Instability of a quasi-1D Luttinger liquid

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(Submitted 26 January 1995; resubmitted 14 March 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **61**, No. 9, 722–727 (10 May 1995)

The properties of an electron liquid in a strong magnetic field and those of a quasi-1D spin-zero Luttinger liquid are analyzed. The homogeneous ground state of such electron liquids is unstable. A charge density wave arises in the system. © *1995 American Institute of Physics.*

Research on high-T_c superconductivity has recently spurred a great deal of interest in the properties of "marginal" Fermi liquids, which cannot, as we know, be described within the framework of Landau's Fermi-liquid theory.[1–3] One such liquid is a 1D electron liquid with an extremely long-range interaction (the Tomonaga model),[4–6] which is a Luttinger liquid. In the present letter we examine an electron liquid with a Coulomb interaction in a strong magnetic field H such that all the electrons are in the lowest Landau level, and transitions to higher-lying levels can be ignored. The properties of an electron–hole liquid in a strong magnetic field were studied in Refs. 7–11 for various density ranges. It was shown there that the ground state of such an electron–hole liquid is an insulating state with a gap at the Fermi surface. The properties of quasi-1D electron liquids are quite different from those of electron–hole liquids. In particular, it has been shown[6] for the Tomonaga model that the ground state of such an electron liquid is metallic, without a gap at the Fermi surface. However, Landau's Fermi-liquid theory does not apply to this case. The properties of an electron liquid in a strong magnetic field with a model short-range potential were studied in Refs. 12 and 13, among other places. In particular, Yakovenko[13] used the parquet approximation in a form which leans heavily on the short-range nature of the interaction. For an electron liquid with a Coulomb interaction in a strong magnetic field, under the condition $p_F\lambda \ll 1$ (p_F is the Fermi momentum, and λ is a magnetic length), however, the long-range nature of the Coulomb potential is important.[11] It leads to results which are qualitatively different from those of Ref. 13. In the present letter we incorporate the long-range nature of the Coulomb interaction. We also use a nonperturbative method, which is exact in a certain stage of the calculations and which makes it possible to go beyond the scope of the parquet approximation.

In the first part of this paper we show that an electron liquid in a strong magnetic field is equivalent to a quasi-1D spin-zero electron liquid, which is a system of regularly arranged metal wires directed along the magnetic field. The forward scattering of two electrons is taken into account exactly, as is the backscattering of two electrons on the same wire. When only this part of the interaction is taken into account, it turns out that a quasi-1D electron liquid is a Luttinger liquid. However, the incorporation of the backscattering of two electrons on different wires renders a quasi-1D Luttinger liquid unstable with respect to a transition to an inhomogeneous state of the charge-density-wave type,

with charge modulation both along and across the wires. In this case the ground state is an insulating state with a gap at the Fermi surface.

We assume that the parameters characterizing the magnetic field and the electron liquid satisfy the conditions

$$\hbar/a_B \ll p_F \ll \hbar/\lambda, \tag{1}$$

where $\lambda = \sqrt{c/eH}$, $a_B = \hbar^2/me_*^2$ is the effective first Bohr radius, and $e_*^2 = e^2/\epsilon_0$ (ϵ_0 is the dielectric constant of the medium). We set $\hbar = 1$.

We consider a system of coherent states for an electron in the lowest Landau level.[14,15] We adopt a cylindrical gauge for the vector potential of the external magnetic field H, which is directed along the z axis $A = (-Hy/2, Hx/2, 0)$. These states $\chi_R(\rho)$ are found as eigenstates of the operator representing the complex coordinate of the center of the orbit [$\boldsymbol{\rho} = (x,y)$; R is an eigenvalue of the complex coordinate of the center of the orbit, which specifies the particular coherent state].[15] The wave function of these states is

$$\chi_{\mathbf{R}}(\rho) = (\sqrt{2\pi}\lambda)^{-1} \exp\{-(2\lambda)^{-2}[(\boldsymbol{\rho}-\mathbf{R})^2 - 2i\boldsymbol{\rho}\wedge\mathbf{R}]\}, \tag{2}$$

where $\boldsymbol{\rho}\wedge\mathbf{R} = x\mathbf{R}_y - y\mathbf{R}_x$; $\mathbf{R}_x = \mathrm{Re}R$, and $\mathbf{R}_y = \mathrm{Im}R$.

In general, the system of coherent states is overfilled. According to a theorem originally formulated by von Neumann,[16,17] there exists an "optimally" complete system of coherent states $\chi_{\mathbf{R}_n}(\rho)$ (i.e., a system of states such that the removal of more than one state from the system leaves the latter incomplete) if the $\mathbf{R}_n$ form a lattice in the complex R plane in which the area of the unit cell is $S = 2\pi\lambda^2$. We expand the electron field $\psi(r,t)$ in the optimally complete system of coherent states $\chi_{\mathbf{R}_n}(\rho)$:

$$\psi(r,t) = \sum_{\mathbf{R}_n} a_{\mathbf{R}_n}(t,z)\chi_{\mathbf{R}_n}(\rho), \quad \bar{\psi}(r,t) = \sum_{\mathbf{R}_n} \bar{a}_{\mathbf{R}_n}(t,z)\bar{\chi}_{\mathbf{R}_n}(\rho).$$

Ignoring the change in the Coulomb potential over distances on the order of λ [this simplification is legitimate under conditions (1)], and renormalizing the fields $a_{\mathbf{R}_n}$ and $\bar{a}_{\mathbf{R}_n}$ in accordance with $a_{\mathbf{R}_n} \to a_{\mathbf{R}_n}$, $\Sigma_{\mathbf{R}_m}\bar{a}_{\mathbf{R}_m}\langle\chi_{\mathbf{R}_m}|\chi_{\mathbf{R}_n}\rangle \to \bar{a}_{\mathbf{R}_n}$, we find that the action of an electron liquid in a strong magnetic field is equivalent to the action of a quasi-1D spin-zero electron liquid with a Coulomb interaction in a system of regularly positioned metal wires directed along the magnetic field. Going over to fields which describe quasiparticles near the regions of the Fermi surface with $p_z = +p_F$ and $p_z = -p_F$, which we denote by $a_{\mathbf{R}}^{(+)}(t,z)$ and $a_{\mathbf{R}}^{(-)}(t,z)$, respectively (we assume that the scale of the change in the fields $a_{\mathbf{R}}^{(+,-)}$ along the z axis is much larger than $\hbar/p_F$), we can write the effective action for the quasiparticles as follows:

$$S = S_0 + S_{\mathrm{int}}^{(1)} + S_{\mathrm{int}}^{(2)}, \tag{3}$$

$$S_0 = \sum_{\mathbf{R}} \int dz dt \bar{a}_{\mathbf{R}}(t,z)(\hat{\gamma}^\mu \hat{p}_\mu) a_{\mathbf{R}}(t,z), \tag{4}$$

$$S_{\text{int}}^{(1)}=-\left\{\frac{1}{2}\sum_{\mathbf{RR}'}\int dzdz'dt(\bar{a}_{\mathbf{R}}(t,z)a_{\mathbf{R}}(t,z))V_{r-r'}(\bar{a}_{\mathbf{R}'}(t,z')a_{\mathbf{R}'}(t,z'))\right.$$
$$\left.+U_0\Sigma_{\mathbf{R}}(\bar{a}_{\mathbf{R}}(t,z)\bar{\sigma}+a_{\mathbf{R}}(t,z))(\bar{a}_{\mathbf{R}}(t,z)\hat{\sigma}_-a_{\mathbf{R}}(t,z)),\right\}. \quad (5)$$

$$S_{\text{int}}^{(2)}=-\sum_{\mathbf{R}\neq\mathbf{R}'}(\bar{a}_{\mathbf{R}}(t,z)\hat{\sigma}_+a_{\mathbf{R}}(t,z))U_{\mathbf{R}-\mathbf{R}'}(\bar{a}_{\mathbf{R}'}(t,z)\hat{\sigma}_-a_{\mathbf{R}'}(t,z)). \quad (6)$$

In expressions (4)–(6) we have $a_{\mathbf{R}}=(a_{\mathbf{R}}^{(+)},a_{\mathbf{R}}^{(-)})$; $\bar{a}_{\mathbf{R}}=(\bar{a}_{\mathbf{R}}^{(+)},\bar{a}_{\mathbf{R}}^{(-)})$; $\mu=0,1$; $\mathbf{r}=(\mathbf{R},z)$; $\hat{\gamma}^0=\hat{1}$; $\hat{\gamma}^1=\hat{\sigma}_z$; $\hat{\sigma}_\pm=1/2(\hat{\sigma}_x\pm i\hat{\sigma}_y)$; $\hat{p}_0=i\partial_t$; $\hat{p}_1=iV_F\partial_z$; σ_{xyz} are the Pauli matrices;

$$U_{\mathbf{R}-\mathbf{R}'}=e_*^2\int dze^{iQz}[z^2+(\mathbf{R}-\mathbf{R}')^2]^{-1/2},\quad U_0=e_*^2\int e^{iQz}\frac{dz}{|z|}=e_*^2\ln\frac{1}{Q\lambda}. \quad (7)$$

In (7) we have $Q=2p_F$. The potential $V(r-r')$ determines forward scattering with a momentum transfer $k_z\ll p_F$. The potentials $U_{\mathbf{R}-\mathbf{R}'}$ determine backscattering with a momentum transfer $k_z=2p_F$. The potential U_0 determines the scattering of two electrons on the same wire, while $U_{\mathbf{R}-\mathbf{R}'}$ determines that of two electrons on different wires.

We represent the interaction $S_{\text{int}}^{(1)}$ as the exchange of a virtual field $\phi_{\mathbf{R}}(zt)$, which describes plasmons. The total action becomes

$$S=S_{a,\phi}+S_\phi^{(0)}+S_{\text{int}}^{(2)}, \quad (8)$$

$$S_{a,\phi}=\sum_{\mathbf{R}}\int dzdt\bar{a}_{\mathbf{R}}(t,z)[\hat{\gamma}^\mu(\hat{p}_\mu-A_\mu)]a_{\mathbf{R}}(t,z), \quad (9)$$

$$S_\phi^{(0)}=\frac{1}{2}\sum_{\mathbf{RR}'}\int dzdz'dt\phi_{\mathbf{R}}(t,z)\hat{g}^{-1}(\mathbf{r}-\mathbf{r}')\phi_{\mathbf{R}'}(t,z'). \quad (10)$$

In (9) and (10) we have $A_0=\phi_{\mathbf{R}}(zt)$; $A_1=0$; and the Fourier transform of $\hat{g}^{-1}(\mathbf{r}-\mathbf{r}')$ is $\hat{g}^{-1}(k,\mathbf{k}_\perp)=(V(\mathbf{k})-U_0)^{-1}$, where $V(\mathbf{k})$ is the Fourier transform of the Coulomb potential on the lattice of wires.

We consider the generating functional $Z_0[J]$, ignoring the scattering of two electrons in different wires, i.e., ignoring $S_{\text{int}}^{(2)}$

$$Z_0[J]=\int DaD\bar{a}D\phi\exp\{i(S_{a,\phi}+S_\phi^{(0)})+i(\bar{J}a+\bar{a}J)\}. \quad (11)$$

We note that the action $S_{a,\phi}$ is actually the action of a massless Dirac electron in an external electromagnetic field in a space of dimensionality (1+1), whose properties were studied by Schwinger.[18] The Green's function of such an electron satisfies the equation

$$\gamma^\mu(\hat{p}_\mu-A_\mu)G_{\mathbf{R}}(x,y|A_{\mathbf{R}})=\delta(x-y), \quad (12)$$

where $x=(t,z)$ and $y=(t',z')$. A solution of this equation can be written

$$G_{\mathbf{R}}(x,y|A_{\mathbf{R}})=\exp\{\hat{\theta}_{\mathbf{R}}(x)-\hat{\theta}_{\mathbf{R}}(y)\}G_0(x,y), \quad (13)$$

$$\hat{\theta}_{\mathbf{R}}(k)=[k_0+(V_Fk_1)\hat{\sigma}_3][k_0^2-(V_Fk_1)^2+i\delta]^{-1}[A_0(k)-\hat{\sigma}_3A_1(k)] \quad (14)$$

$(A_0=\phi_{\mathbf{R}},\ A_1=0)$. Here $\hat{\theta}_{\mathbf{R}}(k)$ is the Fourier transform of $\hat{\theta}_{\mathbf{R}}(x)$ in the variables x;

$$G_0(x,y)=-\frac{1}{2\pi}[V_F(t-t')-\hat{\sigma}_3(z-z')-i\delta s_{t-t'}]^{-1}, \tag{15}$$

where

$$s_t=1 \text{ at } t>0,\ s_t=-1 \text{ at } t<0.$$

An evaluation of functional integral (11) for $Z_0[J]$ yields

$$Z_0[J]=\exp\left\{\mathrm{Tr}\ \ln\left[\gamma^{\mu}(\hat{p}_{\mu}-A_{\mu})\right]+iS_{\phi}^{(0)}-i\sum_{\mathbf{R}}\int dxdy\bar{J}_{\mathbf{R}}(x)G_{\mathbf{R}}(x,y|\phi_{\mathbf{R}})J_{\mathbf{R}}(y)\right\}.$$

The quantity $\mathrm{Tr}\ \ln[\gamma^{\mu}(\hat{p}_{\mu}-A_{\mu})]$ can be evaluated exactly.[18] From this evaluation, we find the following expression for $Z_0[J]$:

$$Z_0[J]=\exp\left\{iS_{\phi}-i\sum_{\mathbf{R}}\int dxdy\bar{J}_{\mathbf{R}}(x)G_{\mathbf{R}}(x,y|\phi_{\mathbf{R}})J_{\mathbf{R}}(y)\right\}. \tag{16}$$

Here S_{ϕ} is the plasmon action, given by

$$S_{\phi}=\int d^2k_{\perp}d^2k(2\pi)^{-4}\phi(k,\mathbf{k}_{\perp})D^{-1}(k,\mathbf{k}_{\perp})\phi(-k,-\mathbf{k}_{\perp}). \tag{17}$$

The field $\phi(k,\mathbf{k}_{\perp})$ is the Fourier transform of $\phi_{\mathbf{R}}(z,t)$ in the variables $x=(z,t);\mathbf{R}$ ($\mathbf{k}_{\perp}$ lies in the Brillouin cell corresponding to the lattice $\mathbf{R}_n$). Here

$$D^{-1}(k,\mathbf{k}_{\perp})=g^{-1}(k,\mathbf{k}_{\perp})-\Pi_0(k), \tag{18}$$

where

$$\Pi_0(k)=(2\pi\lambda^2V_F)^{-1}(V_Fk_1)^2[k_0^2-(V_Fk_1)^2+i\delta]^{-1}. \tag{19}$$

With the help of generating functional $Z_0[J]$ in (16), any correlation function of the electron fields $a,\bar{a}$ can be expressed in terms of a functional integral over the field ϕ. It is easy to see from (13) and (14) that this integral is Gaussian for a correlation function of arbitrary degree in the fields $a,\bar{a}$. We can thus evaluate any Green's functions while ignoring $S_{\mathrm{int}}^{(2)}$. We can also calculate diagrams of any order in the interaction $S_{\mathrm{int}}^{(2)}$. In particular, a calculation of the one-particle Green's function ignoring $S_{\mathrm{int}}^{(2)}$, i.e., ignoring backscattering by different wires, leads to the familiar result.[6] The behavior of the one-particle Green's function is characteristic of a Luttinger liquid. In particular, there is a weakening of the pole in the Green's function, and there is no jump in the distribution function n_p at the Fermi surface.

We consider the vertex representing the electron–electron interaction, dealing with $S_{\mathrm{int}}^{(2)}$ by perturbation theory. Figure 1 shows some examples of diagrams corresponding to the terms of a perturbation-series in $U_{\mathbf{R}-\mathbf{R}'}$. The wavy lines correspond to $U_{\mathbf{R}-\mathbf{R}'}$, and the dashed lines are the fields $\phi_{\mathbf{R}}(t,z)$ over which an integration should be carried out. When the interaction with the field ϕ in second order in $U_{\mathbf{R}-\mathbf{R}'}$ is ignored, only diagrams a and b are singular. The two diagrams have an identical singularity $\sim\ln(\epsilon_F/\omega)$, where

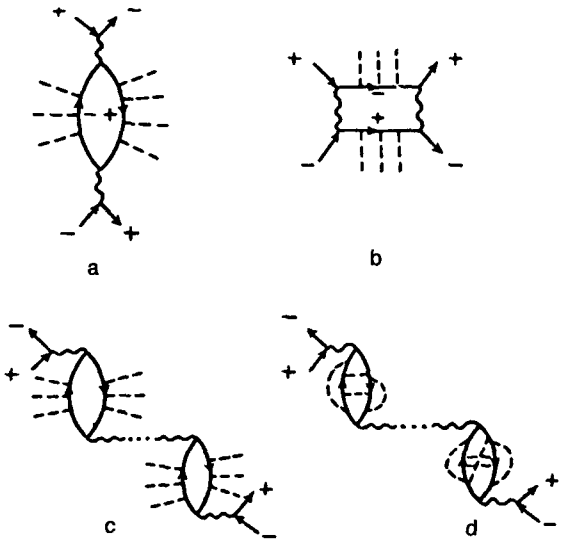

FIG. 1.

ω is the transferred or resultant frequency. Taking account of the interaction of electrons with the ϕ field, we can describe the polarization loop $\Pi^{(-+)}$ (diagram a) by means of a Gaussian integral over the field ϕ:

$$\Pi_{\mathbf{R}}^{(-+)}(xy)=\int D\phi\exp\{iS_\phi\}G_{\mathbf{R}}^{(+)}(xy|\phi_{\mathbf{R}})G^{(-)}(xy|\phi_{\mathbf{R}}). \tag{20}$$

Substituting expression (17) for S_ϕ and expression (13) for the Green's functions into (20), and evaluating the functional integral, we find

$$\Pi_{\mathbf{R}}(k)=\int dtdz\ \exp\{F(x)+ik_\mu x^\mu\}\Pi_0(x), \tag{21}$$

$$F(x)=-i\times 8(2\pi)^{-4}\int d^2k_\perp d^2k(V_Fk_1)^2[k_\mu k^\mu+i\delta]^{-2}D(k,\mathbf{k}_\perp)\sin^2[k_\mu x^\mu/2],$$

where $k_\mu x^\mu=k_0t-k_1z$, $k_\mu k^\mu=k_0^2-(V_Fk_1)^2$, and $x=(t,z)$. Evaluating the integral in (21), we see that the singularity in the polarization operator $\Pi^{(-+)}(\omega)$ at a small frequency transfer, $\omega\to 0$, is strengthened:

$$\Pi^{(-+)}(\omega)=-(2\pi\lambda^2V_F)^{-1}(\epsilon_F/\omega)^\alpha, \tag{22}$$

where $\alpha=e_*^2(\pi V_F)^{-1}\ln(1/p_F\lambda)$. In contrast with the polarization operator, the incorporation of the interaction of the electrons with the field ϕ in the evaluation of a step of the ladder $\Delta(\omega)$ (Fig. 1b) leads to a weakening of the singularity, and it gives us $\Delta(\omega)\sim\omega^\alpha$. We can therefore assume that diagrams of the *RPA* type (Fig. 1c) constitute the main sequence of diagrams for the electron vertex. Each diagram in this sequence must be integrated over the fields ϕ. If condition (1) holds, we can restrict the analysis

to diagrams in which the ϕ fields are paired only within an individual electron loop, and we can ignore a pairing of five fields corresponding to different electron loops (Fig. 1d). As a result, the diagrams for the total vertex, $\Gamma(\omega \mathbf{k}_\perp)$, with a momentum transfer $k_z = 2p_F$ can be summed. The result is

$$\Gamma(\omega \mathbf{k}_\perp) = \tilde{U}(\mathbf{k}_\perp)[1 - \Pi^{(-+)}(\omega)\tilde{U}(\mathbf{k}_\perp)]^{-1}. \tag{23}$$

The seed vertex $\tilde{U}(\mathbf{k}_\perp)$ in (23) is the Fourier transform of the interaction $\tilde{U}_{\mathbf{R}-\mathbf{R}'} = U_{\mathbf{R}-\mathbf{R}'} - U_0\delta_{\mathbf{R}-\mathbf{R}'}$ in the variable $\mathbf{R}-\mathbf{R}'$[by subtracting a term $U_0\delta_{\mathbf{R}-\mathbf{R}'}$ from $U_{\mathbf{R}-\mathbf{R}'}$, we incorporate the condition $\mathbf{R} \neq \mathbf{R}'$ in expression (6) for $S_{\text{int}}^{(2)}$]. The quantity $\Pi^{(-+)}(\omega)$ is given by (22). Since we have $\tilde{U}_{\mathbf{R}-\mathbf{R}'} = 0$ at $\mathbf{R} = \mathbf{R}'$, it is clear that we have $\int d^2\mathbf{k}_\perp \tilde{U}(\mathbf{k}_\perp) = 0$. Accordingly, $\tilde{U}(\mathbf{k}_\perp)$ is a function of varying sign, which goes negative at certain values $|\mathbf{k}_\perp| \sim 1/\lambda$. It can be seen from (23) and (22) that for those values of $\mathbf{k}_\perp$, for which the condition $\tilde{U}(\mathbf{k}_\perp) < 0$, the vertex $\Gamma(\omega, \mathbf{k}_\perp)$ has a pole in the variable ω, which corresponds to an instability of the homogeneous ground state. This assertion means that in the ground state of the system, a charge density wave with a wave vector $K_z = 2p_F$ and with $\mathbf{K}_\perp = \mathbf{k}_\perp^{(\max)}$ arises. Here $\mathbf{k}_\perp^{(\max)}$ is such that the condition $\tilde{U}(\mathbf{k}_\perp^{(\max)}) < 0$ holds, and $|\tilde{U}(\mathbf{k}_\perp^{(\max)})|$ reaches a maximum value. It turns out that we have $|\mathbf{k}_\perp^{(\max)}| \sim 1/\lambda$. As a result of the restructuring of the ground state, an insulating gap arises at the Fermi surface. The size of this gap can be estimated to be $\Delta_0 \sim \epsilon_F(\tilde{U}(\mathbf{K}_\perp)/2\pi\lambda^2 V_F)^{1/\alpha}$.

We wish to thank B. Aronzon for a discussion of the experimental situation and N. Prokof'ev for a discussion of the results.

This study was supported by the Russian Fund for Fundamental Research (Grant 93-02-2538), the International Science Foundation (Grant MBH000), and the grant INTAS-93-285.

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Translated by D. Parsons