Preparation of squeezed spin states of atoms for subquantum interferometry

G. F. Grom and A. M. Kuz'mich
Institute of Nuclear Problems, Belarus State University, 220080 Minsk, Belarus

(Submitted 20 April 1995)
Pis'ma Zh. Éksp. Teor. Fiz. **61**, No. 11, 883–887 (10 June 1995)

A method is proposed for preparing squeezed spin states of atoms for interferometric measurements with an accuracy exceeding the standard quantum limit. © *1995 American Institute of Physics.*

Squeezed states of light have attracted intense interest[1,2] primarily because they might be used for interferometric measurements with an error better than the standard quantum limit $\delta\phi \sim 1/\sqrt{N}$, where N is the number of photons involved in the measurement.[3] As was shown in Refs. 3 and 4, the best sensitivity which has been achieved with the help of squeezed states of light is $\delta\phi \sim 1/N$. Yurke[5] demonstrated that states which increase the sensitivity of interferometric measurements to $\delta\phi \sim 1/N$ exist not only for bosons (photons) but also for fermions. Such states have been termed "squeezed spin states."[6] Kitagawa and Ueda proposed a recipe for generating squeezed spin states for *charged* fermions through the use of the Coulomb interaction between them.[7] On the other hand, no method has so far been found for creating squeezed spin states for atoms (either bosons or fermions), despite the major interest which atomic interferometry has attracted in recent years. The present letter is an effort toward the solution of this problem. The generation recipe which we are proposing here is based on an absorption of two orthogonally polarized electromagnetic-field modes by atoms. One of these field modes is in a coherent state, while the other is in a squeezed-vacuum state. The atoms are in a coherent spin state; i.e., the spins of all the atoms are oriented in a common direction. The photons which have been absorbed and which have gone into excited states of the atoms are in a squeezed spin state. We will discuss the most transparent case, of spin-1/2 fermion atoms in the ground and excited states.

We consider the following situation: We place N two-level atoms with a total spin $F=1/2$ in the ground state, and $F^e=1/2$ in the excited state, in a resonator which has two orthogonally polarized electromagnetic field modes, 1 and 2. Mode 1 is linearly polarized along the X axis, and mode 2 along the Y axis. We adopt the axis of the resonator as the quantization axis, Z. These modes are described by the operators a_i, a_i^+, with the commutation relations

$$[a_i, a_j^+] = \delta_{ij}, \qquad [a_i, a_j] = 0, \qquad i,j = 1,2. \tag{1}$$

For use below we introduce linear combinations of the operators a_i, a_i^+, which correspond to circularly polarized photons: $c_{R,L} = 1/\sqrt{2}(a_1 \pm i a_2)$. It is easy to verify that commutation relations (1) remain in force. The selection rules in terms of the angular-momentum projection lead to the scheme of transitions shown in Fig. 1. We see that the four-level system can be represented as two independent two-level systems. We assign

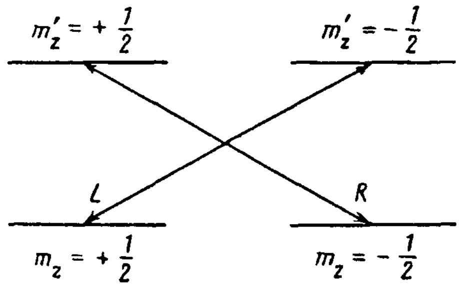

FIG. 1. Scheme of the $(F=1/2)-(F^e=1/2)$ transition. Here R (L) corresponds to right-hand (left-hand) polarized light.

the index L to system of levels 1, 4, and R to levels 2, 3. We assume that all the atoms are in a volume whose linear dimensions are small in comparison with the wavelength of the electromagnetic field. The operators

$$F_z^R=\tfrac{1}{2}\sum_i (|3\rangle\langle 3|-2\rangle\langle 2|),\quad F_z^L=\tfrac{1}{2}\sum_i (|4\rangle\langle 4|-|1\rangle\langle 1|),$$

$$F_+^R=\sum_i |3\rangle\langle 2|,\quad F_+^L=\sum_i |4\rangle\langle 1|, \tag{2}$$

$$F_-^R=\sum_i |2\rangle\langle 3|,\quad F_-^L=\sum_i |1\rangle\langle 4|$$

then give a complete description of this set of atoms. The Hamiltonian of the system consisting of the atoms and the two field modes can be written

$$H=\hbar\omega(c_R^+c_R+c_L^+c_L)+\hbar\omega_0(F_z^R+F_z^L)+H_I. \tag{3}$$

The Hamiltonian of the interaction of the atoms with the field is given in the rotating-wave approximation by

$$H_I=-\hbar g(F_+^R c_R+\text{H.c.})-\hbar g(F_+^L c_L+\text{H.c.}). \tag{4}$$

Here g is the interaction constant. The equations of motion in the case $\omega=\omega_0$ can be written as follows in a coordinate system rotating at a frequency ω:

$$\frac{dc_{R,L}}{dt}=igF_-^{R,L},\quad \frac{dF_-^{R,L}}{dt}=-2igc_{R,L}F_z^{R,L}. \tag{5}$$

The equation for $F_+^{R,L}$ is found from the latter equation by complex conjugation. We consider the case in which the number of atoms, N, is much larger than the average number of photons:

$$N\gg\langle n_1+n_2\rangle. \tag{6}$$

(It can be shown that this condition is a necessary condition for producing excited atoms in squeezed spin states.) Equations (5) have the solution

$$c_{R,L}(t)=c_{R,L}(0)\cos(g\sqrt{2F_z^{R,L}(0)}t)+\frac{iF_-^{R,L}(0)}{\sqrt{2F_z^{R,L}(0)}}F_-^{R,L}(0)\sin(g\sqrt{2F_z^{R,L}(0)}t),$$

$$F_-^{R,L}(t)=F_-^{R,L}(0)\cos(g\sqrt{2F_z^{R,L}(0)}t)+2i\frac{c_{R,L}(0)}{\sqrt{2F_z^{R,L}(0)}}F_z^{R,L}(0)\sin(g\sqrt{2F_z^{R,L}(0)}t). \tag{7}$$

We introduce a spin operator of the excited atoms, $\mathbf{F}^e$:

$$F_x^e=\tfrac{1}{2}\sum_i(|3\rangle\langle 4|+|4\rangle\langle 3|),\quad F_y^e=\frac{-i}{2}\sum_i(|3\rangle\langle 4|-|4\rangle\langle 3|),\quad F_z^e=\tfrac{1}{2}\sum_i(|3\rangle\langle 3|-|4\rangle\langle 4|). \tag{8}$$

It is simple to show that under condition (6), with a large fraction of the atoms in the ground state, the transverse components of the spin operator of the atoms in the upper state, in which we are interested, can be written

$$F_x^e=\frac{1}{N}(F_+^R F_-^L+F_+^L F_-^R),\quad F_y^e=\frac{-i}{N}(F_+^R F_-^L-F_+^L F_-^R). \tag{9}$$

Analysis of the sensitivity of measurements of the angle through which the spin of 1/2 rotates yields the following expression for this angle: [4-6]

$$\delta\phi=\frac{\sqrt{(\Delta F_y)^2}}{\langle F_x\rangle}. \tag{10}$$

We assume that all the atoms are initially (at $t=0$) in the ground state, with spins directed along the X axis, that mode 1 is in the coherent state $|\alpha\rangle_1$, and that mode 2 is in the squeezed-vacuum state $|0,\eta\rangle_2=S_2(\eta)|0\rangle_2$, which arises when the squeezing operator $S_2(\eta)=\exp(1/2\eta^* a_2^2-1/2\eta a_2^{+2})$, α,η acts on the vacuum state. Here α and η are arbitrary complex numbers. We choose α and η to be real, with $\eta=-r$, where $r>0$ is the degree of squeezing. Uncertainty ellipses for both field modes are shown in Fig. 2. Now using Eqs. (7), (9), and (10), we can show that at times t_1 satisfying $g\langle\sqrt{2F_z^{R,L}(0)}\rangle\ t_1=\pi/2+\pi k$, where $k=0,1,2,\ldots$, we have $\langle F_x\rangle=n/2$, $(\Delta F_y)^2=\frac{1}{4}\times n\ \exp(-2r)$ and

$$\delta\phi=\frac{\exp(-r)}{\sqrt{n}}. \tag{11}$$

This result is extremely transparent: The number of excited atoms is equal to the number of photons which are initially present, n, and the factor $\exp(-r)$ is precisely the same as the corresponding factor by which the sensitivity of the photon interferometry is increased. This particular configuration of the electromagnetic field makes it possible to break through the standard quantum limit on the error in measurements of phase shifts in

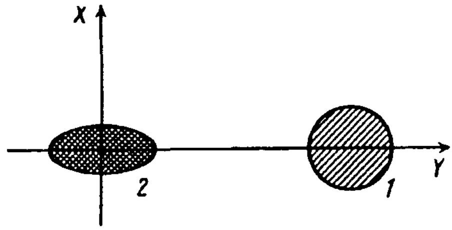

FIG. 2. Uncertainty ellipses for field modes 1 and 2.

polarized-optics interferometry.[8] During the absorption of the photons, the atoms which are in the squeezed spin state at the bottom acquire the correlations which exist between the photons. This is the reason for the improvement in the sensitivity of interferometric measurements with these atoms.

The transitions most appropriate for our purposes are M1 and E2 transitions, since in these cases the lifetime of the atom in its excited state, τ, determined by spontaneous emission, is $\sim 10^{-3}-10^{-2}$ s. Over this time, atoms moving at a velocity $v \sim 5\times 10^4$ cm/s traverse a distance of 0.5 to 5 m; such distances would usually be quite sufficient for purposes of atomic interferometry. For an E1 transition, in contrast, τ is only 10^{-7} s. A ^{207}Pb atom with the M1 transition ($^3P_0$, $F=1/2$)–($^3P_1$, $F^e=1/2$), of frequency $\omega_0 \simeq 2.35\times 10^{14}$ Hz, is suitable for preparing squeezed spin states of atoms. After the preparation procedure has been completed, it is necessary to separate the excited atoms from the ground-state atoms, by any type of quantum nondemolition atomic-level measurements (e.g., the method described in Ref. 9). This element is not absolutely necessary, however, since one could send all the atoms to the interferometer, carrying out a selection by level in the final step of the detection of the atoms.*

We sincerely thank V. G. Baryshevskiĭ for interest in this study and for a useful discussion. One of us (A.K.) wishes to thank the Soros International Science Foundation for financial support.

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Translated by D. Parsons