Optical pulse collapse in defocusing active medium

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A phase-gradient mechanism of a pulse compression in the Kerr-lens mode locking laser systems, which can be described by a complex Ginzburg–Landau equation, has been studied. A pulse collapse has been found to occur in a *defocusing* gain medium, in contrast with the collapse arrest in a focusing active medium. © *1995 American Institute of Physics.*

1. The self-focusing of a powerful light beam that propagates through a nonlinear Kerr medium is a classical example of the wave collapse in a distributed Hamiltonian system (for a review see Ref. 1 and the references cited there). The possibility of a collapse (sometimes called a blow-up) in such systems significantly depends on the dimension of the problem. By contrast, the singularity formation in a finite time due to nonconservative mechanisms demonstrates rather different behavior. First of all, the collapse in this case depends slightly on the dimensionality factor and therefore for its study it is enough to consider low-dimensional systems. In the present paper we treat this problem for a one-dimensional complex Ginzburg–Landau (GL) equation, when the nonlinearity does not saturate linear exponential growth but, on the contrary, leads to an explosive growth of the amplitude for the homogeneous states. Such situation, in particular, is found in Rayleigh–Benard convection in binary mixtures[2–4] and in laser mode-locking laser systems.[5–8] In spite of the existence of such tendency for the homogeneous generation, the pulse onset and its dynamics represent as we shall show, the result of a specific interference of the Hamiltonian and non-Hamiltonian terms, which is a *phase-gradient mechanism.* This mechanism was initially suggested in Refs. 3 and 4 in order to explain the observed dynamics of the coherent structures in a convection of binary mixtures.

In this letter we study a pulse compression due to the phase-gradient mechanism in passively mode-locking laser systems. We demonstrate that a collapse takes place in a *defocusing* gain medium in contrast with the collapse arrest in a focusing active medium. On the basis of the GL model for the passive mode-locking laser systems we explain how a quasi-stable pulse can be formed without limiting action of the fast saturable absorber (quintic term in the GL equation). We also show that the sign of the pulse frequency chirp (i.e., the increase or the decrease of the local pulse frequency from the forward pulse front to the backward one) is a one-to-one correlated with the sign of the Kerr nonlinearity, i.e., it is the focusing or the defocusing.

2. A theory of passive mode-locking developed in Refs. 5 and 6 has highlighted the generic nature of many mode-locking systems. A large variety of mode-locking laser systems, which exploit different physical effects, can be described by the same model which is close to the complex GL equation. We consider here systems which exploit additive pulse mode-locking (APM) or Kerr lens locking (KLM).[6,7] In the KLM, the pulse-shortening mechanism is provided by the saturable absorber. The saturable-absorber action is based on the self-phase modulation effect which occurs due to the Kerr-type nonlinearity. A similar mechanism is realized in the APM, where a pulse narrowing results from the coherent addition of self-phase-modulated pulses.

If the relative changes in the pulse parameters during the passage through the elements of the system are small, the pulse evolution can be considered as a propagation through the equivalent medium, which is composed of a nonlinear dispersive part, and a gain medium. In this case the governing equation takes the form of a complex GL equation:

$$\frac{\partial E}{\partial z}=E+(1+iC_1)\frac{\partial^2 E}{\partial t^2}+(1-iC_2)|E|^2E. \qquad (1)$$

Here $E(z,t)$ is the normalized complex envelope of the electromagnetic wave, and z indicates the number of passes through the laser system. The maximal dimensionless amplification coefficient (in front of E on the right-hand-side of the equation) is chosen to be equal to 1; the coefficient C_1 is $C_1=D/g_2$, where D and g_2 account for the group-velocity dispersion and the gain dispersion, respectively; the parameter $C_2=\gamma/\delta$, where γ is a nonlinear Kerr coefficient, and $\delta>0$ describes the nonlinear action of the fast, saturable absorber. The gain contour is approximated by a parabola near the pulse carrier frequency. This equation also can be applied to the long amplifier when z has the meaning of a physical coordinate of an amplified pulse.

We note that an amplitude equation in the form of the GL equation (1) can be derived for many systems by means of a standard method which expands the basic equations near the threshold of an oscillatory instability (for details see, for example, a review[9]). In the limit $C_1,C_2\to\infty$, Eq. (1) tends to the nonlinear Schrödinger equation. We see that the case $C_1,C_2<0$ corresponds to the focusing medium and $C_1,C_2>0$ corresponds to the defocusing medium. Since Eq. (1) is invariant under a simultaneous sign change of C_1 and C_2 and $E\to E^*$, we can restrict the analysis to the case $C_2\geqslant 0$.

Obviously, a vacuum solution ($E=0$) of Eq. (1) is unstable. Any initial noise is amplified. When the amplitude of the solution becomes sufficiently large, the nonlinear term comes into play. This causes an explosive growth of the amplitude and E may develop a singularity at some finite z_0.

To demonstrate the explosive action of the nonlinear term in Eq. (1), we will consider a time-independent solution ($\partial/\partial t=0$). In this case a simple integration of equation (1) gives

$$|E(z)|^2=\frac{1}{\exp[-2(z-z_0)]-1},$$

where $z_0=\frac{1}{2}\ln(1+I_0)I_0^{-1}$, and $I_0=|E(z=0)|^2$.

We thus see that at the point $z=z_0$ the amplitude becomes infinite. Close to this point we have $|E(z)|^2\sim(z_0-z)^{-1}$.

Simultaneously, there is a rapid phase rotation which accelerates when approaching z_0, $\phi\sim-C_2/2\log(z_0-z)$. This means that the nonlinear amplification term leads to the explosive growth of the amplitude. A similar effect takes place in the case of a pulse evolution in a certain region of the (C_1,C_2) plane. In real systems, of course, this growth is limited, because additional effects which are not included in the model equation (1) becomes significant when amplitudes increase. We will show that in a certain region of the parameters (C_1,C_2) the phase-gradient mechanism can provide a generation of pulses with a finite amplitude, without regard for the higher-order saturation terms; i.e., the formation of a pulse can be described in the framework of Eq. (1).

3. The phase-gradient effect manifests itself as a fast phase rotation which results from an explosive increase of the pulse amplitude. As can be shown, a pulse collapse initiated by the nonlinear saturable absorber action [the cubic term $|E|^2E$ in Eq. (1)] can be stopped by a combined action of the Kerr nonlinearity and the gain dispersion. This effect, in contrast to the limiting action of the quintic term, may be considered as an *intrinsic* stabilizing mechanism. To understand this effect, it is convenient to rewrite Eq. (1) in the hydrodynamic form,

$$\partial_z I+\frac{\partial}{\partial t}Iv=2(1+I-\Omega^2)I+2\sqrt{I}\frac{\partial^2}{\partial t^2}\sqrt{I}, \tag{2}$$

$$\partial_z\Omega+v\frac{\partial}{\partial t}\Omega-C_2\frac{\partial}{\partial t}I=C_1\frac{\partial}{\partial t}\frac{(\sqrt{I})_{tt}}{\sqrt{I}}+\frac{\partial}{\partial t}\frac{1}{I}\frac{\partial}{\partial t}\Omega I. \tag{3}$$

Here $E(z,t)=\sqrt{I}\exp(i\phi)$, $\Omega=\partial_t\phi$, and the "velocity" is $v=2C_1\Omega$.

For simplicity let us consider first the limits $C_1\ll 1$ and $C_2\gg 1$. The influence of the Kerr term can then be amplified by an additional large factor C_2. It is natural, therefore, to assume that the frequency Ω is large, which is equivalent to the usual semiclassical approach (the exact criterion of the applicability of this approximation is $|\Omega|^2\gg|E|_{tt}/|E|,|\Omega_{tt}/\Omega|$). In this limit Eqs. (2) and (3) can be reduced to the form

$$\partial_z I=2I+2(I-\Omega^2)I, \tag{4}$$

$$\partial_z\Omega=C_2\frac{\partial I}{\partial t}. \tag{5}$$

For this system it is easy to represent the solution qualitatively. Consider an initial pulse, both weak and broad, specifically, a pulse which has a wide plateau with a sufficiently small amplitude and which decays away from the plateau. Let $\Omega=0$ initially. At the linear stage of the instability and in the blow-up regime that follows the amplitude will be approximately constant inside the plateau region. A sharp gradient of the amplitude forms at the boundary. This gradient, in turn, acts as a source for the generation of a *phase*

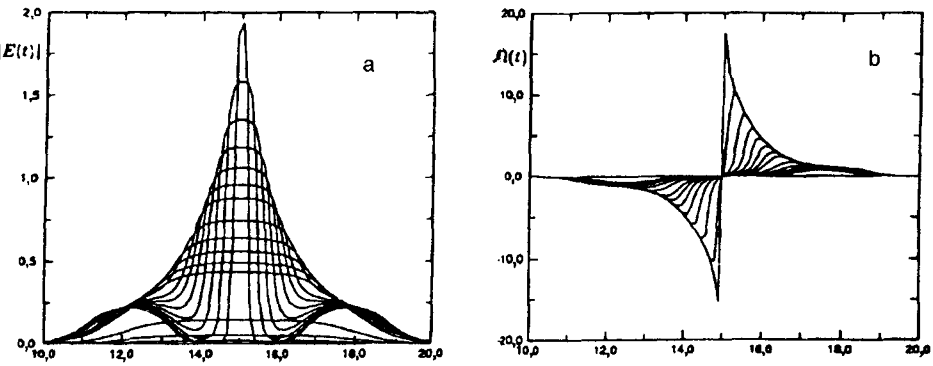

FIG. 1. The z evolution of the pulse amplitude $|E(t)|$ (a) and of the local frequency $\Omega(t)$ (b) for the reduced system (4), (5) with $C_1=0$ and $C_2=15$. The initial distribution is $E(0,t)=0.01$ [tanh$(t-16)$−tanh$(t-24)$]. The lower curves illustrate a motion of two counter-propagating waves of the phase gradient toward the pulse center, and the upper curves demonstrate the simultaneous process of an amplitude growth until some "moment" z and a pulse compression.

gradient, i.e., the Ω in this narrow region. On the other hand, the large value of Ω initially will saturate the blow-up. As a result, the two counter-propagating waves, which represent the moving plateau boundaries that generate the large wave number Ω, will be formed. The wave propagation velocity is not constant; it increases especially rapidly at the blow-up stage. The pulse will be "swallowed" by these waves that move from the edges to the center. This should finally result in a compression of the pulse up to the stage at which the previously ignored effects come into play. The typical evolution of the pulse dynamics in the framework of the reduced system (4), (5) are shown in Figs. 1 and 2. The results of a numerical integration of the system clearly show a pulse compression due to

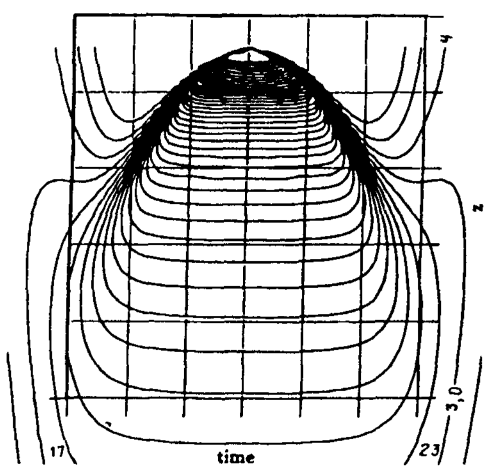

FIG. 2. The level lines of the amplitude for the reduced system in the (t,z) plane. We see that a pulse is almost reduced at $z>3.9$. The initial conditions are the same as for Fig. 1.

the phase-gradient effect. Starting from the initial conditions in the form of a pulse with a plateau we see how the two counter-propagating phase-gradient waves are initially formed and how they subsequently constitute the main factor of the pulse death.

4. From the above qualitative consideration it becomes clear that any gradient or *inhomogeneity* in the amplitude distribution will lead to the generation of a phase gradient, and hence to a suppression of the explosive growth of the amplitude. Therefore, any mechanism that smooths the pulse amplitude distribution works in favor of a collapse (in this case a system is close to the homogeneous state) and any effects leading to the sharpening of the gradients effectively work against the collapse. Numerically, this problem was initially considered in Ref. 10. In particular, the boundary in the (C_1, C_2) plane between the collapsing and the non-collapsing regions was determined. The collapsing region occupies two sectors in this plane for both the focusing ($C_1 C_2 < 0$) and defocusing ($C_1 C_2 > 0$) media. The noncollapsing sector represents the region located near the C_2 axis with the dominant part in the focusing quadrant. The hydrodynamic form of Eq. (1) can give an idea why a focusing medium can prevent a collapse more effectively than a defocusing nonlinearity. Let $C_2 C_1 < 0$. Any gradient in the I distribution will then lead to the generation of an Ω which can suppress the collapse. In this case, however, the "gas" velocity $v = 2C_1\Omega$ is directed to the center of a pulse, which means that a pulse becomes narrower with z, that the gradients increase, and that the amplitude growth is saturated. In the opposite case $C_1 C_2 > 0$, the generation of Ω is accompanied by a "gas" motion away from the pulse center, which leads to the effective smoothing of the solution. The closer the solution becomes to the uniform distribution, the smaller is the effect of pulse amplitude growth suppression by Ω. The numerical integration of the complete equation (1) and the results of Ref. 10 confirm that the collapse is arrested in the focusing medium if $C_2 \leqslant -4C_1$. For a large ratio $|C_2/C_1| \gg 1$ a typical behavior of a pulse is similar to that of a reduced system.

In Fig. 3 we show the evolution of the initial pulse in the case ($C_1 = 6$ and $C_2 > 0$) where a collapse takes place. If the maximal amplitude near the singularity behaves, with a good accuracy, as $\sim (z_0 - z)^\alpha$ with $\alpha \approx -0.50$, then the exponent β of the pulse width $d[d \sim (z_0 - z)^\beta]$ will occur during the collapse stage near 0.3885. It is important to emphasize that the pulse compression takes place in the collapse regime (compare with Ref. 9) and also because of the phase-gradient mechanism. The main difference between the two regimes is that, in contrast with the collapse, the phase-gradient mechanism can provide a finite amplitude pulse together with a pulse compression.

For the mode-locking laser systems this mechanism of stabilization may be described as follows. After the first step, nonlinear action of the saturable absorber dominates, and the pulse approaches a collapse, i.e., its amplitude increases with increasing number of passes z through the laser system. A sharp increase of the pulse amplitude leads to the generation of a large phase-gradient which is located at the pulse boundaries. This results in a decrease of the amplitude growth with z and finally in the stopping of the collapse. At the final stage the pulse is swallowed almost completely. As a result, the output signal will be represented as a pulse train of finite length. At the beginning of this train there will be the initial pulse, then the pulse duration will decrease, and simultaneously its amplitude will increase. For pulses of later generations, near the end of this train, the pulse amplitude will first reach its maximum and then sharply decrease with a

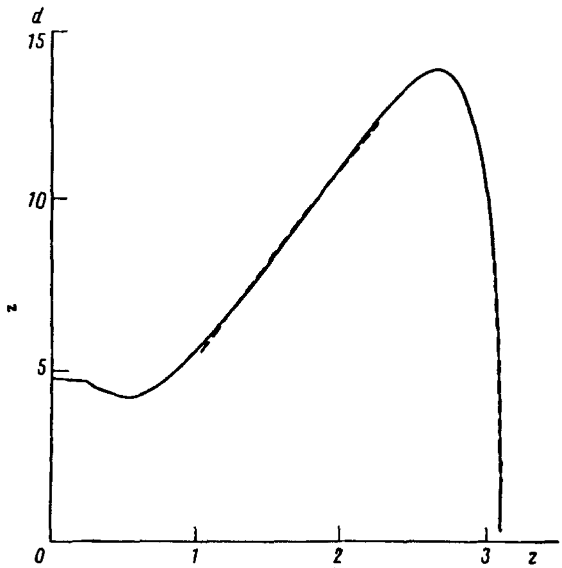

FIG. 3. The dependence of a pulse width d on the distance z due to the collapse mechanism at $C_1=6, C_2=15$. One can distinguish two regimes (the dashed lines): a linear pulse growth when the pulse width is $d=0.0034+5.45z$ and then the collapse regime, $d=26.69(3.09-z)^{0.3885}$.

simultaneous vanishing of the pulse width. The total energy of the last pulses and their intensity will vanish. This corresponds to the complete pulse reduction. Each pulse in this train will have the frequency chirp $\partial\Omega/\partial t$, and its sign will be opposite to the sign of the Kerr constant C_2. In the case of a long amplifier the output pulse, in the dependence of the amplifier size Z, will correspond to a pulse of the Zth generation in the case of mode-locking lasers.

5. In conclusion, we have studied analytically and numerically a model which describes a pulse generation in the complex GL equation describing mode-locking laser systems and other active gain media. We showed that the phase-gradient mechanism can be used to compress pulses in such systems. We have demonstrated that a collapse takes place in the defocusing gain medium and that it is stopped in the focusing amplifying medium in a sufficiently larger region of parameters than in a defocusing case. In any regime the pulse has a chirp and its sign correlates with the sign of the constant C_2.

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