\right]\dot{\phi}(t)\mu_N. \tag{12}$$

Here μ_N is the nuclear magneton. Expression (11) is analogous to the quantum-mechanical representation of the magnetization current, which, according to microscopic calculations, dominates the excitation of the magnetic dipole resonance (see a review[4] and the papers cited there). We believe that this analogy indicates a correspondence between the conclusions reached on the basis of the macroscopic model of an $M1$ resonance under consideration, on the one hand, and the conclusions of microscopic theories, on the other.

The results of the calculations are more conveniently expressed in terms of the geometric parameter $x=R_c/R$, which characterizes the amount of mass (ΔM) which participates in forming the giant 1^+ resonance: $\Delta M=M-M_c=M(1-x^3)$, where $M=(4\pi/3)\rho_0 R^3$ is the total mass of the nucleus, and $M_c=(4\pi/3)\rho_0 R_c^3$ is that part of the nuclear mass which is not affected by the perturbation. Substituting (10) into (4), we find the following expression for the stiffness of the shear vibrations:

$$K=\frac{8\pi}{5}\rho_0 v_F^2 R^3 \frac{x^3}{1-x^3}. \tag{13}$$

The moment of inertia of dipole torsional vibrations can be written

$$J=\frac{8\pi}{15}\rho_0 R^5\left[\frac{1-5x^3+9x^5-5x^6}{(1-x^3)^2}\right]. \tag{14}$$

For $x=0$, the latter expression reduces to the moment of inertia of a solid sphere, $J_0=2/5MR^2$, and the stiffness coefficient vanishes identically. The latter circumstance clearly demonstrates that a macroscopic description of the dynamics of the giant $M1$

resonance is achieved only by virtue of the assumption of a dynamic dissociation of the nucleus into an outer layer, in which oscillations of the magnetic current are excited, and an inner region which is inert with respect to the perturbation.

The collective model which we are discussing here leads to the following analytic estimates for the position of the energy centroid and the total branching ratio for the $M1$ resonance as a function of the mass number and the atomic number:

$$E(M1)=\kappa A^{-1/3}\mathrm{MeV},\ B(M1)=\gamma\ Z^2A^{-2/3}\mu_N^2, \tag{15}$$

$$\kappa=\frac{\hbar^2(9\pi)^{1/3}}{2mr_0^2}\left[\frac{3x^3(1-x^3)}{1-5x^3+9x^5-5x^6}\right]^{1/2}, \tag{16}$$

$$\gamma=\frac{9(9\pi)^{1/3}}{80\pi}\sqrt{\frac{3x^3(1-x^3)}{(1-5x^3+9x^5-5x^6)^3}}\left[1-\frac{5}{2}x^3+\frac{3}{2}x^5\right]^2. \tag{17}$$

It follows from (15) and (17) that for a fixed value of x the model gives a correct description of the experimentally observed $A^{-1/3}$ dependence of the energy position of the peak of the $M1$ resonance as a function of the mass number. This circumstance may also be thought of as a reflection of the fact that the collective dynamics of the magnetic dipole resonance develops in an identical way in all intermediate-weight and heavy nuclei of the periodic table. The calculations of the energies and branching ratios shown in Table I were carried out for the same parameter value $x=0.53$, which was set on the basis of the position of the peak energy of the $M1$ resonance in ^{90}Zr ($r_0=1.15$ fm). For this value of x, about 85% of the total number of nucleons become involved in the dynamics of the magnetic dipole resonance. This result is evidence that the magnetic dipole resonance is substantially a bulk collective excitation. This is a common property of giant nuclear resonances, both electric and magnetic.[9]

The primary prediction of this collective model is an estimate of the total branching ratio for a magnetic dipole resonance, $B(M1)$, as a function of the atomic number Z and the mass number A. Here are the final expressions for the peak energy and of the total branching ratio for the magnetic dipole resonance:

$$E(M1)=40.8\ A^{-1/3}\mathrm{MeV},\ B(M1)=8.5\times10^{-2}\ Z^2A^{-2/3}\mu_N^2.$$

The results calculated for the energy and branching ratio of the magnetic dipole resonance are systematically compared with the existing data in Table I. This comparison reveals that the model satisfactorily conveys the observed tendency for the energy of this resonance to shift in the low-energy direction as we go from intermediate-weight to heavy nuclei, with a simultaneous increase in the total branching ratio.

Our interest was drawn to this problem by some experiments recently undertaken at a linear electron accelerator in Darmstadt (the S-DALINAC installation) on the excitation of a magnetic dipole resonance in inelastic scattering of electrons through 180°. The purpose of those experiments is specifically to identify systematic trends in the behavior of integral characteristic parameters of giant $M1$ resonance (primarily the energy and branching ratio) as a function of the mass number. Accordingly, it can be hoped that the predictions offered in the present letter can be tested experimentally in the very near future.

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