Quasiparticle current of ballistic NcS′S contacts

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An approach developed by Blonder, Tinkham, and Klapwijk[4] for calculating current–voltage characteristics of NcS contacts is generalized to the case of a dirty S′S electrode with spatially nonuniform superconducting properties. The amplitudes for Andreev and normal reflection of electrons from the constriction are calculated. The relationship between the quasiparticle current of the contact and the quasiparticle spectrum in the S′S electrode is studied for arbitrary values of the parameters of the S′ and S materials and for arbitrary values of the superconductivity suppression parameter at the S′S interface.

Analysis of processes which occur in Josephson tunnel junctions with a high critical current density[1] and also certain types of high-T_c contacts[2] shows that their properties should be close to those of structures consisting of a multiple point contact formed by ScIcS′S or SS′cIcS′S constrictions. In general, it is reasonable to assume that the dimensions of these constrictions are much smaller than the electron mean free path in the electrodes and that the transmission of the barrier at the constriction is not too low. The latter circumstance gives rise to subharmonic features on the current–voltage characteristics of the structures. These features are generated by multiple Andreev reflection of the quasiparticles.

The positions of these features have been calculated[3] in a model which assumes thermodynamic equilibrium and spatial uniformity of the superconducting properties of the electrodes (the OTBK model). The electrodes themselves were assumed to be pure metals. The calculations made heavy use of the coefficients for normal and Andreev reflection as calculated in Ref. 4 for an NcS constriction.

A microscopic theory of the current–voltage characteristics of pure NcS and NcN′S contacts was derived in Refs. 5 and 6. Those studies essentially demonstrated the validity of the phenomenological approach of Blonder, Tinkham, and Klapwijk (BTK) and the OTBK model in calculations of quasiparticle currents.[1)]

The properties of "dirty" NN′S structures have been studied previously only in the model of Refs. 7–10, in which all nonequilibrium processes are localized in the N′ material of the bridge connecting the massive electrodes. These electrodes are at thermodynamic equilibrium. That model is valid for describing processes in NcS and NcN′S structures in which the electron mean free path in the constriction (N′) is much smaller than the geometric dimensions of the constriction.[11,12]

In the present letter we look at a fundamentally different situation: a ballistic constriction of finite transmission connecting two dirty electrodes. In this case, the nonequilibrium processes are localized in the constriction. From the physical standpoint, the approach developed below is a direct generalization of the BTK model. Making use of the circumstance that the physical processes which occur in the constriction can be described by both the Bogolyubov–de Gennes equations and by the Green's-function formalism, we write the coefficients for direct and Andreev reflection from the constriction in terms of parameters of the quasiparticle energy spectrum in the electrodes. The spectrum itself will be calculated through the use of Green's functions. That approach retains the simplicity and physical clarity of the BTK model, in combination with the use of the Green's function method for describing the steady-state proximity effect in the electrodes.

MODEL OF THE CONTACT

We assume that the geometric dimensions of the constriction in the NcS′S contact are much smaller than the electron mean free paths in the N (l_n) and S′ ($l_{s'}$) materials. We also assume that the conditions of the dirty limit hold for the S′ and S metals. We impose no restrictions on l_n or the transmission of the constriction, D. Since the dimensions of the constriction are small, we can also assume that the probability for backscattering of quasiparticles after passage through the constriction is negligible. We make use of that circumstance. Also noting that both the order parameter and the normal and anomalous Green's functions, averaged over the Fermi surface [$\langle G_\epsilon(x)\rangle$ and $\langle F_\epsilon(x)\rangle$, respectively], in the Gor'kov[13] equations are independent of the spatial coordinates in the S′ region near the constriction, we write the solution of those equations in region S′ in the form of plane waves:

$$\begin{pmatrix} G_\epsilon(x,x') \\ F_\epsilon(x,x') \end{pmatrix} = c(x')\begin{pmatrix} g(x) \\ f(x) \end{pmatrix} e^{iq_2^+ x} + d(x')\begin{pmatrix} f(x) \\ g(x) \end{pmatrix} e^{-iq_2^- x}. \tag{1}$$

For definiteness, we have selected the propagation of the incident wave to be directed out of the N metal into the composite S′S electrode. Here $g(x)$ and $f(x)$ are semiclassical Green's functions, independent of x in the constriction, which determine the amplitudes of the electron- and hole-like excitations, respectively. A relationship between these functions and an expression for the wave vector in (1),

$$\eta = \frac{f(x)}{g(x)} = \frac{i\langle F_\epsilon(x)\rangle}{1+\langle G_\epsilon(x)\rangle}, \quad q_2^{\pm} = \sqrt{2m_2\left(\mu \pm i\,\frac{\langle G_\epsilon(x)\rangle^2 + \langle F_\epsilon(x)\rangle^2}{2\tau}\right)}, \tag{2}$$

follow directly from the system of nonlinear equations for $g(x)$ and $f(x)$ which are found by substituting (1) into the Gor'kov equations.

At the same time, we can use the Bogolyubov–de Gennes equations in the constriction, and we can seek a solution of these equations in the form of incident waves $\Psi_{\rm inc}$ (e.g., incident from N on S′S), reflected waves $\Psi_{\rm refl}$, and transmitted waves $\Psi_{\rm tran}$ (into the S′S electrode). In particular, we have

$$\Psi_{\rm tran} = c\begin{pmatrix} u \\ v \end{pmatrix} e^{iq_2^+ x} + d\begin{pmatrix} v \\ u \end{pmatrix} e^{-iq_2^- x}, \quad q_2^{\pm} = \sqrt{2m_2(\mu \pm \epsilon)}, \tag{3}$$

where m_2 and ϵ are the effective mass and energy of the quasiparticle, and u and v are the amplitudes of the electron- and hole-like excitations, respectively. Proceeding as in the BTK model, we easily find the following expressions for the coefficients of Andreev reflection, $A(\epsilon)$, and normal reflection, $B(\epsilon)$:

$$A(\epsilon)=\frac{|\eta|^2}{[1+Z^2(1-|\eta|^2)]^2}, \quad B(\epsilon)=\frac{Z^2(1+Z^2)(1-|\eta|^2)^2}{[1+(1-|\eta|^2)Z^2]^2}, \quad \eta=\frac{v}{u}. \tag{4}$$

The so-called Z-factor in (4), which was introduced in Ref. 4, is determined by the transmission coefficient of the constriction: $D^{-1}=1+Z^2$. Now noting that the energy-dependent terms in the expression for the wave vector in (2) and (3) are small in comparison with the chemical potential μ in the contact region, we easily see that solutions (1) and (3) have the same structure, and the parameters η introduced in (2) and (4) have the same meaning and are identical. Substituting the expression for η from (2) into (4), and using the normalization condition $\langle G_\epsilon\rangle^2+\langle F_\epsilon\rangle^2=1$, we find the relationships which we need between (on the one hand) the coefficients $A(\epsilon)$ and $B(\epsilon)$ and (on the other) the semiclassical Green's functions $\langle G_\epsilon(0+)\rangle$ and $\langle F_\epsilon(0+)\rangle$, which characterize the local energy spectrum of the S$'$S electrode in the constriction region:

$$A(\epsilon)=\frac{|\langle F_\epsilon(0+)\rangle|^2}{|1+2Z^2+\langle G_\epsilon(0+)\rangle|^2}, \quad B(\epsilon)=\frac{4Z^2(1+Z^2)}{|1+2Z^2+\langle G_\epsilon(0+)\rangle|^2}. \tag{5}$$

In the spatially uniform case we have $\langle G_\epsilon(0+)\rangle=-i\epsilon/\sqrt{\Delta_0^2-\epsilon^2}$, and $\langle F_\epsilon(0+)\rangle=\Delta_0/\sqrt{\Delta_0^2-\epsilon^2}$, and the BTK result[4] follows from (5).

Equations (5) are the basic result of the present letter. They constitute a natural generalization of the result of the BTK model to the spatially nonuniform superconducting state of the S$'$S electrode. As in Ref. 4, they make it possible to calculate the current through the NcS$'$S constriction:

$$I(V)=\frac{R_0^{-1}}{1+Z^2}\int_{-\infty}^{+\infty}[f_0(\epsilon+eV)-f_0(\epsilon)][1+A(\epsilon)-B(\epsilon)]d\epsilon, \tag{6}$$

$$R_0=[2N_1(0)Se^2v_{F1}]^{-1}.$$

Here $f_0(\epsilon)$ is a Fermi distribution, S is the area of the contact, and $N_1(0)$ and v_{F1} are the density of states and the Fermi velocity of the electrons in the N metal. Correspondingly, using (5), we can generalize the results of the OTBK model[3] to calculate the quasiparticle current of ballistic SS$'$cS$'$S contacts. Expression (6) is written for the 1D model of a contact. In general it should be averaged over angles, and the angular dependence of the constriction transmission coefficient D should be taken into account.

The problem is thus reduced to one of solving a steady-state problem, that of calculating the Usadel functions $\langle G_\epsilon(0+)\rangle$ and $\langle F_\epsilon(0+)\rangle$ at the free boundary of an S$'$S sandwich.

PROXIMITY EFFECT IN A "DIRTY" S'S SANDWICH

To solve this problem, it is convenient to use the functions $\theta(\epsilon,x)$ which are related to the Usadel functions by the equations $\langle G(x)\rangle=\cos\theta(\epsilon,x)$, $\langle F_\epsilon(x)\rangle=\sin\theta(\epsilon,x)$. These functions are the solutions of the boundary-value problem[14–16]

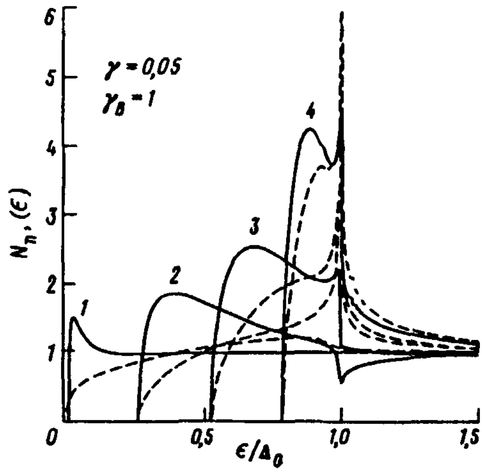

FIG. 1. Density of states in the N′ region of an N′S sandwich at the free surface of the N′ metal (solid curves) and at the N′S interface (dashed curves), normalized to the density of states in the N metal in the normal state. Curves *1–4*) $d/\xi_{s'}=10, 2, 1$, and 0.5, respectively. The suppression parameters are $\gamma=0.05$ and $\gamma_B=1$.

$$\xi^2_{s',s}\theta''_{s',s}(x)+i\epsilon\sin\theta_{s',s}(x)+\Delta_{s',s}(x)\cos\theta_{s',s}(x)=0, \tag{7}$$

$$\Delta_s(x)\ln\frac{T}{T_c}+2\frac{T}{T_c}\sum_\omega\left[\frac{\Delta_s(x)}{\omega}-\sin\theta_s(x,\epsilon-i\omega)\right]=0, \tag{8}$$

$$\gamma_B\xi_{s'}\theta'_{s'}=\sin(\theta_s-\theta_{s'}),\quad \gamma\xi_{s'}\theta'_{s'}=\xi_s\theta'_s,\quad \gamma_B=\frac{R_B}{\rho_{s'}\xi_{s'}},\quad \gamma=\frac{\rho_s\xi_s}{\rho_{s'}\xi_{s'}},$$

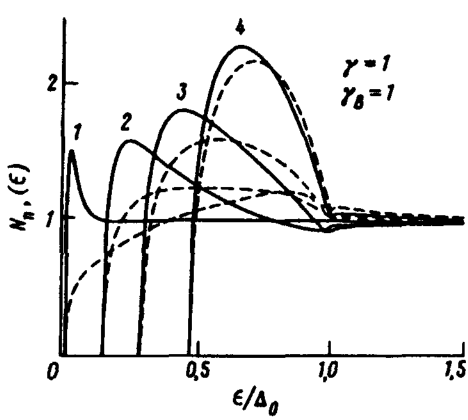

FIG. 2. Density of states in the N′ region of an N′S sandwich at the free surface of the N′ metal (solid curves) and at the N′S interface (dashed curves), normalized to the density of states of the N metal in the normal state. Curves *1–4*) $d/\xi_{s'}=10, 2, 1$, and 0.5, respectively. The suppression parameters are $\gamma=1$ and $\gamma_B=1$.

$$\theta'_{s'}(-d_{s'})=0, \quad \theta_s(\infty)=\arctan(i\Delta_0(T)/\epsilon). \tag{9}$$

Here $\xi_{s',s}, \rho_{s',s}$, and $T_{c',c}$ are the coherence lengths, resistivities, and transition temperatures of the S′ and S materials, respectively; R_B is the resistivity of the S′S interface; d is the thickness of the S′ layer; ω are the Matsubara frequencies; and $\Delta(T)$ is the equilibrium value of the modulus of the order parameter in the interior of the S electrode. The prime means differentiation with respect to the coordinate x, which is reckoned from the plane of the constriction in the direction perpendicular to the S′S interface. In the calculations below we consider only the case $T_{c'}=0$. (There is no serious problem in generalizing the results to the case of nonzero temperatures of the S′ material;[16,17] a corresponding study will be carried out in the future.)

Boundary-value problem (7)–(9) has been solved by numerical methods for arbitrary thicknesses of the S′ layer and for arbitrary values of the suppression parameters γ and γ_B. Figures 1 and 2 show the results of calculations of the density of states at the S′S interface (dashed curves) and at the free boundary of the S′ material (solid curves) under the condition $T \ll T_c$. Significantly, at all values of d there is nonzero energy gap Δ_g in the S′ region. This gap is induced by a proximity effect. As a result, at small values of γ and under the condition $d \lessdot \xi_{s'}$, there are two structural features in the density of states at the free boundary, with energies ϵ which are equal to the equilibrium value of the order parameter in the S electrode, $\varepsilon=\Delta(T \ll T_c)=\Delta_0$, and the effective gap in the S′ material, $\varepsilon=\Delta_g$. The first of these is suppressed with increasing d or γ. The second, in contrast, becomes sharper with increasing d. The conclusion that there is a nonzero energy gap Δ_g in a normal metal with $T_{c'}=0$ in an NS system has previously been reached only in the limit $d \lessdot \xi_{s'}$ (in the McMillan model,[18] for a low transmission of the NS interface, and in Ref. 17 for an arbitrary transmission).

The two features in the density of states give rise to corresponding peaks on the

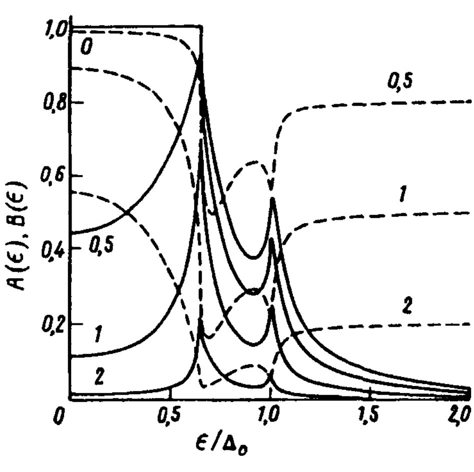

FIG. 3. Coefficients of Andreev reflection, $A(\epsilon)$ (solid curves), and of normal reflection, $B(\epsilon)$ (dashed curves), of a ballistic NcN′S constriction ($\gamma_M=\gamma d/\xi_{s'}=0.1$, $\gamma_{BM}=\gamma_B d/\xi_{s'}=1$) for various values of the factor Z ($Z=0, 0.5, 1$, and 2).

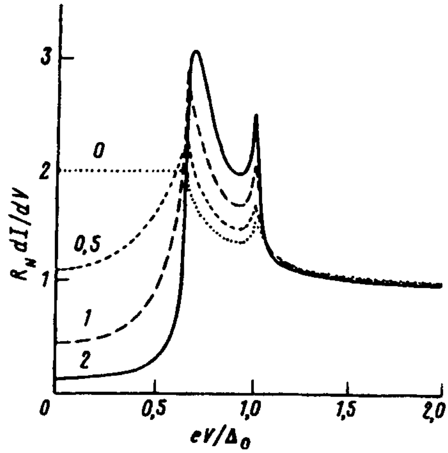

FIG. 4. Conductance of a ballistic NcN′S constriction ($\gamma_M=\gamma d/\xi^*_{s'}=0.1$, $\gamma_{BM}=\gamma_B d/\xi_{s'}=1$) for various values of the factor Z ($Z=0, 0.5, 1$, and 2) at a temperature $T \ll T_c$.

plots of $A(\varepsilon)$, $B(\varepsilon)$, and the differential conductance of an NcS′S contact with a thin S′ layer ($d \ll \xi_{s'}$; Figs. 3 and 4). At $Z=0$ and at low voltages, the conductance is doubled by Andreev reflection, as in the case of the BTK model. In dirty constrictions, this doubling does not occur.[11,12] With increasing Z, the conductance at a zero bias voltage decreases and depends on Z only. In accordance with the discussion above, the structural feature at large energies or voltages is easily smoothed over by any para-destroying mechanism (e.g., an increase in γ or d). The position of the first peak, in contrast, is stable. It might be utilized to study the proximity effect in an S′S sandwich for arbitrary combinations of the materials making up the sandwich.

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[1)]The BTK model gives the correct expression for only the quasiparticle component of the current. It cannot be used in calculations of the superconducting and interference components.

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