

Interference effects in mesoscopic superconductor structures

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Interference effects which arise during the flow of a quasiparticle current through “hybrid” mesoscopic systems, in which a normal conductor (N) is in contact with superconducting (S) and normal (N) regions (banks), are analyzed. A Josephson effect is manifested in such systems, despite the absence of a current through one or both of the S banks. This effect becomes a time-varying effect at currents (or voltages) above a certain threshold. Some nontrivial structural features induced on the current–voltage characteristic by the interference effects are analyzed. © 1995 American Institute of Physics.

Interference transport phenomena in mesoscopic superconductor systems, in which a normal conductor (N) of one of several possible topologies is in contact with two or more S (superconducting) and N electrodes (banks), have recently attracted considerable experimental and theoretical interest.^{1–6} One reason is a manifestation of the wave nature of quasiparticles in some oscillations observed on the plot of the conductance of the system versus the phase difference between the superconductors. Another reason for the interest is the diversity of interference phenomena in complex S/N systems, where “complex” means a system which has several different current-flow channels. The difference is seen, in particular, in the circumstance that a current can flow through the s (“superconducting”) channels of an N conductor connecting S banks, even if there is no voltage across the ends of the conductor, while a current can flow through the n (“normal”) channels connecting S and N banks, or exclusively N banks, only when a voltage is applied. The properties of complex S/N systems, like the functional relationship between the current and the voltage, on the one hand, and the phase difference between the superconductors, φ , on the other, depend strongly on the particular channel through which the current flows (and on how many channels are “working”). In this letter we examine interference phenomena which arise in complex S/N systems (Fig. 1) during current flow through the n channel.²⁾ We derive equations relating the current and the voltage to the phase difference for such systems. The problem of finding these relations is reminiscent of the corresponding problem for weak links (in particular, of $S-N-S$ structures), although the meaning of the quantities which appear in the relations being sought, like the relationship between the voltage across the system, V , and φ , is not the same as usual. We derive and analyze a “resistive model” of such systems, which can be derived for gapless superconductors. We analyze the deviations from this model for low-transmission S/N interfaces for superconductors which do have a gap. In particular, we show that Josephson phenomena arise in the system, even though there is no current

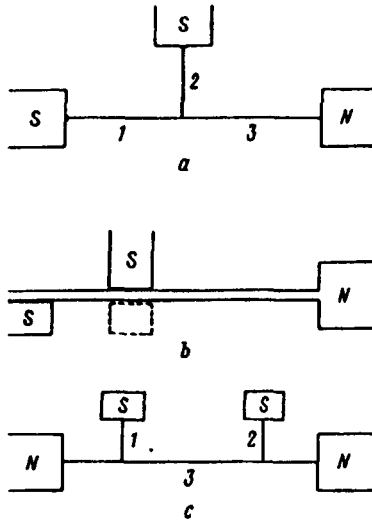


FIG. 1. Schematic diagrams of the structures under consideration. a, c—Quasi-1D N conductor connecting S and N banks; b—system of two tunnel junctions formed between S banks and a thin N film. The heavy lines at the S – N interfaces represent tunnel barriers.

through one of the superconductors (a, b) or through both of them (c). These phenomena become time-varying when the current or voltage exceeds a certain threshold.

In analyzing the interference phenomena we take the approach of Refs. 5 and 6, starting from a system of equations for semiclassical Green's functions.⁷⁻⁹ We write this system of equations in a compact form using the supermatrix \check{g} , which consists of retarded (\hat{g}^R), advanced (\hat{g}^A), and Keldysh (\hat{g}) Green's functions.^{8,9} We consider the dirty case [in which the mean free path satisfies $l \ll \sqrt{D}/T_c = \xi_N(T_c)$, where D is the diffusion coefficient in the N conductor, and $\hbar = 1$]. In this case the equations can be derived⁸ for the Green's function averaged over directions, \check{g}_{av} . In a quasi-1D N conductor for which the length scale of the cross section is assumed small in comparison with $\xi_N(T_c)$, the function \check{g}_{av} depends on the one coordinate u ($=x$ or y) and satisfies a second-order equation in the derivative $\partial/\partial u \equiv \partial_u$ of the type $\partial_u(\check{g}^* \partial_u \check{g}) \equiv \partial_u(\check{J}) = \check{L}(\check{g})$ (below we omit the subscript "av"). The operator \check{L} is given in Refs. 8 and 9. This function also satisfies the normalization condition $(\check{g}^* \check{g})(t, t') = \check{1} \delta(t - t')$, where $\check{g} = \check{g}(u, t, t')$, and the asterisk (*) means an integration over the internal variable.⁹ The Keldysh component of the matrix, \check{J}_x , determines the current (the x axis is in the same direction as the current):

$$I = \frac{\pi}{2e\rho_N} (2\partial_x f_1 - Sp \hat{\tau}_z \hat{g}^R * \partial_x f_1 * \hat{\tau}_z \hat{g}^A)(t, t) + \frac{\pi}{2e\rho_N} (J_x^R * f - f * J_x^A)(t, t) \equiv I_n + I_s, \quad (1)$$

where $J_x^\alpha = Sp \hat{\tau}_z \hat{g}^\alpha * \partial_x \hat{g}^\alpha$ ($\alpha = R, A$), f and f_1 are distribution functions,⁸ and ρ_N is the resistance per unit length of the N conductor. The equation for the function \check{g} can be supplemented with boundary conditions at the points (u_b) corresponding to the S/N

junctions. The joining conditions for the semiclassical Green's functions were derived in Ref. 10 for the case with potential barriers at the boundaries of metals. In the dirty limit under consideration here, those conditions become¹¹ (see also Refs. 12 and 13)

$$(2R_b/\rho_N)\check{J}_u(u_b) = \check{g}(u_b) * \check{g}_{\text{eq}} - \check{g}_{\text{eq}} * \check{g}(u_b), \quad (2)$$

where R_b is the resistance of the barrier, and \check{g}_{eq} is the equilibrium Green's function corresponding to the S or N bank, which is associated with its own value of the potential. At the points $x = x_o, y = 0$, at which three N -conductor branches converge, we need to consider the relation⁵

$$\check{J}_x(x_o + 0) - \check{J}_x(x_o - 0) + \check{J}_y(x_o) = \check{0}. \quad (3)$$

A consequence of this relation is Kirchhoff's law for the current at these points.

The equation for the function \check{g} , along with Eqs. (1)–(3), describes a proximity effect and scattering of quasiparticles (ordinary scattering and “Andreev” scattering) at the S – N junctions. The nonequilibrium nature of their distribution function is taken into account. Analytic expressions for the current based on these equations can be derived easily in cases in which the amplitude of the condensate Green's function, \hat{f}^R , is small in the N conductor. One of these cases is realized when there is a gapless superconductivity in the S banks, in which the depairing rate (γ_s) in S , determined by (for example) paramagnetic impurities,¹⁴ is large: $\gamma_s \gg \Delta$. In particular, the Fourier component $\hat{f}_{\epsilon, \epsilon'}^R$ satisfies the equation

$$D \partial_u^2 \hat{f}_{\epsilon, \epsilon'}^R - [-i(\epsilon + \epsilon') + 2\gamma] \hat{f}_{\epsilon, \epsilon'}^R = \hat{0} \quad (4)$$

in this case, where γ is the depairing rate in the N conductor. In this stage we focus on the case of direct S – N junctions (there are no barriers). In the leading approximation in the small parameter $(\Delta/\gamma_s)^2$, the current through the j th segment of the N conductor can be written as the sum of an ohmic component $I_{nj} = G_j V_j$ and a superconducting component I_s , which is nonzero (and the same) in the first and second segments: $I = G_1 V_1 + I_s = G_3 V_3$, $G_2 V_2 + I_s = 0$, where $G_j = 1/R_j = 1/\rho_N d_j$, $V = V_1 + V_3$. The condition that the banks be at equilibrium means that the potential difference across the superconductors is $V_s = V_1 + V_2 = \partial_t \varphi / 2e$. We also note that in the voltage region of interest here ($V \sim V_*$; see the discussion below) we can ignore the difference between the distribution function f and the equilibrium distribution function in finding the current I_s for the gapless case. To avoid overburdening the equations, we also assume $eV \ll \max \{\epsilon_{dj}, \gamma\}$, where $\epsilon_{dj} = D/d_j^2$. The solution of (4) then leads to a result of the standard form, $I_s = J_c \sin \varphi$, where J_c is the “critical current” in the s -channel (which determines the actual critical current for a closed superconducting chain corresponding to the condition $I = 0$). For the system in Fig. 1a we find

$$J_c = \frac{\pi T \Delta^2}{e \rho_N} \sum_{k=0}^{\infty} \sqrt{2(\omega_n + \gamma)/D} / (\omega_n + \gamma)^2 [\coth \kappa_{n1} + \coth \kappa_{n2} + \coth \kappa_{n3}] \sinh \kappa_{n1} \sinh \kappa_{n2}. \quad (5)$$

Here $\omega_n = \pi T(2n + 1)$ and $\kappa_{n1} = d_j \sqrt{2(\omega_n + \gamma)/D}$. Using the relations written above, we find the following simple system of equations to describe the interference phenomena³:

$$I = \frac{G_1}{2e} \partial_t \varphi + I_* \sin \varphi, \quad V = \frac{G_1 R_n}{2e} \partial_t \varphi + V_* \sin \varphi. \quad (6)$$

Here $I_* = J_c(1 + G_1 R_2)$ and $V_* = I_* R$ are respectively the threshold current and threshold voltage above which Josephson oscillations arise, and a voltage across the superconductors arises, and $R_n = R_1 + R_3$ is the resistance of the system in the normal state. These equations can be called the "resistive model" of the system under consideration, in view of the similarity of the first equation in (6) to the familiar weak-link model.¹⁵ This similarity has its limits, however, since Eqs. (6) contain the voltage and the phase difference measured between different banks. Accordingly, the relationship between the voltage V and $\partial_t \varphi$ differs from the Josephson relationship (remaining the same as that between V_S and $\partial_t \varphi$). The threshold current, which determines (in particular) the amplitude of the oscillations in $I(t)$ (for a given V), is greater than J_c . At $I \leq I_*$, a steady-state phase difference arises between the superconductors. This difference is determined by the relation $\sin \varphi = I/I_*$. In this regime, the resistance of the system, $R = (R_n R_2 + R_1 R_3)/(R_1 + R_2)$, is smaller than R_n . Above the threshold the result depends on which of the quantities (the current or the voltage) is given. In the regime of a given and constant current I , by solving (6) we find an oscillatory time dependence of the voltage across the system, $V(t)$, and also of the voltage across the superconductors, $V_S(t)$:

$$V(t) = IR + (R_n - R)V_S(t), \quad V_S(t) = \frac{R_1(I^2 - I_*^2)\theta(I - I_*)}{I - I_* \cos(\Omega_I t)}, \quad (7)$$

where the oscillation frequency is $\Omega_I = 2e\bar{V}_S = 2eR_1\sqrt{I^2 - I_*^2}\theta(I - I_*)$. From (7) we find the current-voltage characteristic, i.e., the current dependence of the average voltage:

$$\bar{V} = RI + (R_n - R)\sqrt{I^2 - I_*^2}\theta(I - I_*). \quad (8)$$

It follows from (8), in particular, that at $I > I_*$ the resistance satisfies $R > R_n$.

In the regime with a given constant voltage V satisfying $V > V_*$, the current oscillates:

$$I(t) = GV - (G - G_n)V_S(t), \quad V_S(t) = G_n R_1 \frac{(V^2 - V_*^2)\theta(V - V_*)}{V - V_* \cos(\Omega_V t)}, \quad (9)$$

where $\Omega_V = 2e\bar{V}_S = 2eG_n R_1\sqrt{V^2 - V_*^2}\theta(V - V_*)$, and $G_{(n)} = 1/R_{(n)}$. In this case the current-voltage characteristic is described by

$$\bar{I} = GV - (G - G_n)\sqrt{V^2 - V_*^2}\theta(V - V_*). \quad (10)$$

It follows from this result that the conductance of the system, $G(V) = d\bar{I}/dV$, may be negative at $V > V_*$ (Fig. 2).

Corresponding results can be derived for the system in Fig. 1c. In particular, a functional relationship of the type in (6) and also Eqs. (7)–(10) remain valid. In the latter, we should replace G_1 by $G_3 = 1/\rho_N d_3$ (the conductance of the segment connecting the branch points of the N conductor). In this case the resistance R is given by $R = R_n - R_3 + 1/[1/R_3 + 1/(R_1 + R_2)]$, and the threshold values are related to J_c by $I_* = [1 + (R_1 + R_2)G_3]J_c = V_*/R$. The specific functional dependence of these quantities

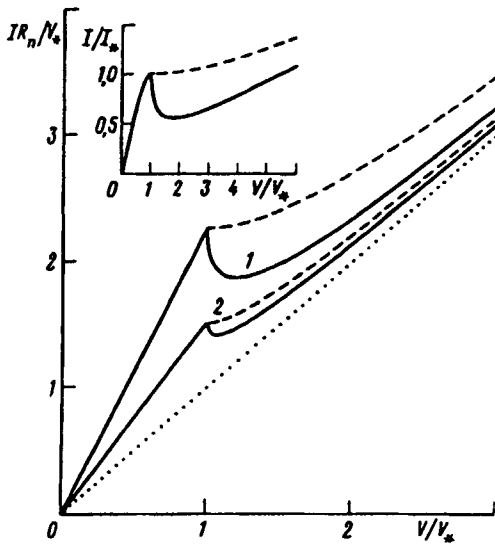


FIG. 2. Current-voltage characteristics of the systems in Fig. 1, a-c, according to the "resistive model." Solid curves: $\bar{I}(V)$ in the given-voltage regime. Dashed curves: $I(V)$ in the given-current regime. The different curves correspond to the values $R/R_n=2.25$ (1) and 1.5 (2). Dotted curve: Ohm's law, $I=V/R_n$. The inset shows curves for systems a and b when there are tunnel barriers at the S/N interfaces, as found for the case $G_{11}=G_{12}=0.2G_{22}$ at $T=0$.

on the geometric parameters of the structure is unimportant for our purposes at this point. We simply note that, as in the case in Fig. 1a, the quantities I_* and V_* fall off exponentially with increasing total length of the segments connecting the superconductors (d_S) at $d_S > \xi_N(T)$. Note that a phase coherence of the superconductors is manifested for the resistance of the system at a zero bias voltage. It may lead to a significant decrease in the resistance in comparison with R_n , even at large lengths $d_S \gg \xi_N(T)$ (a "long-range proximity effect").⁴⁾ This result of course also holds for superconductors with a gap, for which the equations given above for $R(V=0)$ become the same as the exact result at $T=0$, as can be shown.

The "resistive model" in (6) is valid (at low voltages) for gapless superconductors. For superconductors with a gap, the properties of this system can be studied analytically when there are potential barriers at the $S-N$ junctions whose resistances R_{bj} are significantly greater than the resistance of the N conductor. For the case of a steady-state phase difference between the superconductors, we find the following expressions for the current I_j through the j th barrier (from the S conductor to the N conductor) for the system in Fig. 1a, making use of results found in Ref. 5:

$$I_j = \int_0^\infty d\epsilon [F_{jj}(\epsilon) + F_{12}(\epsilon)\cos\varphi - (-1)^j F(\epsilon)\sin\varphi] = J_{jj} + J_{12}\cos\varphi - (-1)^j J\sin\varphi. \quad (11)$$

At low temperatures ($T \ll \Delta$) and at low voltages ($eV \ll \Delta$), to which we restrict the discussion, we find

$$F_{ij}(\epsilon) = (1/4e)G_{bi}G_{bj}\rho_N[\tanh(\beta\epsilon_+) - \tanh(\beta\epsilon_-)](\text{Im}f_{s\epsilon}^R)^2 \times \text{Re}[\delta_{ij}\tanh(k_\epsilon d_i)/k_\epsilon + q_{ij}(k_\epsilon)], \quad (12)$$

$$F(\epsilon) = (-1/2e)G_{b1}G_{b2}\rho_N(-1)\text{Im}\{\tanh(\beta\epsilon)\text{Re}f_{s\epsilon}^R + (i/2)\text{Im}(f_{s\epsilon}^R)[\tanh(\beta\epsilon_+) + \tanh(\beta\epsilon_-)]\}f_{s\epsilon}^R q_{ij}(k_\epsilon),$$

where

$$k_\epsilon = \sqrt{2(-i\epsilon + \gamma)/D}, q_{ij}(k) = 1/k[\tanh(kd_1) + \tanh(kd_2) + \coth(kd_3)]\cosh(kd_i)\cosh(kd_j),$$

where $\epsilon_\pm = \epsilon \pm eV$, $f_{s\epsilon}^R$ is the Green's function of the superconductor ($\text{Im}f_{s\epsilon}^R = -1$ for $\epsilon \ll \Delta$), $\delta_{ii} = 1$, and $\delta_{12} = 0$. An analog of the system in Fig. 1a is the planar structure in Fig. 1b, in which tunnel junctions are formed between the S banks and an N film which is thin [in comparison with $\xi_N(T_c)$]. We find the functions F_{ij} for this geometry by a method like that used in Ref. 6 in a study of the conductance of an $S-N$ quantum interferometer. We write the result for the important particular case in which the tunnel junctions, of identical width w , lie one on top of the other. In this case we have $F_{11} = r^2 F_{22} = r F_{12}$, where $r = R_{b1}/R_{b2}$, and F_{12} and F are defined as in (12), with the function

$$q_{12}(k) = \{kw - \sinh(kw) + 2\sinh^2(kw/2)[1 + \exp(-2kL - kw)]\}/k^3w^2. \quad (12a)$$

Here L is the distance from the left edge of the tunnel junction to the edge of the film, whose length is assumed to be large in comparison with L and $\tilde{\xi}_N = \sqrt{D/\gamma}$. Noting, as above, that the current through one of the barriers (with the resistance R_{b1}) is $I_1 = I$, while there is no current through the other (with a resistance R_{b2}), we find an expression for the current-voltage characteristic and an equation determining the functional dependence $\varphi(V)$ from (12):

$$I(V) = J_{11}(V) + J_{22}(V) + 2J_{12}(V)\cos\varphi(V),$$

$$\cos\varphi = [J(J^2 + J_{12}^2 - J_{22}^2)^{1/2} + J_{22}J_{12}]/(J^2 + J_{22}^2). \quad (13)$$

Expressions (11)–(13) can be evaluated analytically in several limiting cases. In particular, for the system in Fig. 1a, at absolute zero, under the condition $d_3 \gg d_1 = d_2 = d \gg \xi_N(T_c)$, we find

$$J_{ij}(V) = J_1 r^{i+j-2} (-1)\text{Im}\{2\delta_{ij}\ln \cosh\kappa(V) + \ln[1 + 2\tanh\kappa(V)]\},$$

$$J(V) = J_1 r \text{Re} \ln \left[\frac{3}{1 + 2\tanh\kappa(V)} \right], \quad (14a)$$

where $J_1 = G_{b1}^2 \rho_N D / 4ed$, and $\kappa(V) = (k_\epsilon d)$ ($\epsilon = eV$). For the system in Fig. 1b, for wide junctions ($w \gg \tilde{\xi}_N$), we find (assuming $\gamma \ll \Delta$)

$$J_{ij}(V) = J_1 r^{i+j-2} \arctan(eV/\gamma), \quad J(V) = J_1 r \ln(\Delta/\sqrt{(eV)^2 + (\gamma)^2}). \quad (14b)$$

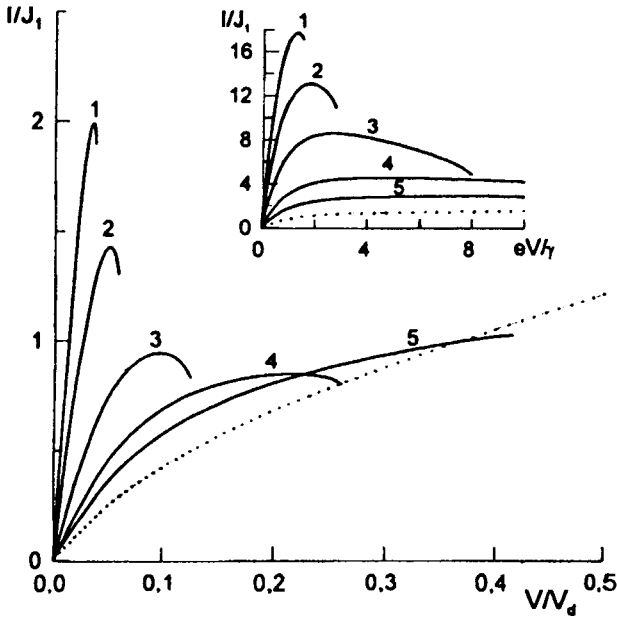


FIG. 3. Current-voltage characteristics found at voltages $V \leq V_*$ in the systems in Fig. 1a with tunnel barriers at the $S-N$ junctions. These results were calculated for $T=0$ and the values $r=R_{b1}/R_{b2}=3$ (1), 2 (2), 1 (3), 0.5 (4), and 0.3 (5). $V_d=D/ed^2$. The inset shows similar curves found for systems like that in Fig. 1b for the case of wide, overlapping tunnel junctions at $T=0$ and $\Delta/\gamma=100$. 1— $r=4$; 2—3; 3—2; 4—1; 5—0.5. The dotted curve represents the behavior corresponding to the case $r=0$ ($R_{b2} \rightarrow \infty$), in which there are no interference effects.

Here $J_1 = G_{b1}^2 \rho_N D / 4ew$. Figure 3 shows current-voltage characteristics corresponding to the steady state (voltages $V \leq V_*$), as calculated from (13) and (14) for various values of $r = R_{b1}/R_{b2}$. We thus see that, in contrast with the resistive model, the $I-V$ is nonlinear in the steady-state regime. In general, the threshold current does not correspond to the threshold voltage [$I_* \geq I(V_*)$ and $\varphi(V_*) \neq \pi/2$], and a negative resistance may arise even at $V \leq V_*$.

The equations describing a time-varying regime in the systems of Fig. 1, a and b, with tunnel barriers (a detailed analysis of this case will be reported separately) take a particularly simple form when the threshold voltage is low: $eV_* \ll \epsilon_{dj}, \gamma$. At voltages $V \sim V_*$ we then have

$$I = V(G_{11} + G_{12} \cos \varphi) + J_c \sin \varphi, \quad (V - \partial_t \varphi / 2e)(G_{22} + G_{12} \cos \varphi) = J_c \sin \varphi, \quad (15)$$

where $G_{ij} = \partial_V J_{ij}(V=0)$ and $J_c \equiv J(V=0)$. For the system in Fig. 1b with overlapping $S-N$ tunnel junctions (under the conditions $eV, T \ll \Delta$), we have $G_{ij} = G_{11} r^{i+j-2}$. Equations (15) and (11) differ from (6) in that they contain terms with $\cos \varphi$ which become important at a conductance $G_{12} \sim \min\{G_{ii}\}$. In particular, at $G_{12} > G_{22}$, there is no threshold voltage in the region $eV \ll \epsilon_{dj}, \gamma$, and there is a steady-state solution described by

expressions (13) and (15). In the case $G_{12} < G_{22}$, there is a transition to Josephson oscillations in this interval. The functional relationship between the current and the phase,

$$I = (\partial_t \varphi / 2e)(G_{11} + G_{12} \cos \varphi) + J_c \sin \varphi [2 + (G_{11} - G_{22}) / (G_{22} + G_{12} \cos \varphi)],$$

has a rich spectrum of harmonics (if $G_{22} \neq G_{11}$), even at “high” currents ($I \gg I_*$, with $\varphi \approx 2eVt$). As an example, the inset in Fig. 2 shows current–voltage characteristics found from (15) for the case $r \gg 1$.

There is the interesting question of the response of these systems to an oscillating voltage $V(t) = \bar{V} + V_{\sim} \sin(\omega t)$. A detailed analysis of this question will be published in a separate paper; here we simply note that (under the condition $\bar{V} > V_*$) the presence of an alternating voltage may lead to an increase in the conductance of this system on certain parts of the current–voltage characteristic, and it may fix the average voltage across the S banks at the value $\omega/2e$ (or at a multiple of this value at a large amplitude V_{\sim}).

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²Results for the “hybrid” systems with mesoscopic N rings studied experimentally in Ref. 2, in which the phenomena under discussion here are complicated by the Aharonov–Bohm effect, will be published in a separate paper.

³Incorporating barriers with a resistance R_{bj} at the $S-N$ junctions reduces to the replacement of R_j by $\rho_n d_j + R_{bj}$ and changes in the threshold values I_* and V_* .

⁴Here, of course, the lengths d_S must not exceed the values at which the phase fluctuations become important. For thermal fluctuations, this condition means¹⁵ $eT \ll J_c(d_S) \sim J_c(0) \exp(-d_S/\xi_N)$.

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