# Femtosecond parametric excitation of electromagnetic field in a cavity

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The method of parametric excitation of electromagnetic waves in a cavity (particularly from the vacuum state) is discussed. Parametric excitation of electromagnetic waves is accomplished creating a dense plasma layer which represents the "mobile" wall of the cavity. The mobile plasma layer can be produced by irradiating a semiconductor film with femtosecond laser pulses. Parametrically excited radiation can be resolved from radiation of another origin by analysis of its squeezing, specific angle distribution, and by time-resolved recording. © *1995 American Institute of Physics.*

## 1. INTRODUCTION

We discuss parametric excitation of electromagnetic waves in a cavity (in particular, from the initial vacuum state) by using power femtosecond laser pulses. The idea is to create a dense electron-hole $(e-h)$ plasma in a thin semiconductor film at small time intervals by irradiating it with power femtosecond laser pulses. The surface of such a plasma may be considered as the mobile wall of the cavity. This plasma layer and the fixed metal mirror represent a cavity with one mobile wall (see Fig. 1). The effect of parametric excitation of electromagnetic field due to the motion of one of the cavity walls is essential for an ultrashort time $(\tau)$ of displacement of the mobile wall. The use of femtosecond laser pulses in the proposed experiment is therefore justifiable. We consider the new regime of parametric excitation when $\tau$ is comparable to the time of propagating of light from one wall of the cavity to another: $\tau \sim \frac{L}{c}$ where $L$ is the length of the cavity.[2)] This regime is opposite to the case $\tau \gg \frac{L}{c}$ which was considered before (see, e.g., Refs. 2–4 and the references cited there).

In this case no stationary picture of the distribution of electromagnetic energy in the cavity exists. It appears that the value of the excited energy $\bar{E}$ in the cavity depends on the total history of the process, specifically, the total displacement of the mobile wall of the cavity.

In the present article we calculate in the framework of quantum electrodynamics in the cavity the parametric excitation of the electromagnetic field, in particular, from the vacuum state in the case of instantaneous excitation and also slow, adiabatic excitation. The classic approach is also considered. The angular distribution and the quantum statistical properties (squeezing, etc.) of parametrically excited radiation are described.

  

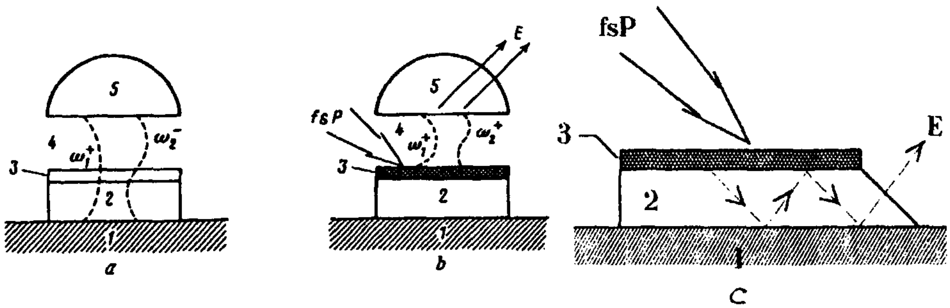


FIG. 1. Emission of cavity modes due to the parametric excitation: a) before excitation, b) after excitation; *1*—metal layer; *2*—dielectric slab; *3*—semiconductor film, *4*—gap; *5*—dielectric prism; $\omega_1^-$, $\omega_2^-$ ($\omega_1^+$, $\omega_2^+$) are vacuum cavity modes before (after) excitation; $fsP$ is exciting femtosecond laser pulse; $E$ is the light radiation due to the parametric excitation, c) another realization.

## 2. BASIC RELATIONS

Let us consider the electromagnetic field in a rectangular cavity with a time-dependent length $L_z = L_z(t)$, but $L_x$=const and $L_y$=const. Let us assume that the quality $Q$ of the cavity is infinitely high. [We also assume that after times $t < t_0$ and $t > t_0 + \tau$ the wall is at rest, and that at $t_0 < t < t_0 + \tau$ the velocity $L_z < 0$, i.e., the size of the cavity is smaller. Note that the phenomena considered below take place in the case of a general dependence $L_z(t)$. This leads to simple boundary conditions for those cavity modes whose vector **E** is parallel to the $xy$ plane (see also Ref. 2): $\mathbf{A}|_{z=L_z(t)} = 0$, where **A** is the vector potential.

The expression for the energy in the cavity with a time-dependent length $L_z(t)$ in the case div **A**=0 is

$$E = \frac{L_x L_y}{4\pi} \sum_q \int_0^{L_z(t)} \left( |\dot{\mathbf{a}}_\mathbf{q}|^2 + \omega_q^2 |\mathbf{a}_\mathbf{q}|^2 + c^2 \left| \frac{\partial \mathbf{a}_\mathbf{q}}{\partial z} \right|^2 \right) dz - c^2 \frac{L_x L_y}{4\pi} \sum_q \frac{\partial |\mathbf{a}_\mathbf{q}|^2}{\partial z} \Big|_{z=0}^{z=L_z(t)}, \tag{1}$$

where $\mathbf{a}_\mathbf{q}$ are the 2D Fourier transforms of **A** in the $xy$ variables and the vector $\mathbf{q} = (k_x, k_y)$.

To determine the time-dependent boundary conditions, it is natural to use the variables $u = \xi(t) z$, where $\xi(t) = L_z^- / L_z(t)$. In these variables the wave equation for the components $\mathbf{b}_\mathbf{q}(u,t) = \mathbf{a}_\mathbf{q}(z,t)$ of the vector potential **A** on the condition div **A**=0 takes the form

$$\hat{A}\hat{B}\mathbf{b}_\mathbf{q} = -\omega_q^2 \mathbf{b}_\mathbf{q}, \tag{2}$$

where

$$\hat{A} = \frac{\partial}{\partial t} + [u\eta(t) + c\xi(t)] \frac{\partial}{\partial u}; \quad \hat{B} = \frac{\partial}{\partial t} + [u\eta(t) - c\xi(t)] \frac{\partial}{\partial u} \quad \text{and} \quad \eta(t) = \frac{\dot{\xi}}{\xi}.$$

## 3. ONE-DIMENSIONAL CASE $|\mathbf{q}|\ll k_z$

In the case $|\mathbf{q}|\ll k_z$ Eq. (2) can be transformed to the form $\hat{A}\hat{B}b=0$, and therefore can be reduced to the first-order equations in partial derivatives $\hat{A}b=0$ and $\hat{B}b=0$. For the modes with $\omega\ll 1/\tau$ we find

$$b(u,t)=C^{-}\exp\left[ik_z u\mp i\int_{t_0}^{t}\omega(t')dt'\right],\quad t_0<t<t_m,$$

$$b(u,t)=C^{+}\exp\left[ik_z u\mp i\int_{t_m}^{t}\omega(t')dt'\right],\quad t_m<t<t_0+\tau,$$

where $C^{\pm}$ are constants ($C^{+}\neq C^{-}$); the value $t_m$ is defined by the equation $\bar{L}_z(t)=0$ ($t_0<t_m<t_0+\tau$); $\tau$ is the time of displacement of the mobile wall, and $\omega(t)=\cos k_z\xi(t)$. The upper sign corresponds to the equation $\hat{A}b=0$ and the lower sign corresponds to $\hat{B}b=0$.

Because of $\hat{A}\hat{B}=\hat{B}\hat{A}$, the linear combination of the solutions of equations $\hat{A}b=0$ and $\hat{B}b=0$ is also the solution of Eq. (2). Taking into account the boundary condition $\mathbf{A}|_{z=L_z(t)}=0$, we write

$$b=\left(\frac{4\pi}{L\omega^{-}}\right)^{1/2}\sum_{k_z}C_{k_z}^{-}\sin(k_z u)e^{i\omega^{-}t},\quad t<t_0,$$
$$b=\left(\frac{4\pi}{L\omega^{+}}\right)^{1/2}\sum_{k_z}(C_{1k_z}^{+}\sin(k_z u)e^{i\omega^{+}t}+C_{2k_z}^{+}\sin(k_z u)e^{-i\omega^{+}t}),\quad t>t_0+\tau, \tag{3}$$

where $\omega^{-}=\cos k_z$ and $\omega^{+}=\cos k_z(L^{-}/L^{+})$. The value $L$ is the length of the cavity in the $z$ direction in the coordinate $(u,t)$ [in the coordinate $(u,t)$ we have $L=\text{const}$]. By comparing two solutions (3) at $t=t_m=0$ we find

$$C_{1k_z}^{+}=C_{k_z}^{-}\frac{1}{2}\left(\sqrt{\frac{\omega^{+}}{\omega^{-}}}+\sqrt{\frac{\omega^{-}}{\omega^{+}}}\right),$$

$$C_{2k_z}^{+}=C_{k_z}^{-}\frac{1}{2}\left(\sqrt{\frac{\omega^{+}}{\omega^{-}}}-\sqrt{\frac{\omega^{-}}{\omega^{+}}}\right).$$

If we substitute the solution (3) at the time $t>t_0+\tau$ into Eq. (1) on condition that $b(u,t)=a(z,t)$, we find

$$E=\sum_{k_z}|C_{k_z}^{-}|^2\omega^{+}\gamma_k, \tag{4}$$

where

$$\gamma_k=\frac{1}{4}\frac{(\omega^{+})^2+(\omega^{-})^2}{\omega^{+}\omega^{-}}.$$

In the opposite case $\omega\gg\frac{1}{\tau}$ we find the exponentially small effect of the excitation $\gamma_k\sim\frac{1}{2}(1\;+\;e^{-2\pi\tau\omega^{-}})$.

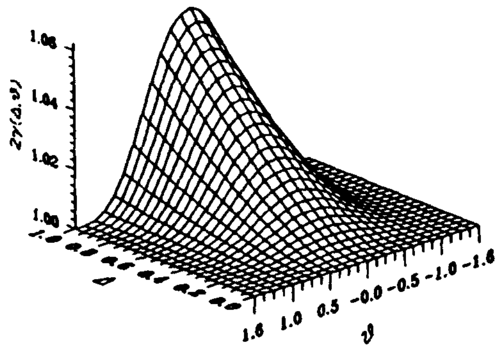

FIG. 2. Relative spectral density of radiation $U_\omega/U_{0\omega}=2\gamma_k$ plotted as a function of $\Delta$ and observation angle $\theta$ in the instantaneous approach.

These results are obtained only when $\tau\sim L/c$ and in principle they differ from the one found in Refs. 2–4 for the case $\tau\gg L/c$.

## 4. THREE-DIMENSIONAL CASE $|\mathbf{q}|\sim k_z$

If $|\mathbf{q}|\sim k_z$, we cannot divide Eq. (2) into two first-order differential equations. But for long-wavelength modes, in which the polarization is parallel to the $xy$ plane in the case $\delta L/L\ll 1$, we can use Eq. (4), but now we have $\omega_k^-= c\sqrt{|\mathbf{q}|^2+k_z^2}$ and $\omega_k^+ = c\sqrt{|\mathbf{q}|^2+(L^-/L^+)^2k_z^2}$. We thus find

$$\bar{E}=\sum_{\mathbf{k}} |C_{\mathbf{k}}|^2\omega_k^+\gamma_k\sim V^+\int d\Omega\int_{\omega_{\min}}^{\omega_{\max}}2\gamma_k\sqrt{1+\Delta\cos^2(\theta)}U_{0\omega}d\omega, \tag{5}$$

where $U_{0\omega}= (|C_{\mathbf{k}}|^2/2(2\pi c)^3)\omega^3$ is the initial spectral density of the radiation, $V^+$ is the final volume of the cavity, $\omega=\omega_k^-$, $\Delta=L^{+2}+L^{-2}/L^+L^-$, $d\Omega$ is the differential of the solid angle, and $\theta$ is the angle between the wave vector of the cavity mode and the direction of displacement of the mobile wall. The low limit $\omega_{\min}$ of integration in Eq. (5) depends on the size of the cavity $L^-$: $\omega_{\min}=\pi c/L^-$. The upper limit $\omega_{\max}$ is different in the two cases, $\omega\ll 2\pi/\tau$ and $\omega\gg 2\pi/\tau$.

In the case $\omega\ll 2\pi/\tau$ (the "instantaneous" approach) we have $\omega_{\max}=2\pi/\tau$ and the value $\gamma_k$ is

$$\gamma_k=\frac{1}{4}\frac{2+\Delta\cos^2(\theta)}{\sqrt{1+\Delta\cos^2(\theta)}}. \tag{6}$$

The quantity $\gamma_k(\Delta,\theta)$ shown in Fig. 2 describes the angular distribution and effectiveness of excitation of the eigenmodes of the cavity.

In the other case $\omega\gg 2\pi/\tau$ (the "adiabatic" approach) we have $\omega_{\max}=\infty$ and the value $\gamma_k$ is

$$\gamma_k \sim \tfrac{1}{2}\left(1+e^{-2\pi\tau\omega^-}\right).$$

## 5. QUANTUM CONSIDERATION

1. Instantaneous approach $\omega \ll 2\pi/\tau$. Let us assume that the initial state of the field is the vacuum state. The quantum calculation for the model in which the frequency of each of the cavity modes changes instantaneously from the initial value $\omega^- = \omega^-(L^-)$ to the final value $\omega^+ = \omega^+(L^+)$ (i.e., $\omega \ll 2\pi/\tau$, where $\omega$ is the frequency of the cavity mode) gives the expressions having the form of Eqs. (5) and (6), but now $|C_{\mathbf{k}}|^2 = 3\hbar$. The total energy excited from the initial vacuum state is (in order to omit the energy of the ground state we replace $\gamma_k \to \gamma_k - 1/2$):

$$\delta\bar{E} = V^+ \frac{6\pi\hbar}{c^3\tau^4} I_1(\Delta), \tag{8}$$

where $I_1(\Delta) \sim \Delta^2/40$ for $\Delta \ll 1$.

2. Adiabatic approach $\omega \gg 2\pi/\tau$. In the case $\omega \gg 2\pi/\tau$ the electromagnetic field in the cavity is described by the Schrödinger equation for the harmonic oscillator with a slowly changing frequency. It leads to relations (5) and (7) obtained in a classical approach with $|C_{\mathbf{k}}|^2 = 3\hbar$. After integration we have

$$\delta\bar{E} = V^+ \frac{6\hbar}{(2\pi c)^3} \frac{\omega_{\min}^3}{\tau} e^{-2\pi\tau\omega_{\min}} I_2(\Delta),$$

where $I_1(\Delta) \sim I_2(\Delta) \sim \Delta^2/40$ for $\Delta \ll 1$.

The value $\delta\bar{E}$ in each case, $\omega \ll 2\pi/\tau$ and $\omega \gg 2\pi/\tau$, obviously is equal to the work of nonstationary correction to the (stationary) Casimir force during the time of displacement of the cavity wall (see, e.g., Ref. 5).

## 6. SPECIFIC PROPERTIES OF THE FIELD STATE AFTER PARAMETRIC EXCITATION

The state of the field produced by the parametric excitation from the initial vacuum state is different from the electromagnetic field of another origin (which may appear in the experiment) because only even quantum levels are populated and because specific distribution of the wave packet in phase space takes place in the former case. These states are squeezed in the phase space in the direction of the canonical variable $P$:

$$\langle \Delta Q^2 \rangle_k = \frac{\hbar}{\omega_k}\left(\gamma_k + \sqrt{\gamma_k^2 - 1/4}\right), \quad \langle \Delta P^2 \rangle_k = \hbar\omega_k\left(\gamma_k - \sqrt{\gamma_k^2 - 1/4}\right).$$

As a result, the uncertainty relation has the form: $\sqrt{\langle \Delta P^2 \rangle_k \langle \Delta Q^2 \rangle_k} = \hbar/2$ (the last expression for the oscillator with parametrically driven frequency was found in Ref. 6; see also Ref. 7. The states originated from the parametric excitation are similar to the *Schrödinger cat* states, but the corresponding population of the quantum levels is not a *Poisson* one.

## 7. CONCLUSION

Let us now consider the possible experimental manifestation of the analyzed phenomena. By using power femtosecond laser pulses we can produce rather dense $(e-h)$ plasma in the short time interval $\tau$ in a thin semiconductor layer of width $l_s$ (see Fig. 1). The initial length of such a resonator is $l_s+l_i$, where $l_i$ is the thickness of the gap (the insulator film), and the final length is $l_i$. We then have $\tau \sim l_s/v$, where $v$ is the velocity of electrons in the plasma and $\Delta=(l_s+l_i)^2-l_i^2/l_i^2 \sim 2l_s/l_i \ll 1$. The velocity $v \sim 10^8$ cm/sec is attainable at an appropriate laser frequency $[v \sim (2(E_g-\hbar\omega)/m^*)^{1/2}$, where $E_g$ is the energy gap of a superconductor]. If the value $l_s$ is smaller than the extinction length for the excitation pulse, then the plasma is produced simultaneously on the whole film. In the last case the parametric excitation of the electromagnetic field occurs as a result of the change in the transmittance of the semiconductor film (details will be published elsewhere). In the latter case the characteristic time $\tau$ is defined mainly by the form of the laser pulse.

The energy of the field after the parametric pumping [see Eq. (8)] depends strongly on $\tau$:

$$\delta\bar{E} \approx V^+ \frac{3\hbar\pi^2}{5c^3\tau^4}\left(\frac{l_s}{l_i}\right)^2.$$

For example, for $l_s \sim 10^3$ Å, $l_i \sim 10^5$ Å, and $\tau \sim 10^{-14}$ sec (this is an attainable pulse time), we have $\lambda_{\min} \sim 3\times 10^{-4}$ cm. For $l_i \sim 10^5$ Å we find $\bar{E}/V^+ \sim 1.6\times 10^6$ eV/cm$^3$ and $\bar{N}/V^+ \sim 5\times 10^6$ photons/cm$^3$. Note that the estimates given above are actually the upper estimates (for an infinitely high quality, $Q$, of the cavity). By taking into account the dielectric susceptibility $\varepsilon$ of the medium in the cavity we can obtain the additional multiplier $\varepsilon^{5/2}$ in the estimate for the excited energy.

It was found that the angular distribution of the parametrically excited radiation [which is described by the dependence $\gamma=\gamma(\theta)$, where $\theta$ is the angle between the direction of radiation and the $z$ axis; see Fig. 2] is different from the distribution of the radiation of another origin appearing in the experiment. Because of a short pumping time, a parametrically excited light appears in a time interval $\tau$, which is shorter than that required for the appearance of radiation of another origin. These specific properties and the quantum statistical properties described above (the population of even levels and squeezing) can be used to distinguish a parametrically excited field from other electromagnetic radiation in the experiment (see, e.g., Refs. 8 and 9).

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[2)]Different phenomena in QED in the cavity due to the quasi-stationary modulation of the wall (Lamb shift modulation, etc.) see, for example, in Ref. 1 and the references cited there.

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